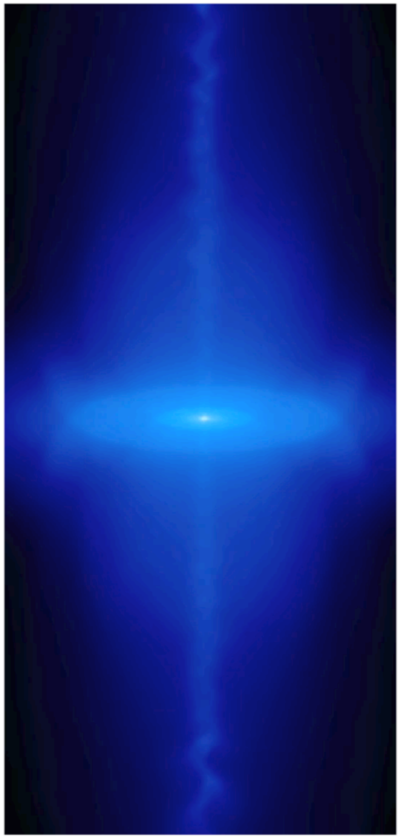
The State of the Art: Smoothed Particle Magnetohydrodynamics



Terrence Tricco
Monash Centre for Astrophysics
Monash University
Melbourne, Australia
terrence.tricco@monash.edu
http://users.monash.edu/~tricco

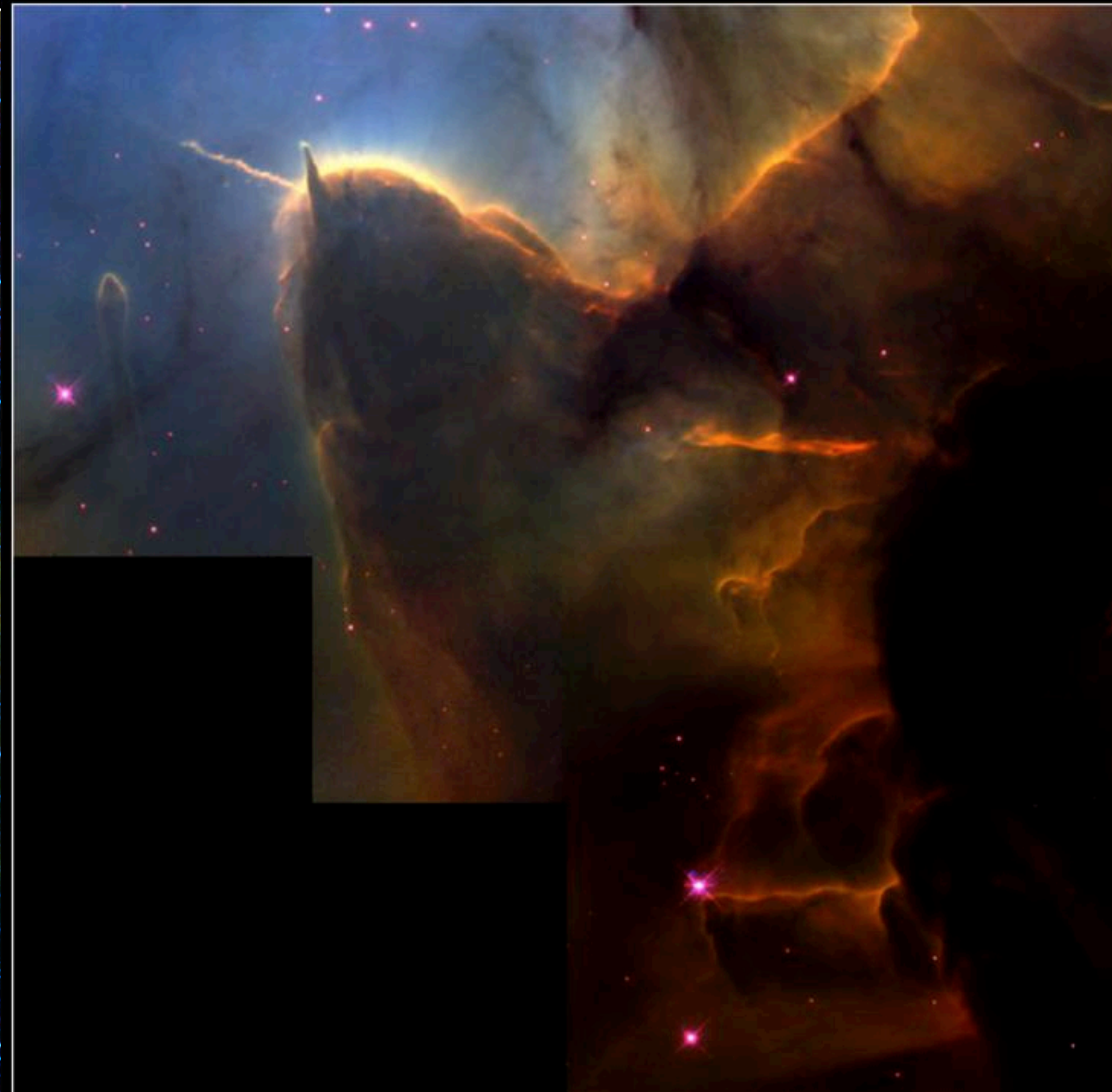
Daniel Price (Monash)
Matthew Bate (Exeter)
Christoph Federrath (Monash)

MoCA Monash Centre for Astrophysics

MONASH University Science

Trifid Nebula/Messier 20

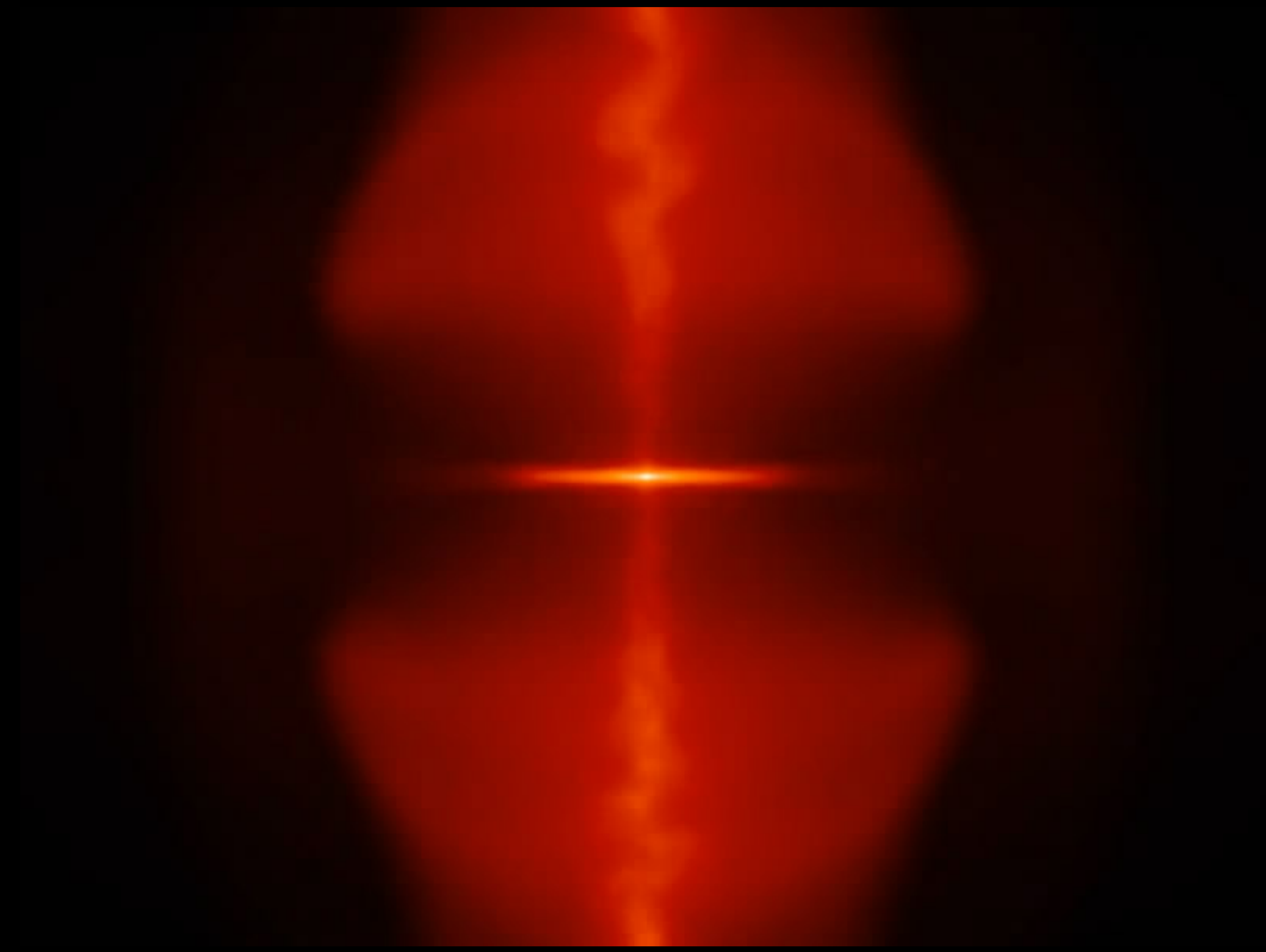
NASA / JPL-Caltech / J. Rho (SSC/Caltech)



Trifid Nebula • M20

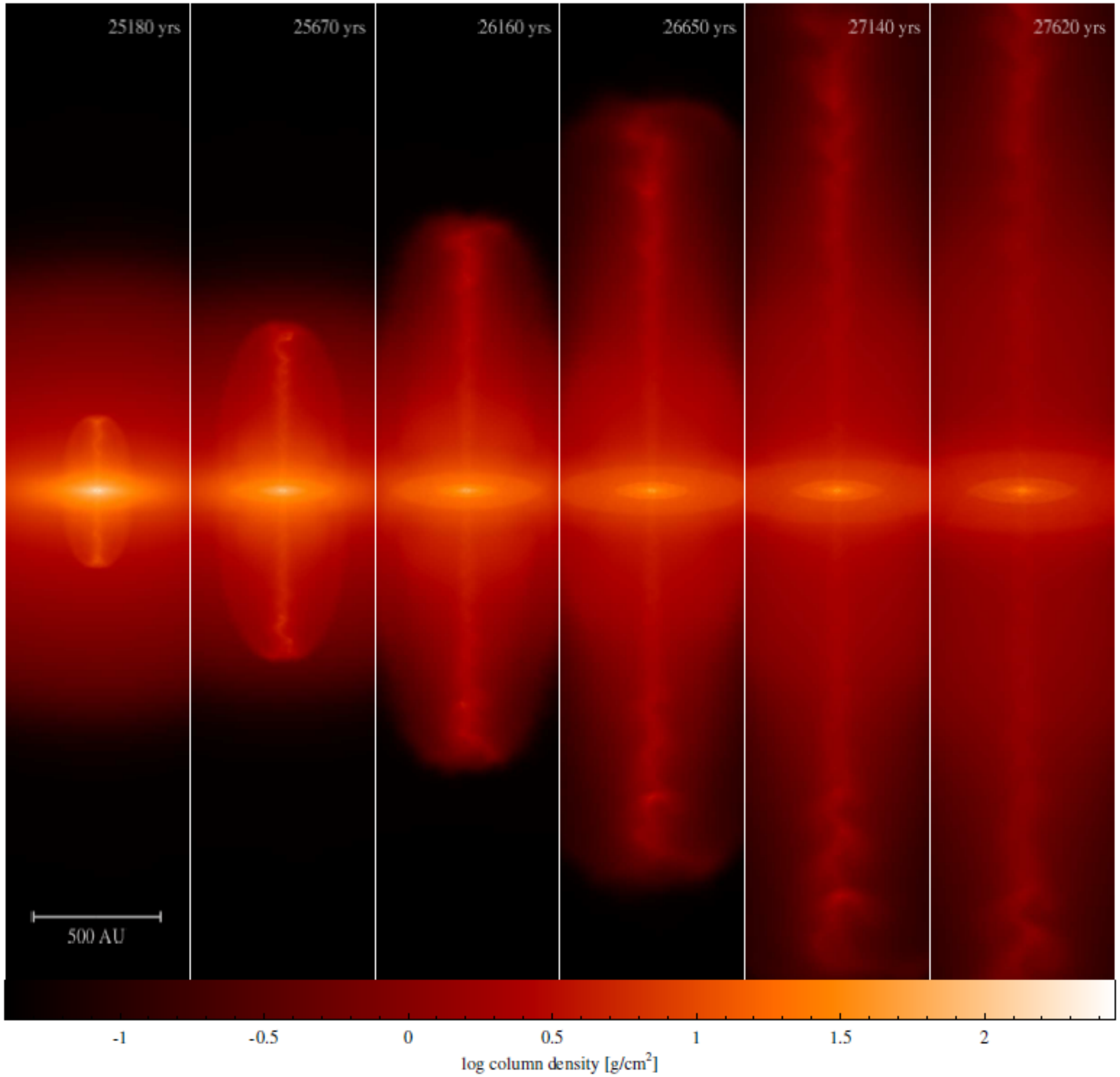
HST • WFPC2

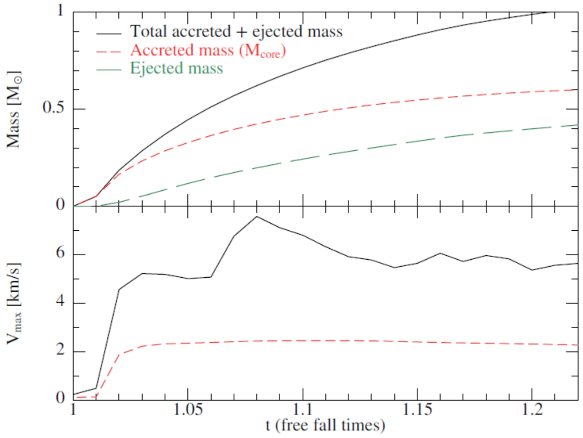
NASA and J. Hester (Arizona State University) • STScI-PRC99-42



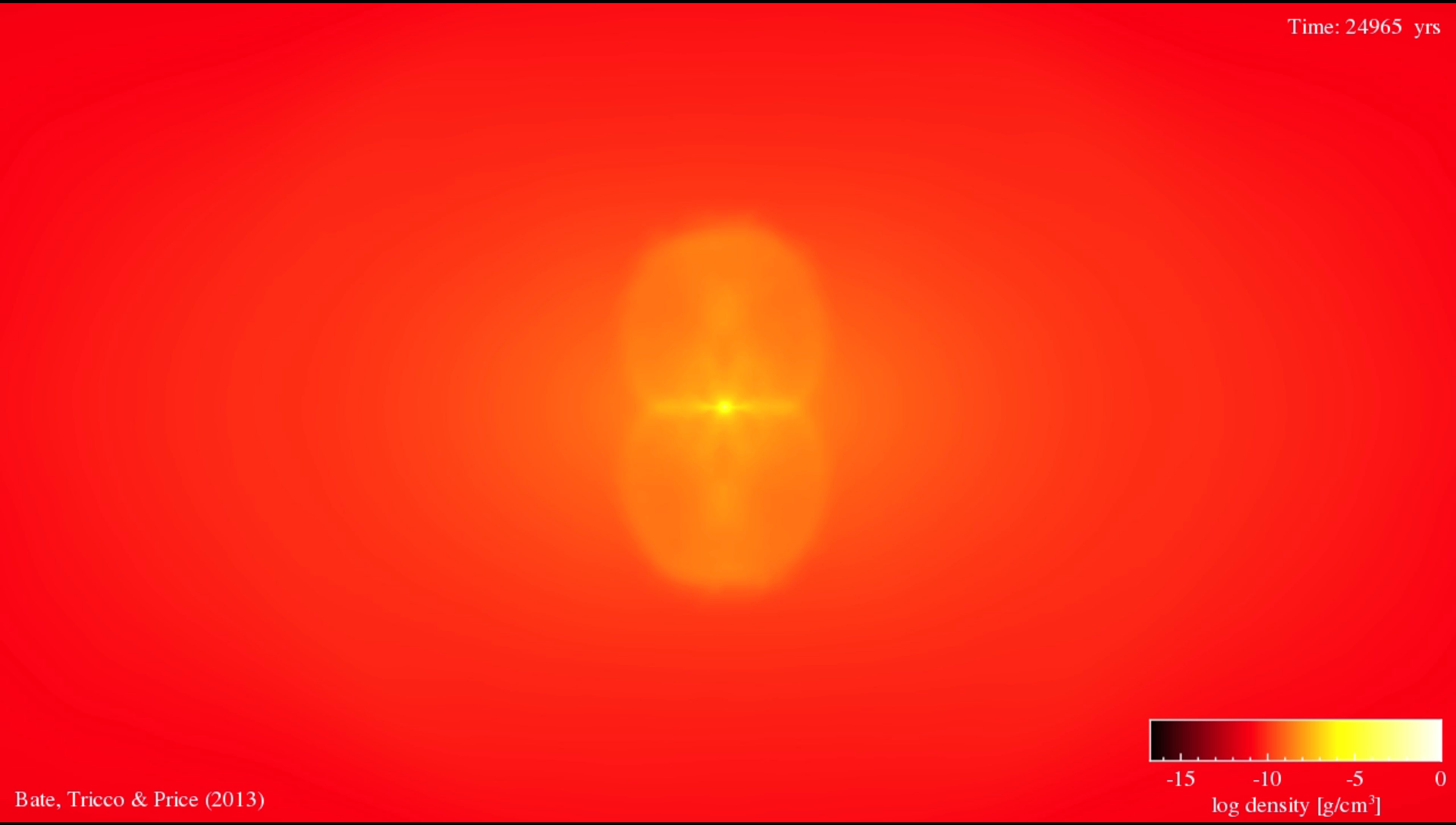
- 1 solar mass rotating dense core, initial density ~10^{-18} g/cm^3
- Uniform vertical magnetic field, ~160 μG (mass-to-flux ratio 5)

Price, Tricco, Bate (2012)





Price, Tricco, Bate (2012)



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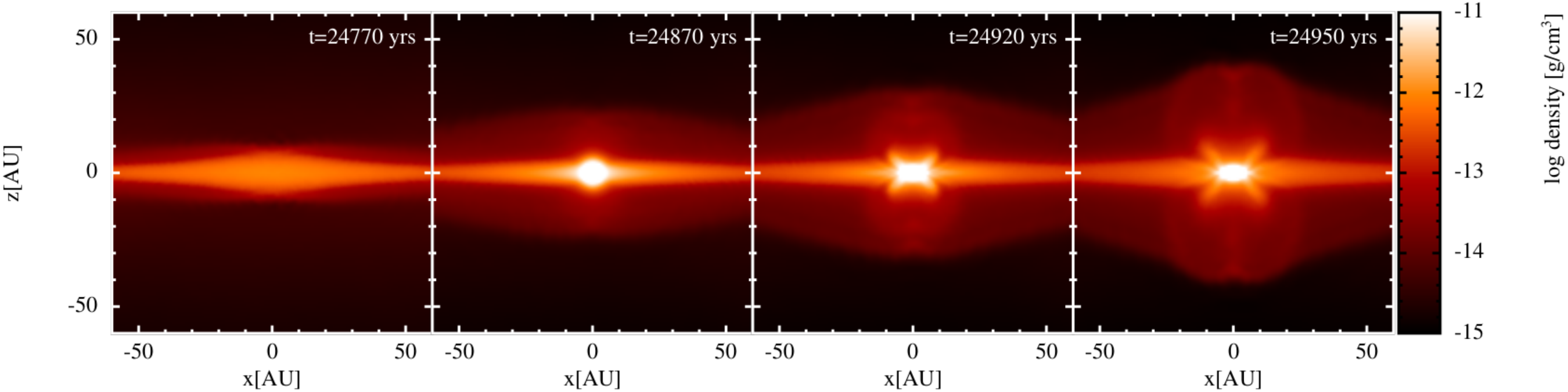
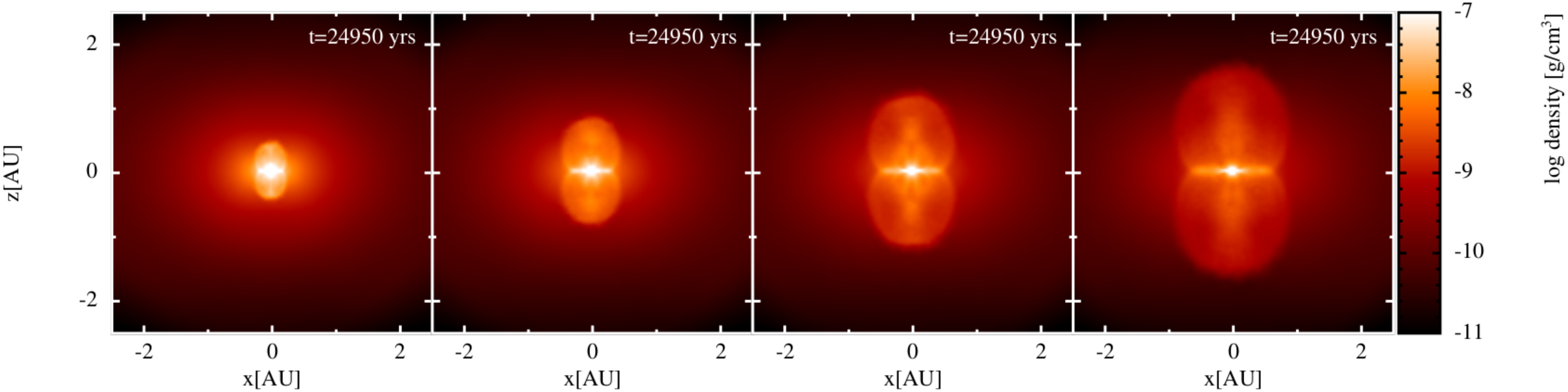
Bate, Tricco, Price, submitted

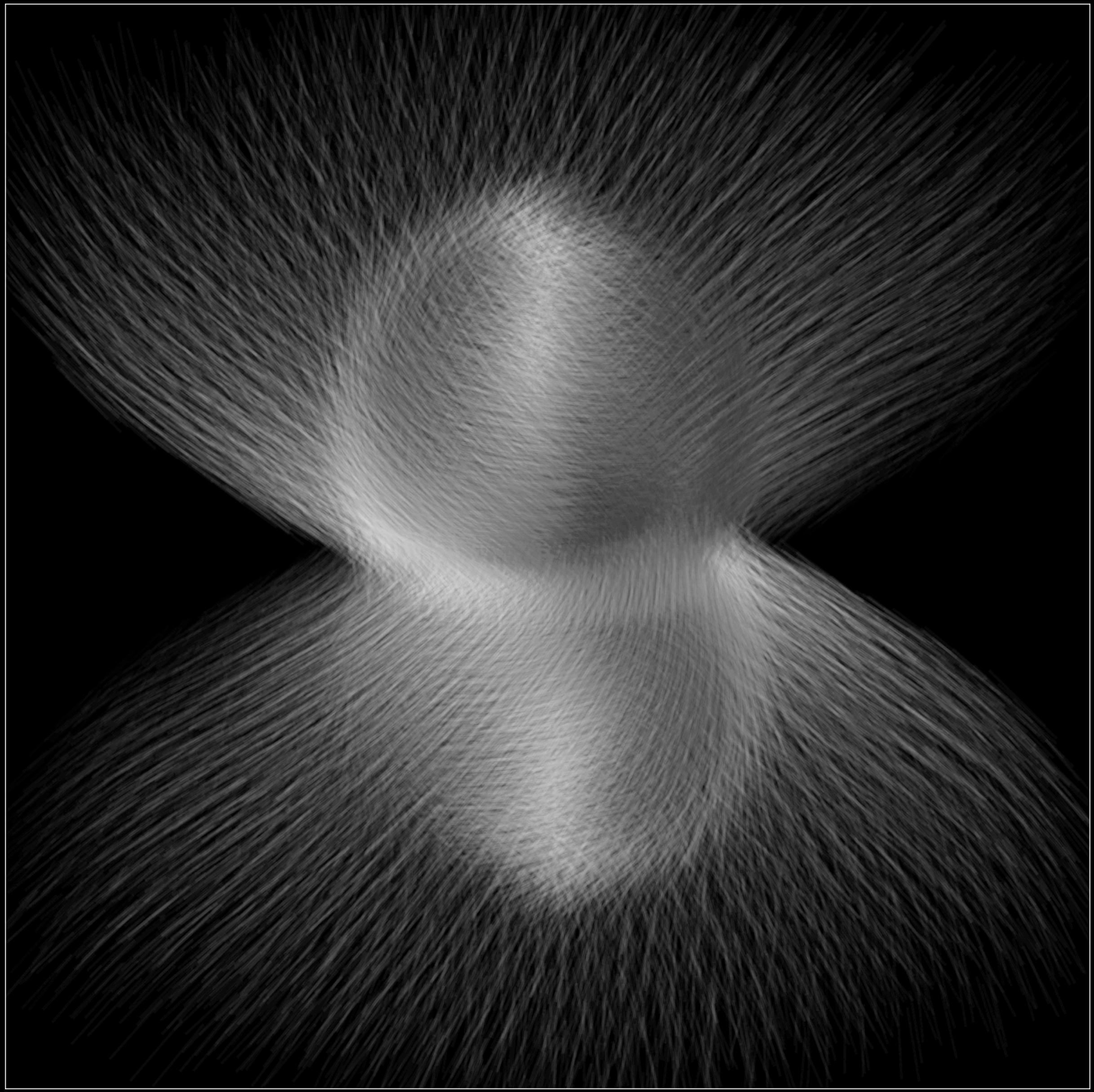
First core outflow:



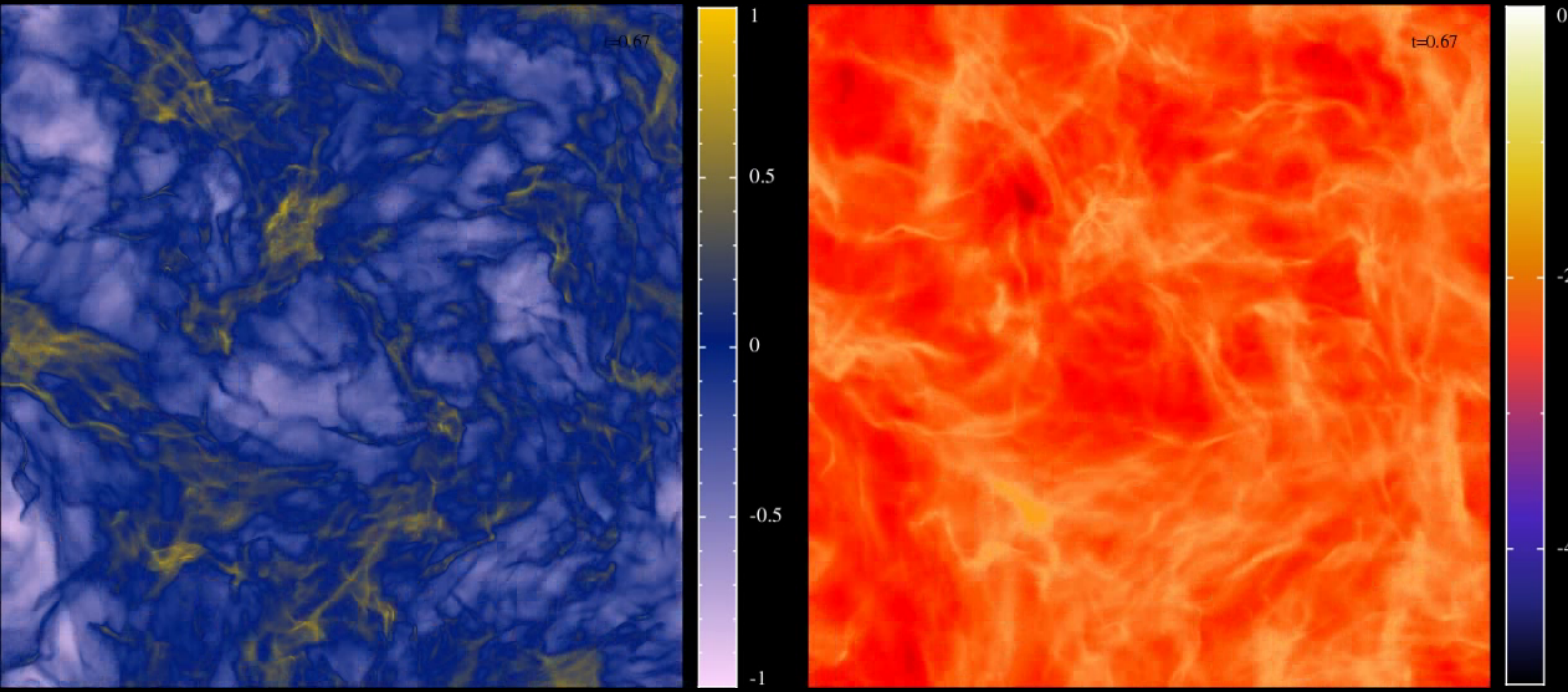
Second core outflow:



Bate, Tricco, Price, submitted



Bate, Tricco, Price, submitted

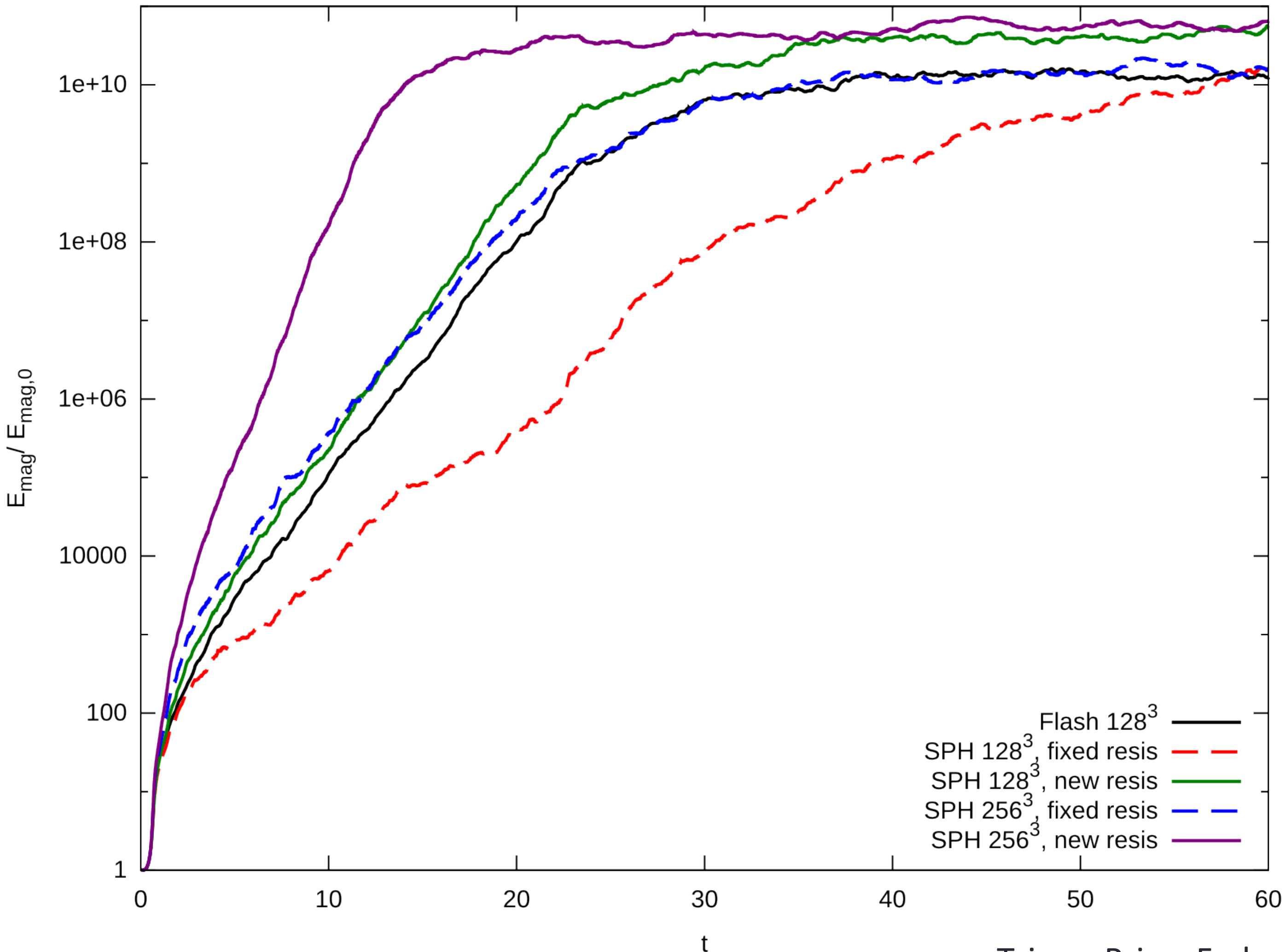


log Density

log Bz

Tricco, Price, Federrath, in prep

Magnetic Energy Growth



Tricco, Price, Federrath, in prep

“I won’t believe SPH can do MHD
until it can do the MRI.”

- James Wadsley

"I won't believe SPH can do MHD until it can do the MRI."
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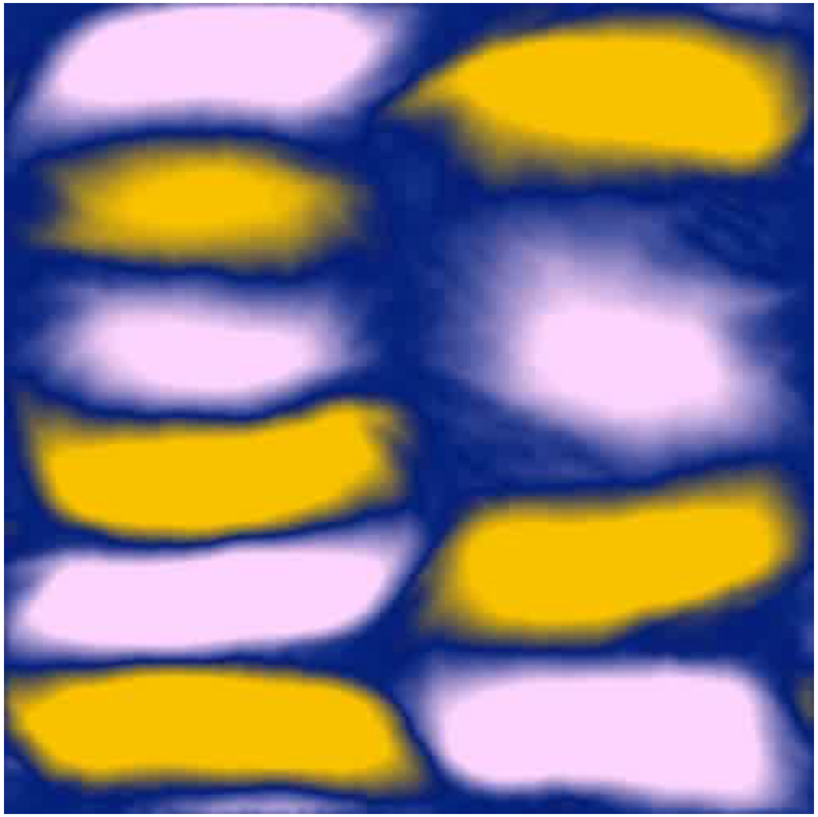
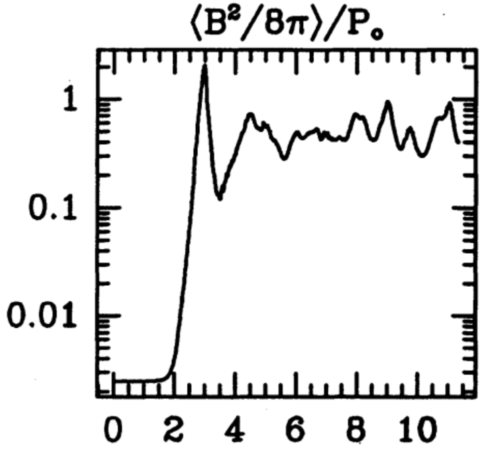
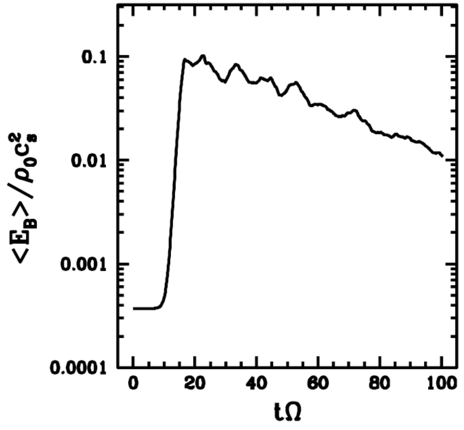
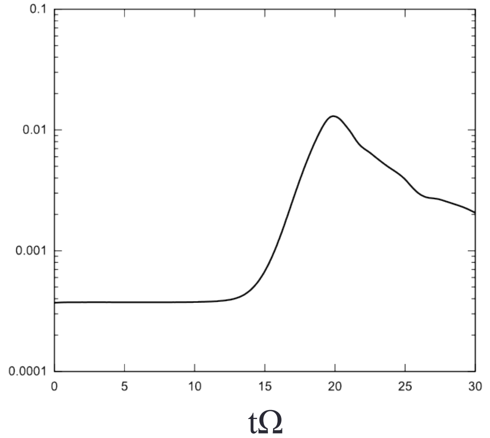








Balbus, Hawley (1998)

Guan, Gammie (2008)

SPMHD

What’s our secret?

- Two things:

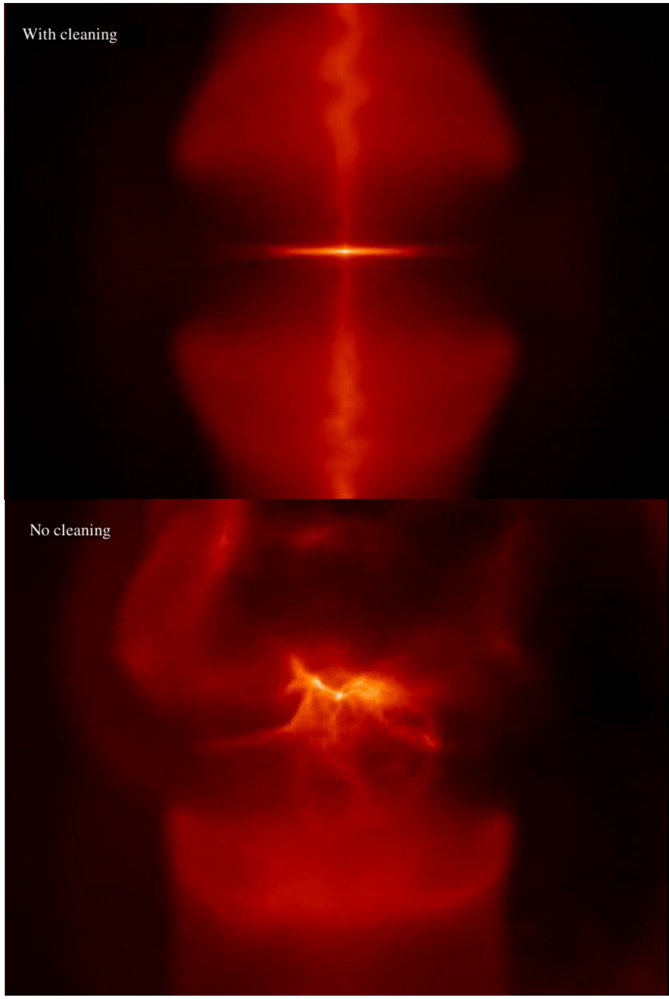
1. “Constrained” Hyperbolic Divergence Cleaning
2. An artificial resistivity switch to reduce numerical dissipation of the magnetic field & capture shocks in weak field regimes

Magnetic Fields in SPH

Most important criterion:

$$\nabla \cdot \mathbf{B} = 0$$

- Real fields have no divergence, so we want our representation of the field to be realistic,
- If large divergence is present, can cause numerical artifacts and stability issues.



Hyperbolic Divergence Cleaning

(Dedner et al, 2002)

No Cleaning

Hyperbolic

Hyperbolic/
Parabolic

- Couple scalar field ψ to magnetic field:

$$\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)_{\psi} = -\nabla\psi$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = -c_h^2 \nabla\cdot\mathbf{B} - \frac{\psi}{\tau}$$

- Produces damped “divergence” waves:

$$\frac{\partial^2(\nabla\cdot\mathbf{B})}{\partial t^2} - c_h^2\nabla^2(\nabla\cdot\mathbf{B}) + \frac{1}{\tau}\frac{\partial(\nabla\cdot\mathbf{B})}{\partial t} = 0$$

Constrained Divergence Cleaning

(Tricco & Price, J Comp Phys, 2012)

- Define energy of ψ field

$$e_\psi \equiv \frac{\psi^2}{\mu_0 \rho c_h^2}$$

- Include as part of system Lagrangian:

$$L = \int \left(\tfrac{1}{2}\rho \mathbf{v}^2 - \rho u - \frac{\mathbf{B}^2}{2\mu_0} - \frac{\psi^2}{2\mu_0 c_h^2} \right) \mathrm{d}V$$

- New evolution equation

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = -c_h^2 \nabla \cdot \mathbf{B} - \frac{\psi}{\tau} - \tfrac{1}{2}\psi \nabla \cdot \mathbf{v}$$

Constrained Divergence Cleaning

- Derive SPH cleaning equations from discretised Lagrangian:
  - Requires conjugate derivative operators for $\nabla \cdot \mathbf{B}$ and $\nabla \psi$:

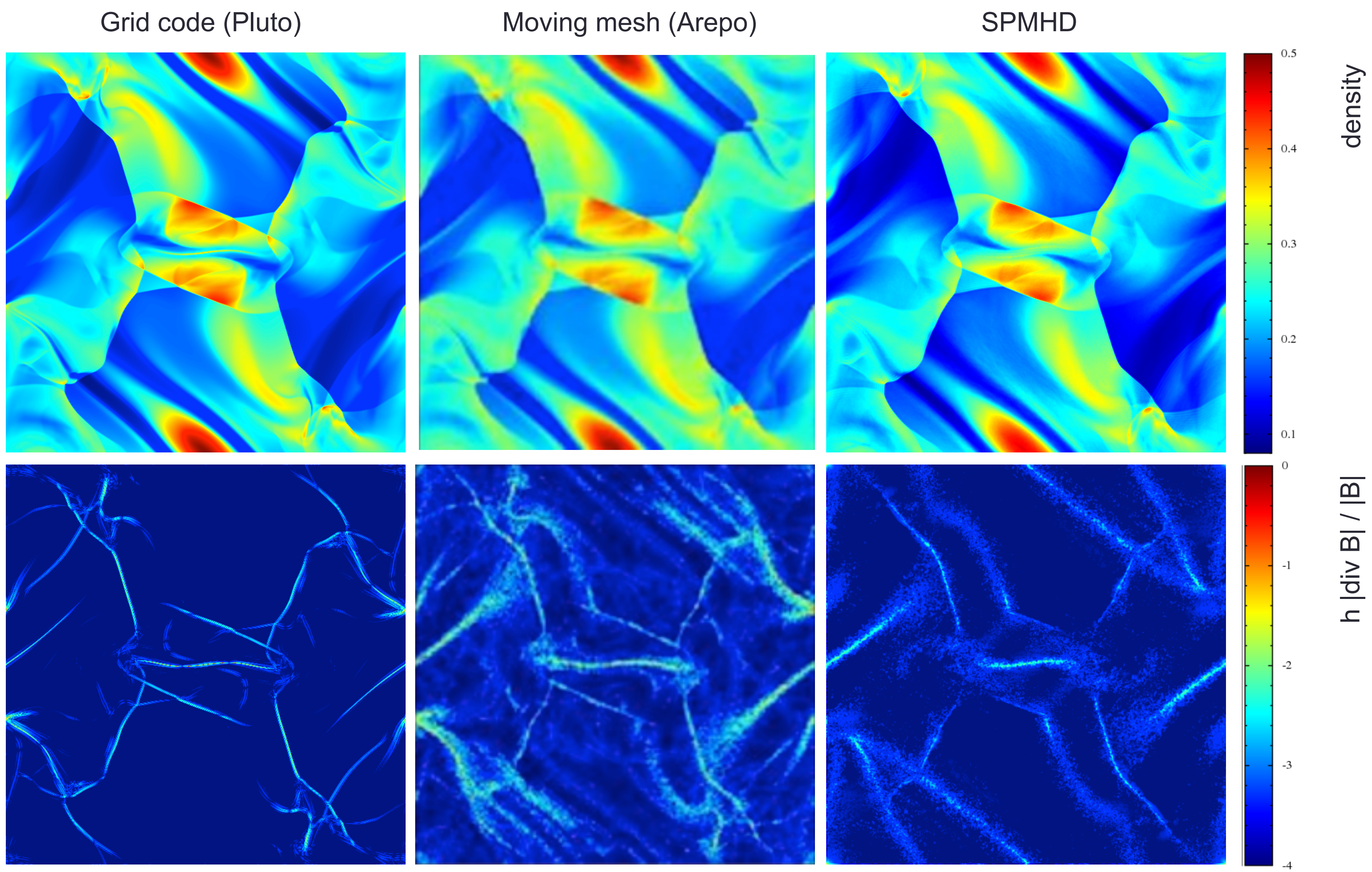
$$(\nabla \cdot \mathbf{B})_a = -\frac{1}{\Omega_a \rho_a} \sum_b m_b \left(\mathbf{B}_a - \mathbf{B}_b \right) \cdot \nabla_a W_{ab}(h_a)$$

$$\left(\frac{\mathrm{d}\mathbf{B}_a}{\mathrm{d}t} \right)_\psi = -\rho_a \sum_b m_b \left[\frac{\psi_a}{\Omega_a \rho_a^2} \nabla_a W_{ab}(h_a) + \frac{\psi_b}{\Omega_b \rho_b^2} \nabla_a W_{ab}(h_b) \right]$$

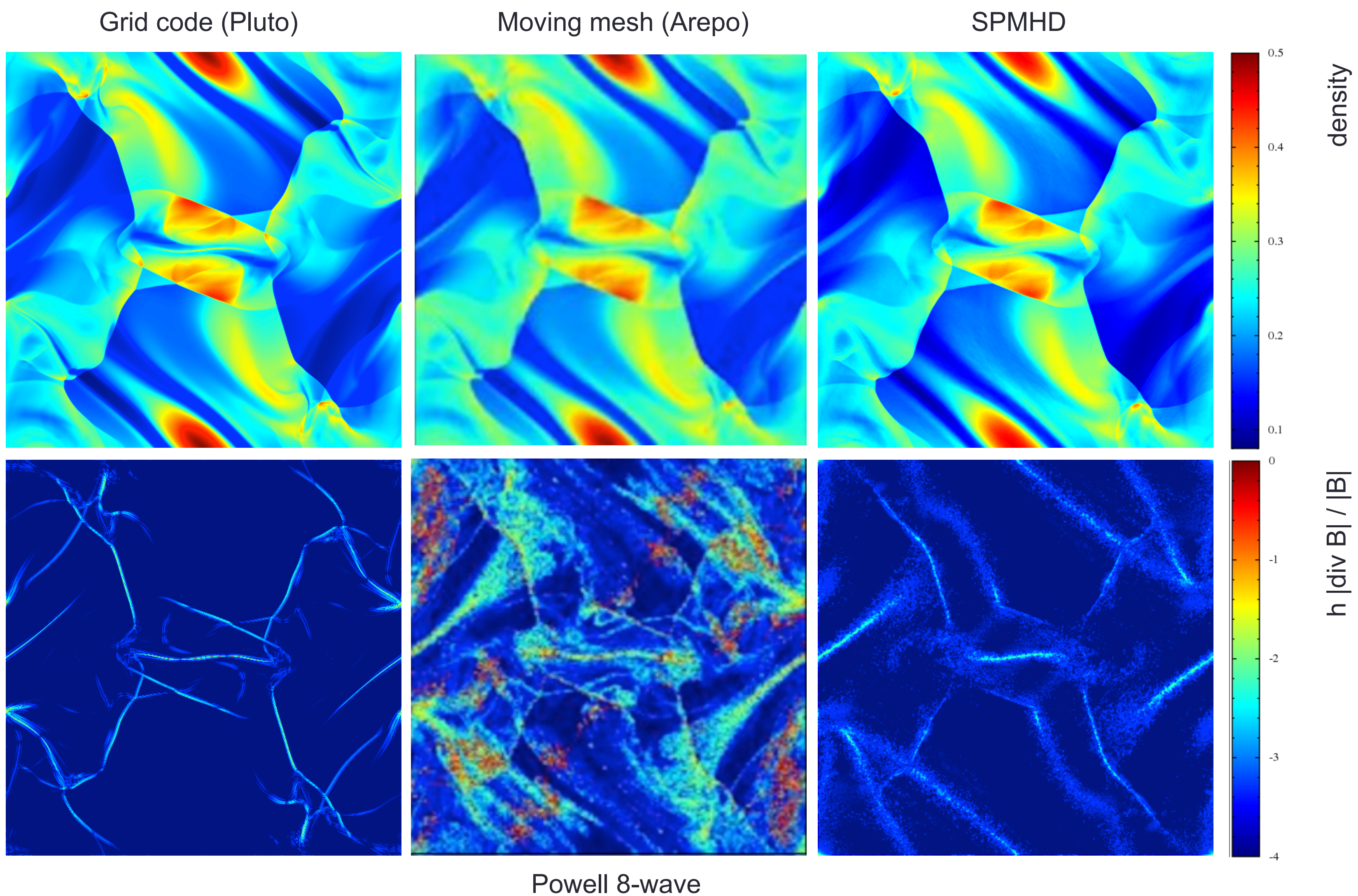
  - Retains conservation and stability properties of SPH
  - Guaranteed to always decrease divergence of the field due to strict energy conservation
  - Provides approx. 10x decrease in divergence error

Tricco & Price, J Comp Phys (2012)

Comparison to other MHD Methods

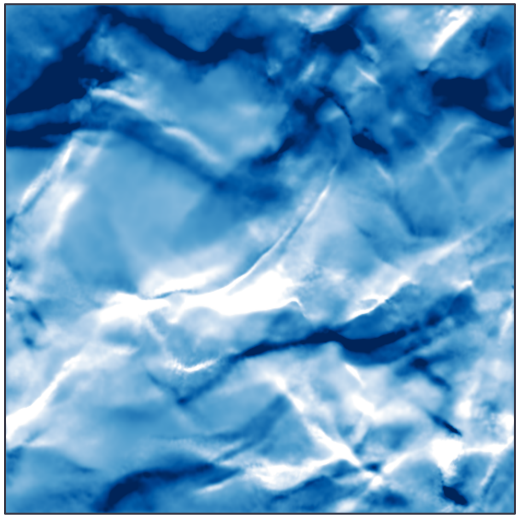


Comparison to other MHD Methods

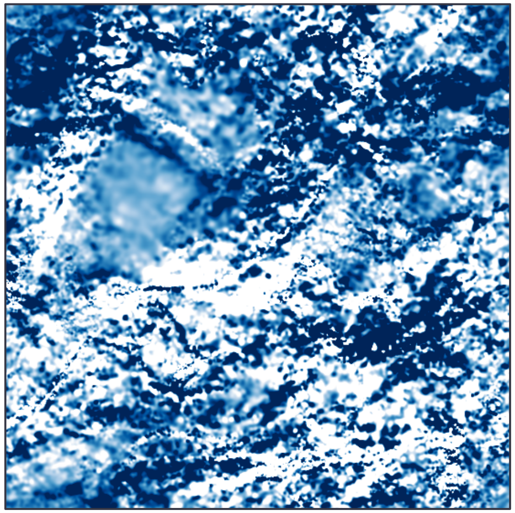


Magnetic Shock Capturing

- Magnetic shocks captured by using Artificial Resistivity
  - Dissipate magnetic field about the shock to correctly model discontinuity
- Use a "switch" to activate resistivity only near shocks to reduce dissipation
- Previous switch: $\propto \nabla \times \mathbf{B}$
  - Failed when magnetic field is very weak



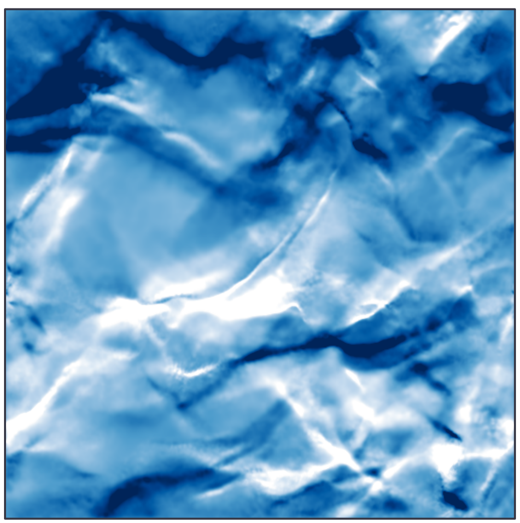
Fixed resistivity



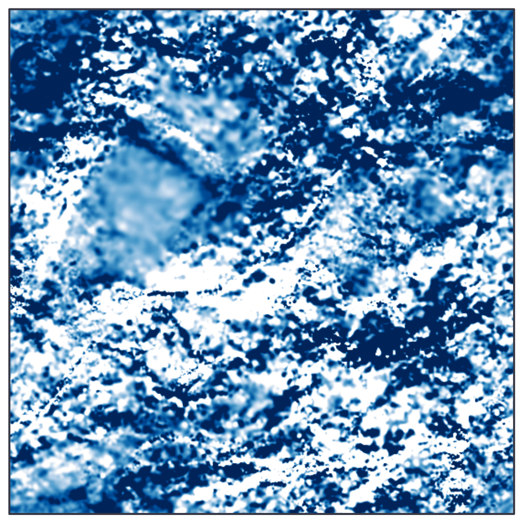
Previous switch

Magnetic Shock Capturing

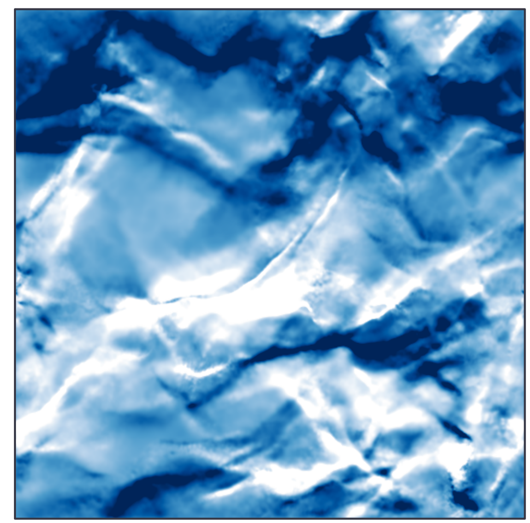
- New switch: sets AR $\propto h|\nabla \mathbf{B}|/|\mathbf{B}|$
  - ie, relative degree of discontinuity in the magnetic field
  - valid for all field strengths, captures shocks as field is amplified



Fixed resistivity

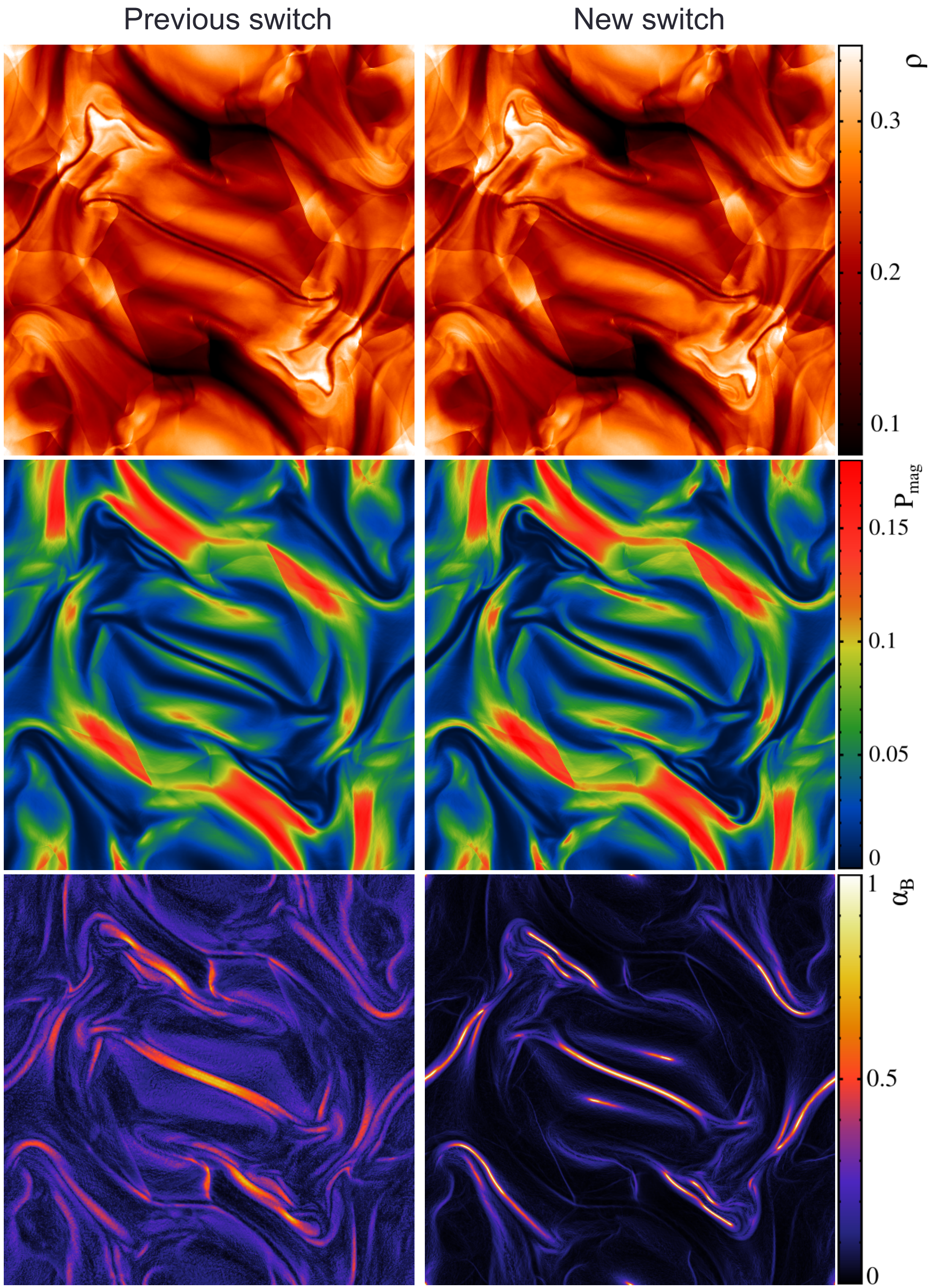


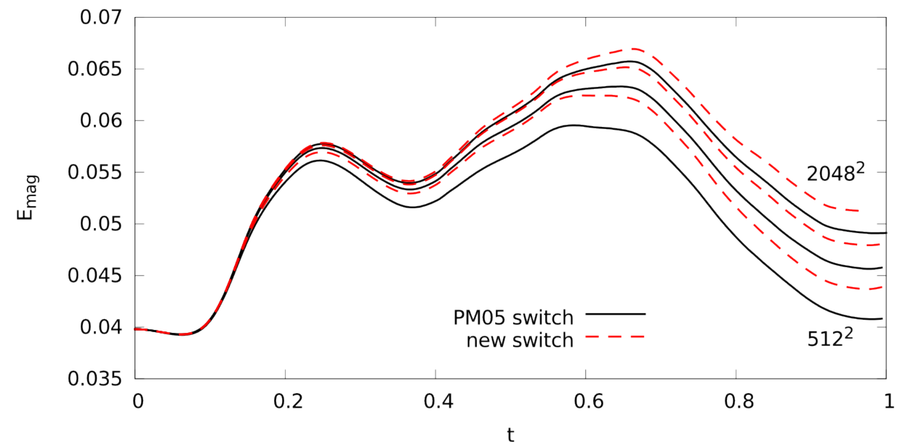
Previous switch



New switch

Tricco, Price, in review


Previous switch
New switch
ρ
0.3
0.2
0.1
P_mag
0.15
0.1
0.05
0
α_B
1
0.5
0


0.07
0.065
0.06
0.055
0.05
0.045
0.04
0.035
E_mag
0
0.2
0.4
0.6
0.8
1
t
PM05 switch
new switch
2048^2
512^2

Summary

- Method enhancements for SPMHD:

1. Divergence cleaning which is stable, effective, robust
2. Switch to reduce dissipation of the magnetic field, can capture shocks in the weak field regime

If you've been hesitating about adding magnetic fields to your SPH simulations, hesitate no more!

Feel free to contact me:

terrence.tricco@monash.edu

http://users.monash.edu/~tricco