A Switch for Artificial Resistivity

And other dissipation terms

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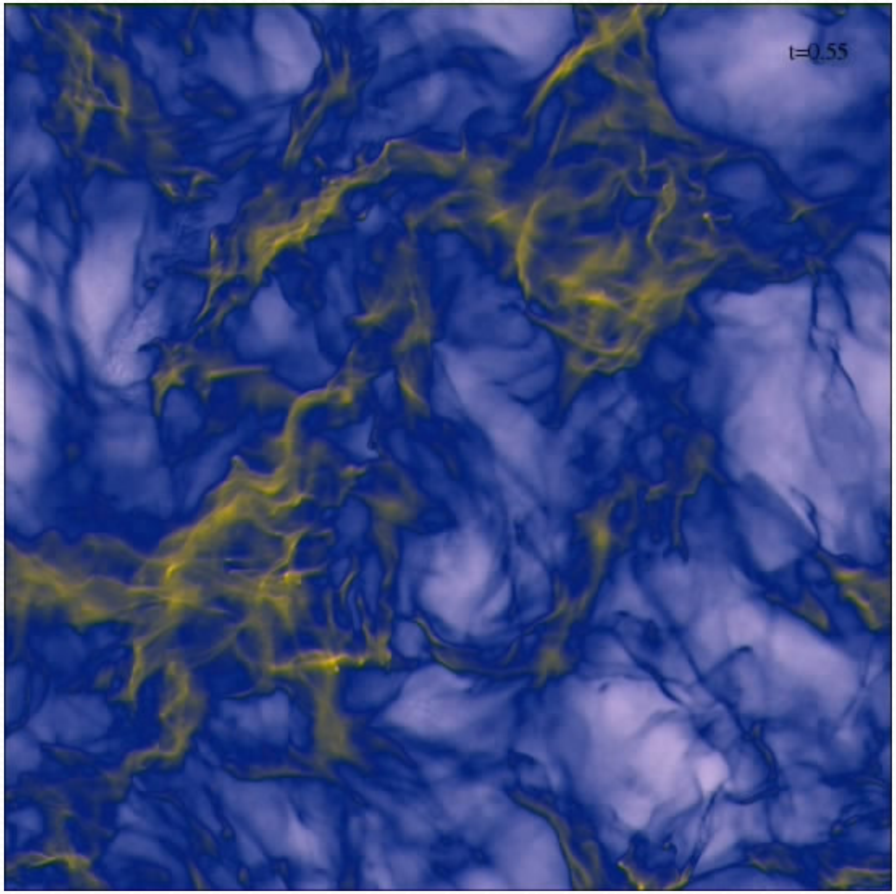
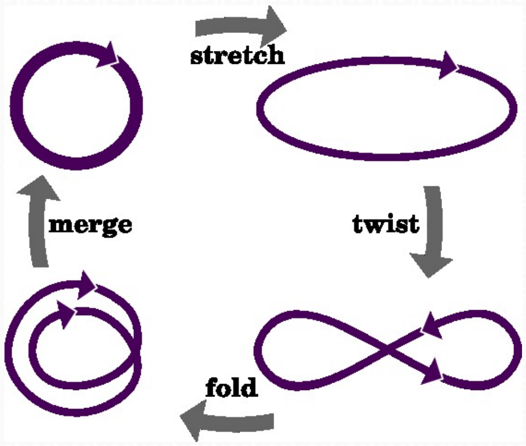
Mach 10 Driven Magnetised Turbulence

- Conditions similar to molecular clouds
- Simulation details:
  - 3D Mach 10 Turbulence in a periodic box
  - Solenoidal stochastic forcing at low wave numbers
  - Isothermal equation of state
  - Initially weak magnetic field (E_{mag} 10^{10} weaker than E_{kin})
- Extends the pure hydro comparison of Price, Federrath (2010)



Magnetic Field Dynamo

- Initially weak magnetic field (E_{mag} 10^{10} weaker than E_{kin})
- Turbulent dynamo causes magnetic field to exponentially grow until reaches equipartition with kinetic energy



1. Strong shocks in a weak field
2. Large range of magnetic field strengths

Capturing Discontinuities in SPH

- Shocks: Artificial viscosity

$$\left(\frac{\mathrm{d}\mathbf{v}_a}{\mathrm{d}t}\right)_{\mathrm{diss}} = \sum_b m_b \frac{\alpha v_{\mathrm{sig}}}{\overline{\rho}_{ab}} (\mathbf{v}_a - \mathbf{v}_b) \cdot \hat{\mathbf{r}}_{ab} \nabla_a W_{ab}$$

- Magnetic discontinuities: Artificial resistivity

$$\left(\frac{\mathrm{d}\mathbf{B}_a}{\mathrm{d}t}\right)_{\mathrm{diss}} = \rho_a \sum_b m_b \frac{\alpha_B v^B_{\mathrm{sig}}}{\overline{\rho}^2_{ab}} (\mathbf{B}_a - \mathbf{B}_b) \hat{\mathbf{r}}_{ab} \cdot \nabla_a W_{ab}$$

Use Switches to reduce Dissipation

- Molecular clouds have **Reynolds numbers of ~10^6-10^9**
- Morris switch for viscosity:

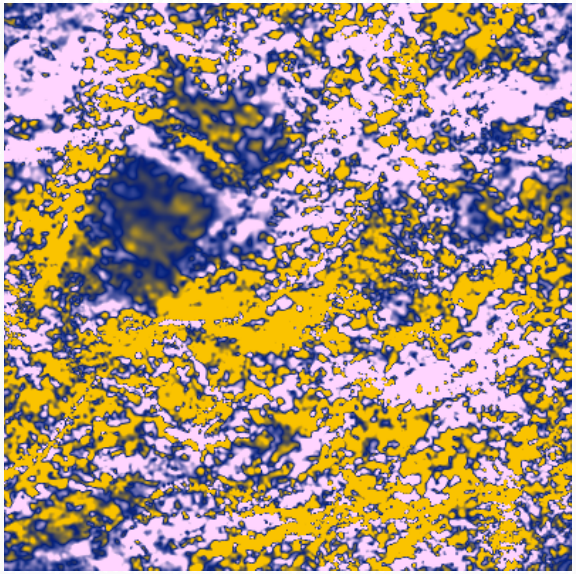
$$\frac{\mathrm{d}\alpha_a}{\mathrm{d}t} = \max(-\nabla \cdot \mathbf{v}_a, 0) - \frac{\alpha_a - \alpha_{\min}}{\tau}$$

- Price & Monaghan (PM05) switch for resistivity:

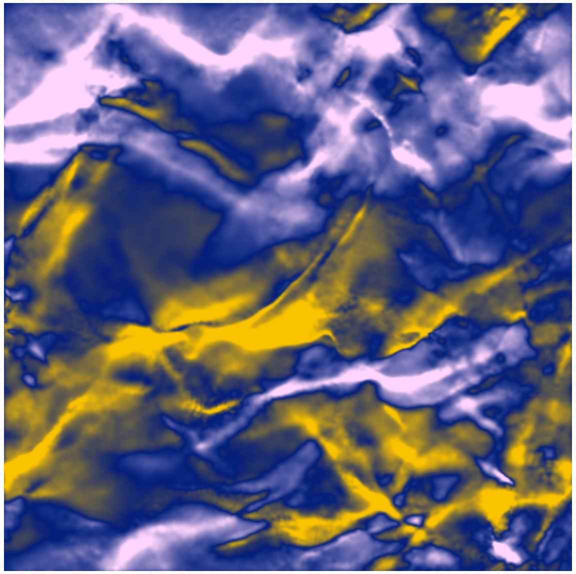
$$\frac{\mathrm{d}\alpha_{B,a}}{\mathrm{d}t} = \max\left(\frac{|\nabla \cdot \mathbf{B}_a|}{\sqrt{\mu_0 \rho}}, \frac{|\nabla \times \mathbf{B}_a|}{\sqrt{\mu_0 \rho}}\right) - \frac{\alpha_{B,a}}{\tau}$$

PM05 Switch & Turbulent Dynamo

- Column integrated x-component of magnetic field after 2 turbulent turnover times
- PM05 fails to work at these low magnetic field strengths!



PM05 switch



Fixed α_B=1

New Resistivity Switch

- Requirement: Robustly work for all magnetic field strengths!

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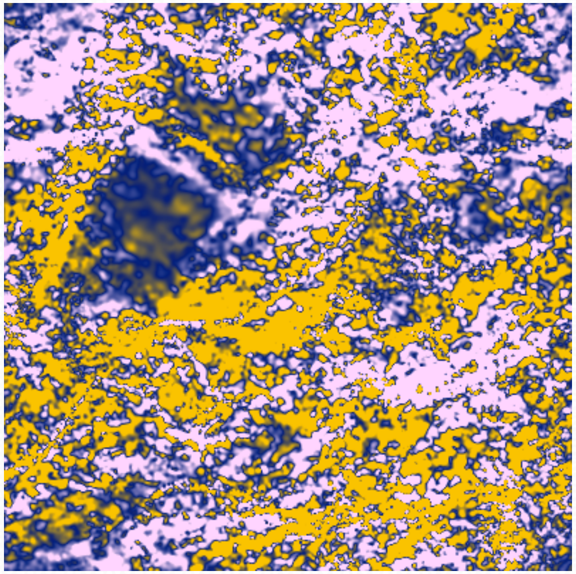
- Directly set: $$\alpha_{B,a} = \frac{h_a |\nabla \mathbf{B}_a|}{|\mathbf{B}_a|}$$
- Use grad B as shock indicator
- Normalise by magnitude of magnetic field to remove B dependence
  - Automatically gives values in desired range, i.e. [0,1]
  - Removes time delay, instantly responds to sharp jumps

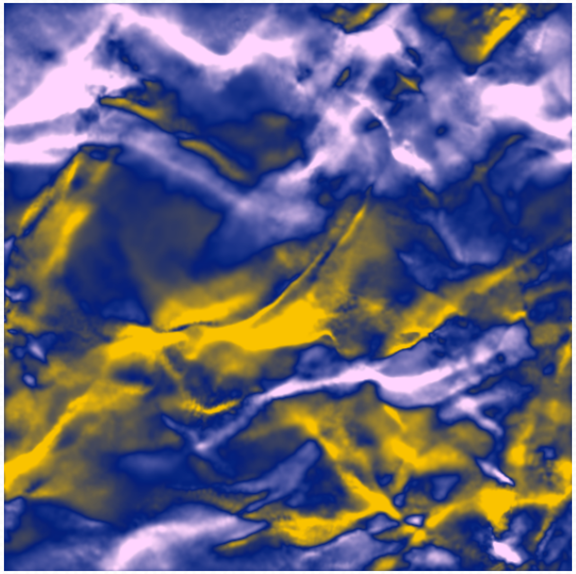
$$\nabla \mathbf{B}_a \equiv \frac{\partial B_a^i}{\partial x_a^j} \approx -\frac{1}{\Omega_a \rho_a} \sum_b m_b (B_a^i - B_b^i) \nabla_a^j W_{ab}(h_a)$$

$$|\nabla \mathbf{B}| \equiv \sqrt{\sum_i \sum_j \left| \frac{\partial B_a^i}{\partial x_a^j} \right|^2}.$$

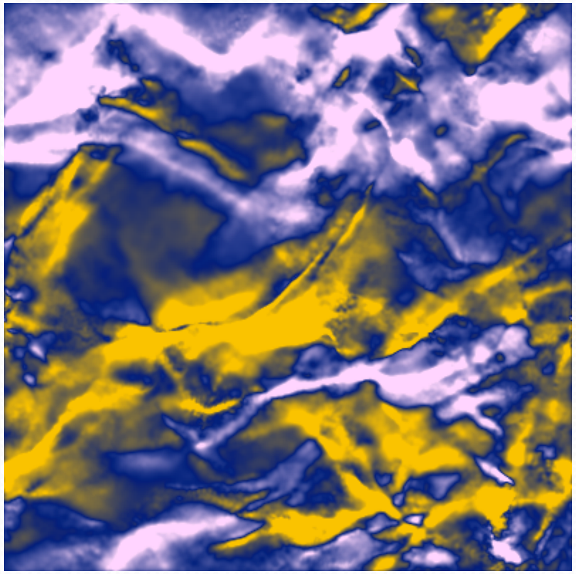
New Switch & Turbulent Dynamo



PM05 switch

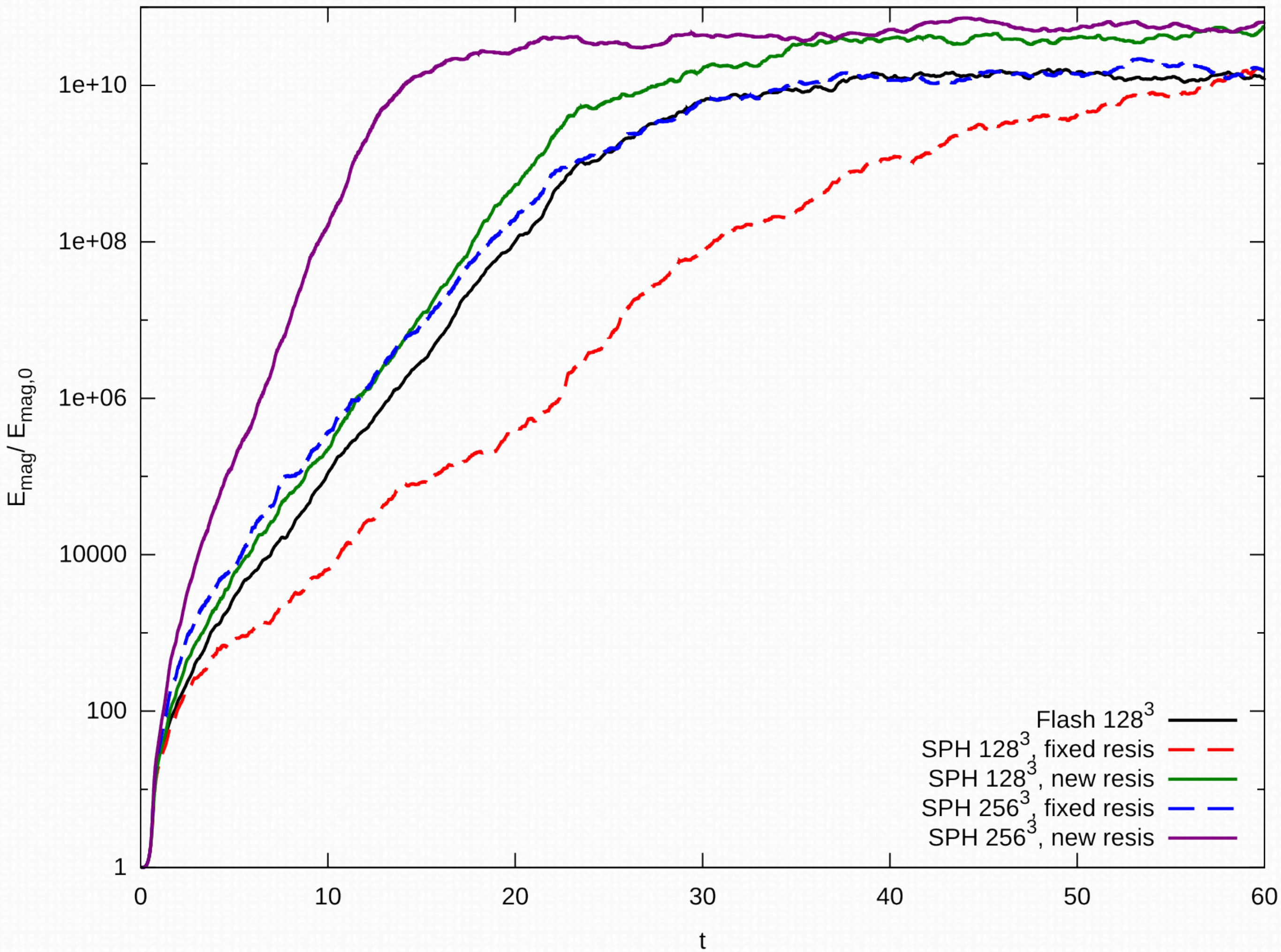


Fixed $\alpha_B=1$

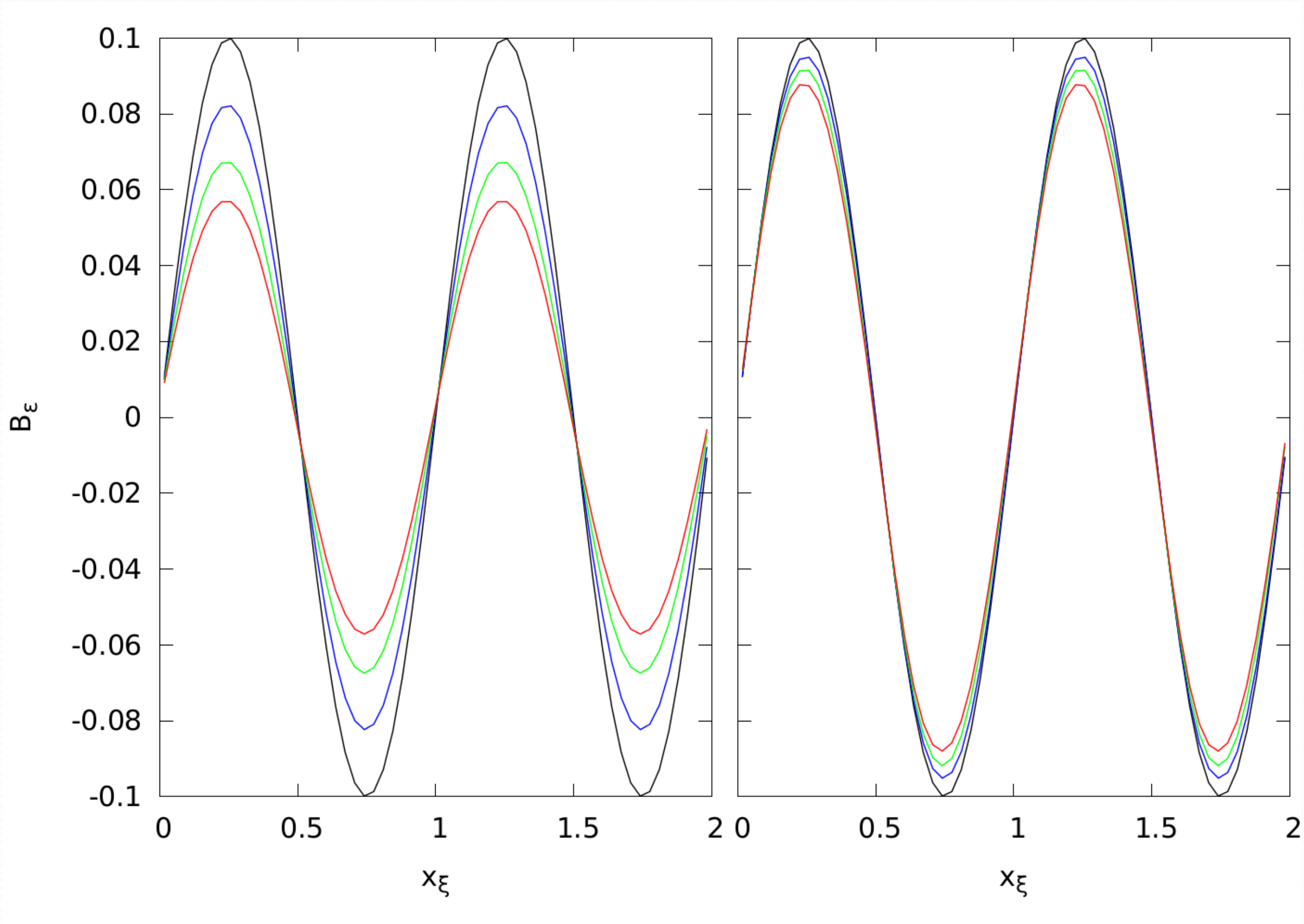


New switch

Magnetic Energy Growth



Magnetic Wave Propagation

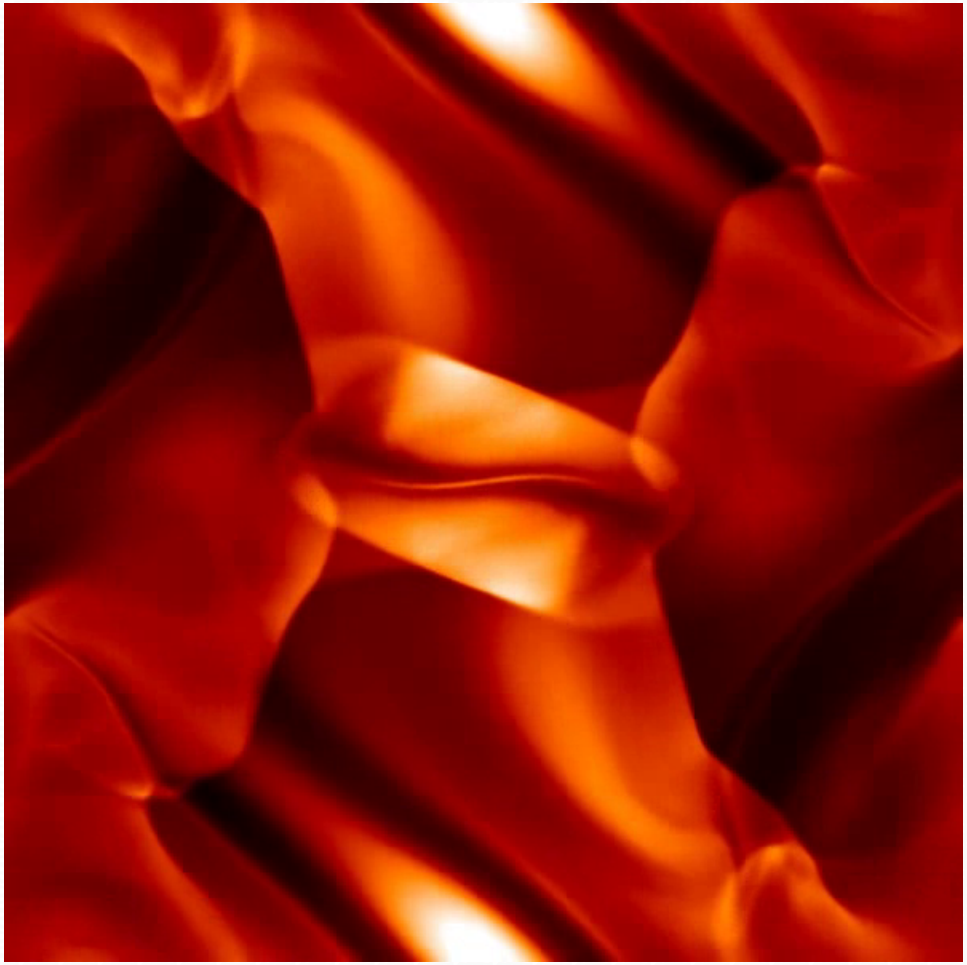


PM05 switch

New switch

Orszag-Tang Vortex

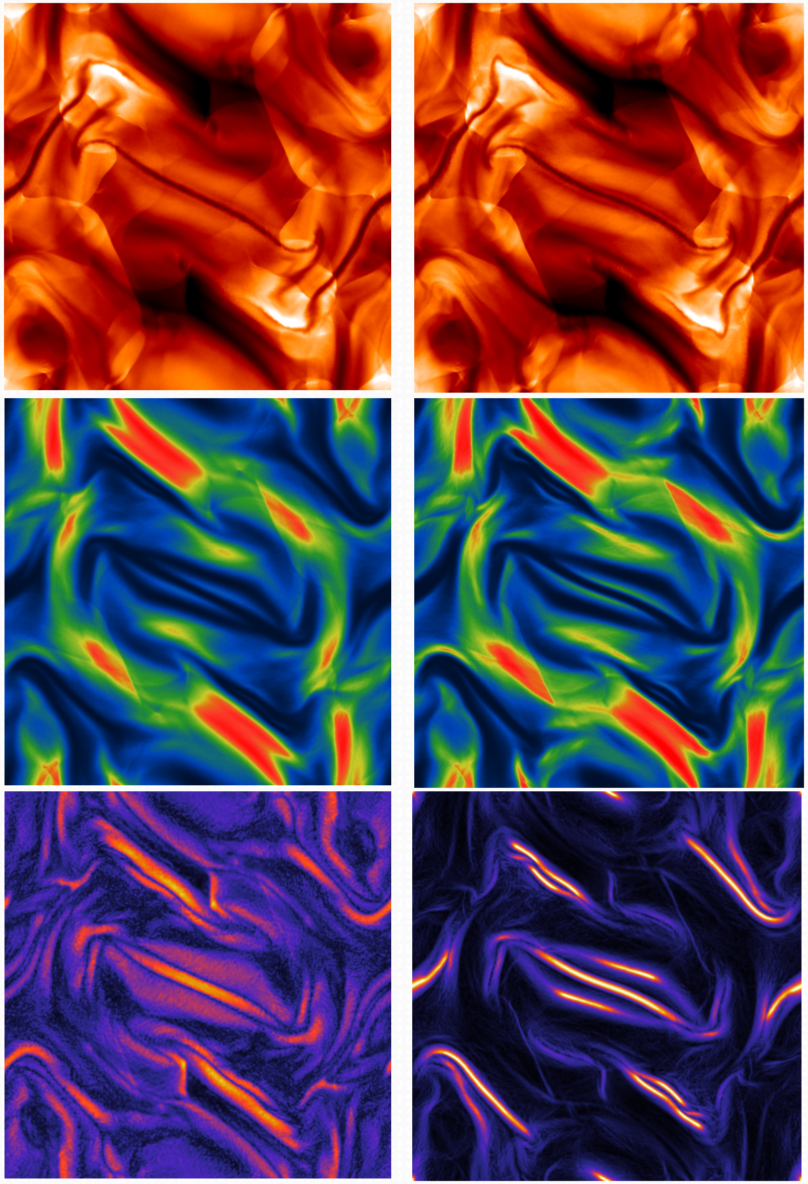
- Velocity vortex super-imposed on magnetic vortex
- Creates several classes of interacting shock waves



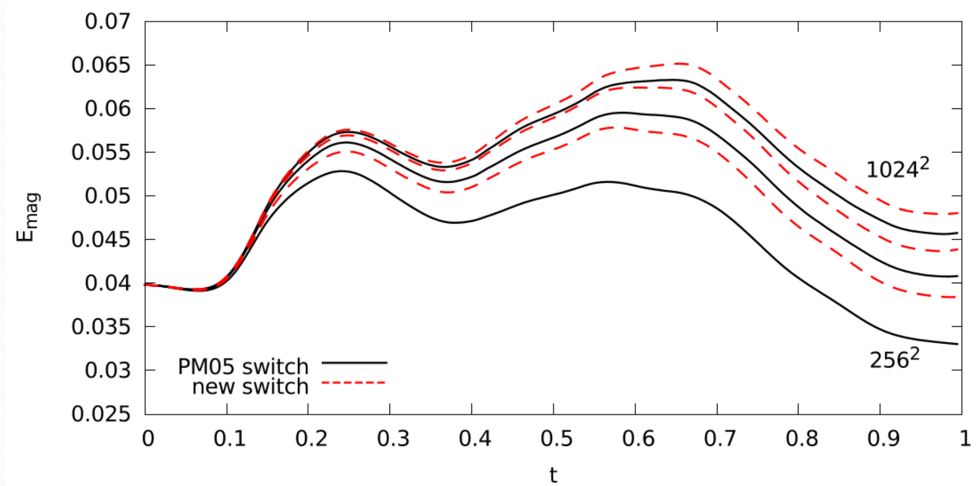
ρ

B^2

α_B



PM05 switch　　New switch



- Magnetic energy vs time for 256^2, 512^2, and 1024^2 particles
- New switch effectively yields higher resolution

New Viscosity Switch

- Shock indicator: -div v
  - Only activates in regions of converging flow
- Normalisation: speed of sound
  - Galilean invariant, relates quantity to Mach number
- Need to treat post-shock oscillations: add a decay term

If $-\frac{h_a \nabla \cdot \mathbf{v}_a}{c} > \alpha_a$ then $\alpha_a = -\frac{h_a \nabla \cdot \mathbf{v}_a}{c}$

else $\frac{\mathrm{d}\alpha_a}{\mathrm{d}t} = -\frac{\alpha_a}{\tau}$

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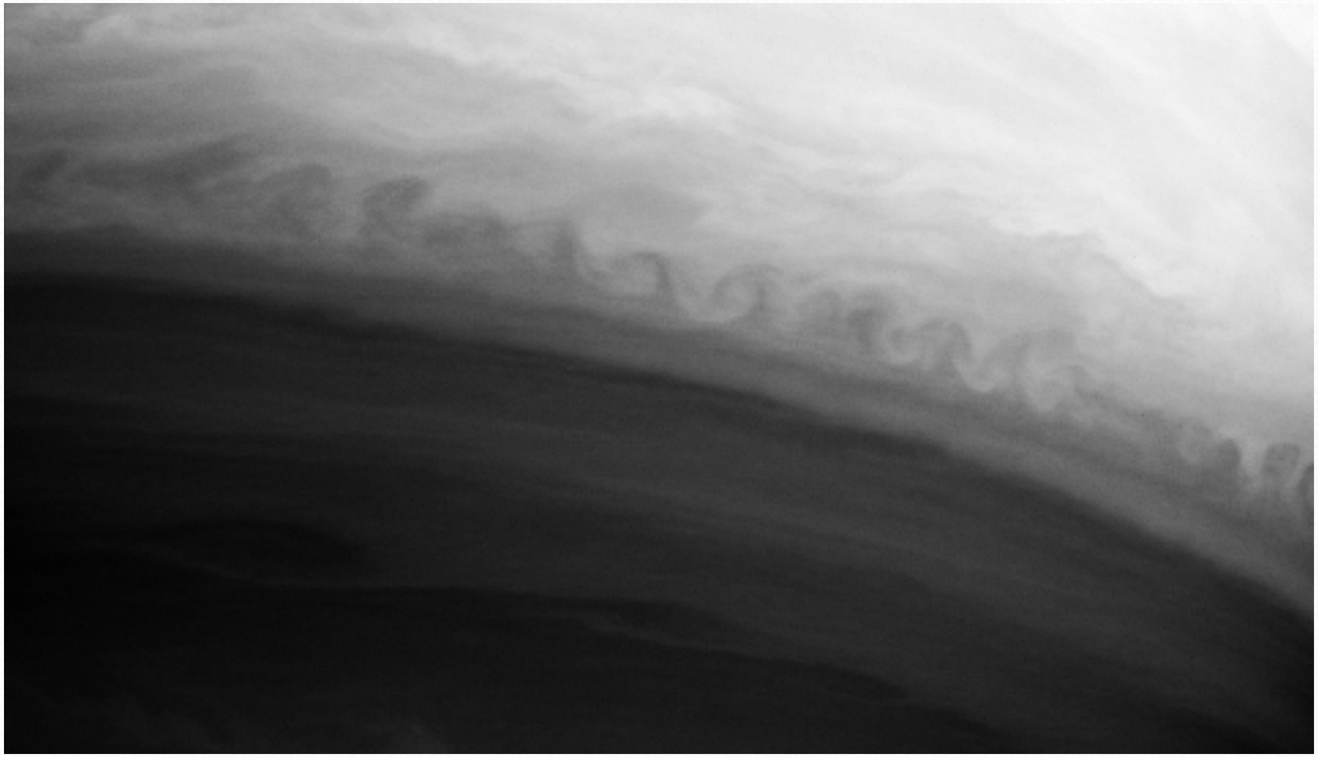
- Cullen & Dehnen (2010):

If $A_a = \xi \max\left(-\frac{\mathrm{d}(\nabla \cdot \mathbf{v}_a)}{\mathrm{d}t}, 0\right) > \alpha_a$ then $\alpha_a = \alpha_{\max} \frac{h_a^2 A_a}{v_{\rm sig}^2 + h_a^2 A_a}$

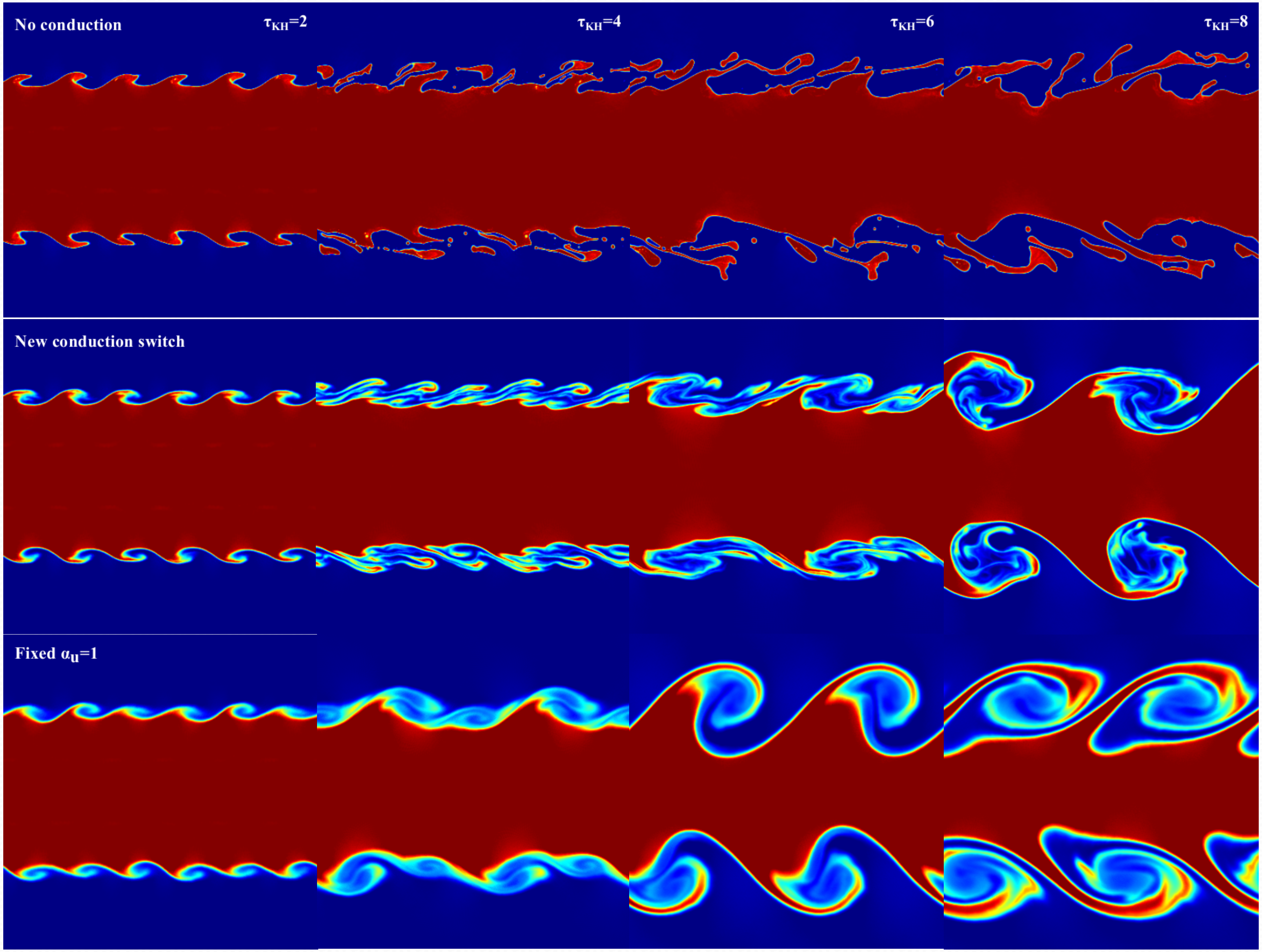
else $\frac{\mathrm{d}\alpha_a}{\mathrm{d}t} = -\frac{\alpha_a - \alpha_{\min}}{\tau}$

Thermal Conduction switch

$$\alpha_{u,a} = \frac{h_a |\nabla u_a|}{|u_a|}$$



Kelvin-Helmholtz instabilities on Saturn



No conduction
τ_{KH}=2
τ_{KH}=4
τ_{KH}=6
τ_{KH}=8
New conduction switch
Fixed α_u=1

Take Home Message

- New artificial resistivity switch supercedes PM05 switch in every respect:
  - Simple
  - Effective
  - Robust
  - Less dissipative