A Switch for Artificial Resistivity

And other dissipation terms

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Mach 10 Driven Magnetised Turbulence

- Conditions similar to molecular clouds

- Simulation details:
  - 3D Mach 10 Turbulence in a periodic box
  - Solenoidal stochastic forcing at low wave numbers
  - Isothermal equation of state
  - Initially weak magnetic field ($E_{\text{mag}} 10^{10}$ weaker than $E_{\text{kin}}$)

- Extends the pure hydro comparison of Price, Federrath (2010)
Magnetic Field Dynamo

- Initially weak magnetic field ($E_{mag} \cdot 10^{10}$ weaker than $E_{kin}$)
- Turbulent dynamo causes magnetic field to exponentially grow until reaches equipartition with kinetic energy

1. Strong shocks in a weak field
2. Large range of magnetic field strengths
Capturing Discontinuities in SPH

• **Shocks:** Artificial viscosity

\[
\left( \frac{d\mathbf{v}_a}{dt} \right)_{\text{diss}} = \sum \limits_b m_b \frac{\alpha \nu_{\text{sig}}}{\rho_{ab}} (\mathbf{v}_a - \mathbf{v}_b) \cdot \hat{r}_{ab} \nabla_a W_{ab}
\]

• **Magnetic discontinuities:** Artificial resistivity

\[
\left( \frac{dB_a}{dt} \right)_{\text{diss}} = \rho_a \sum \limits_b m_b \frac{\alpha_B \nu^B_{\text{sig}}}{\rho_{ab}^2} (\mathbf{B}_a - \mathbf{B}_b) \hat{r}_{ab} \cdot \nabla_a W_{ab}
\]
Use Switches to reduce Dissipation

• Molecular clouds have **Reynolds numbers of $\sim 10^6$-$10^9$**
• Morris switch for viscosity:
  
  $$\frac{d\alpha_a}{dt} = \max(-\nabla \cdot \mathbf{v}_a, 0) - \frac{\alpha_a - \alpha_{\text{min}}}{\tau}$$

• Price & Monaghan (PM05) switch for resistivity:
  
  $$\frac{d\alpha_{B,a}}{dt} = \max \left(\frac{|\nabla \cdot \mathbf{B}_a|}{\sqrt{\mu_0 \rho}}, \frac{|\nabla \times \mathbf{B}_a|}{\sqrt{\mu_0 \rho}}\right) - \frac{\alpha_{B,a}}{\tau}$$
PM05 Switch & Turbulent Dynamo

- Column integrated x-component of magnetic field after 2 turbulent turnover times
- PM05 fails to work at these low magnetic field strengths!

PM05 switch

Fixed $\alpha_B=1$
New Resistivity Switch

• Requirement: Robustly work for all magnetic field strengths!
New Resistivity Switch

- **Requirement:** Robustly work for all magnetic field strengths!

- Directly set: \( \alpha_{B,a} = \frac{h_a |\nabla B_a|}{|B_a|} \)

- Use grad B as shock indicator

- Normalise by magnitude of magnetic field to remove B dependence
  - Automatically gives values in desired range, i.e. [0,1]
  - Removes time delay, instantly responds to sharp jumps

\[
\nabla B_a \equiv \frac{\partial B_a^i}{\partial x_a^j} \approx -\frac{1}{\Omega_a \rho_a} \sum_b m_b (B_a^i - B_b^i) \nabla_a W_{ab}(h_a) \quad |\nabla B| \equiv \sqrt{\sum_i \sum_j \left| \frac{\partial B_a^i}{\partial x_a^j} \right|^2}.
\]
New Switch & Turbulent Dynamo

PM05 switch

Fixed $\alpha_B = 1$

New switch
Magnetic Energy Growth

![Graph showing Magnetic Energy Growth]

- **Flash 128³**
- **SPH 128³, fixed resis**
- **SPH 128³, new resis**
- **SPH 256³, fixed resis**
- **SPH 256³, new resis**

The graph illustrates the growth of magnetic energy over time for different simulations and resolutions.
Magnetic Wave Propagation

PM05 switch

New switch
Orszag-Tang Vortex

- Velocity vortex super-imposed on magnetic vortex
- Creates several classes of interacting shock waves
- Magnetic energy vs time for $256^2$, $512^2$, and $1024^2$ particles
- New switch effectively yields higher resolution
New Viscosity Switch

• Shock indicator: \(-\text{div } \mathbf{v}\)
  • Only activates in regions of converging flow

• Normalisation: speed of sound
  • Galilean invariant, relates quantity to Mach number

• Need to treat post-shock oscillations: add a decay term

\[
\text{If } -\frac{h_a \nabla \cdot \mathbf{v}_a}{c} > \alpha_a \quad \text{then} \quad \alpha_a = -\frac{h_a \nabla \cdot \mathbf{v}_a}{c}
\]

\[
\text{else} \quad \frac{d\alpha_a}{dt} = -\frac{\alpha_a}{\tau}
\]
New Viscosity Switch

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\]

• Cullen & Dehnen (2010):

\[
\text{If } A_a = \xi \max \left(-\frac{d(\nabla \cdot \mathbf{v}_a)}{dt}, 0\right) > \alpha_a : \text{then } \quad \alpha_a = \alpha_{\max} \frac{h_{a}^{2}A_a}{\sqrt{v_{\text{sig}}^{2} + h_{a}^{2}A_a}}
\]

\[
\text{else } \quad \frac{d\alpha_a}{dt} = -\frac{\alpha_a - \alpha_{\min}}{\tau}
\]
Thermal Conduction switch

\[ \alpha_{u,a} = \frac{h_a |\nabla u_a|}{|u_a|} \]

Kelvin-Helmholtz instabilities on Saturn
Take Home Message

• New artificial resistivity switch supercedes PM05 switch in every respect:
  • Simple
  • Effective
  • Robust
  • Less dissipative