Comparing SPMHD and Grid-based MHD on the Turbulent Dynamo Amplification of Magnetic Fields

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Goal

To investigate the ability of Smoothed Particle Magnetohydrodynamics to simulate the small-scale turbulent dynamo amplification of magnetic fields.

Initial Conditions

The calculations use conditions representative of molecular clouds. To keep the comparison fair, we have started from as simple a state as possible -- uniform density, zero velocity, and uniform magnetic field. The gas uses an isothermal equation of state. The turbulence is initiated and sustained at r.m.s velocity of Mach 10 by a stochastic driving force (Federrath et al, 2010; Price & Federrath, 2010). The initial magnetic energy is 10 orders of magnitude smaller than the mean kinetic energy of the turbulence.

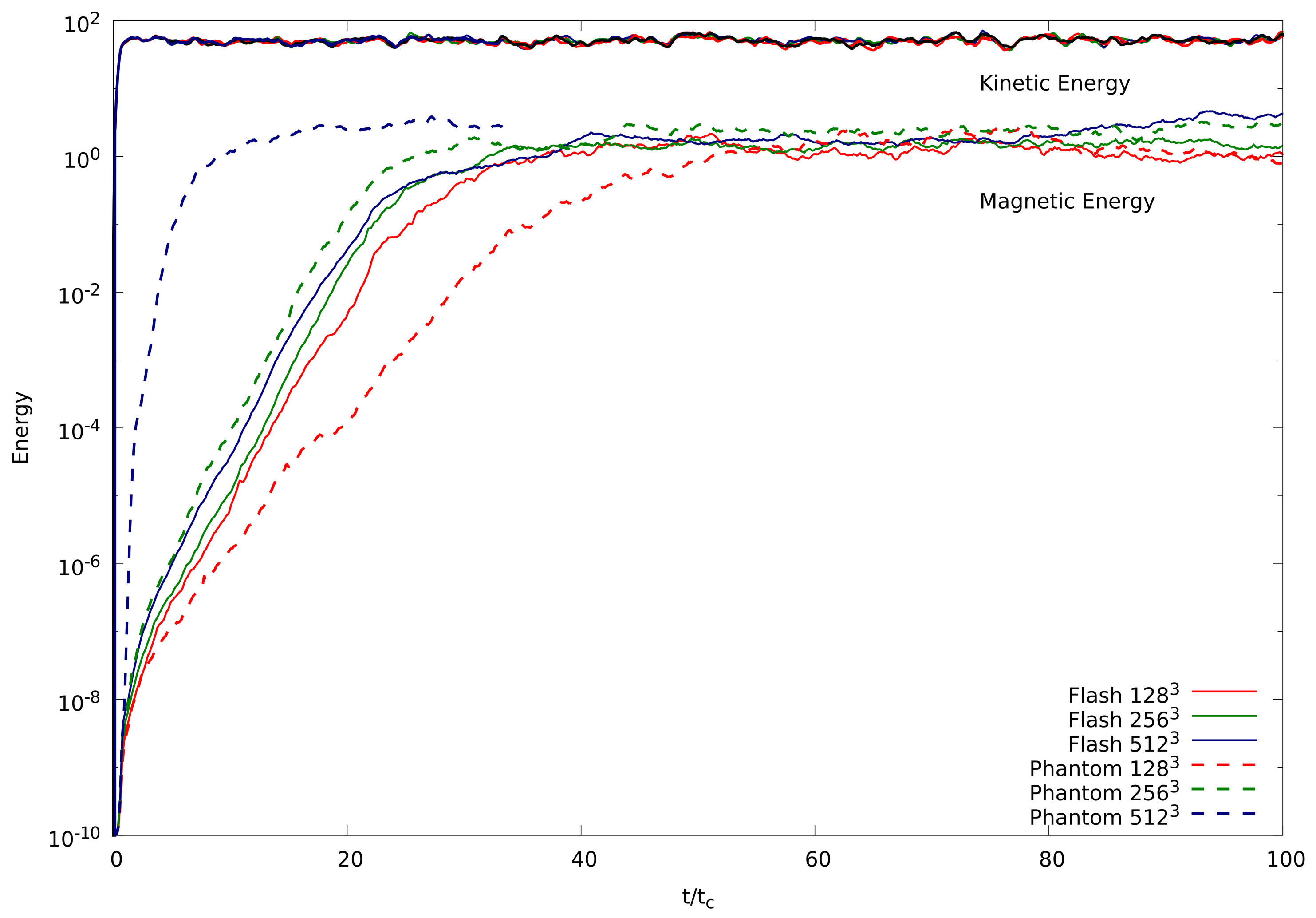
Numerical Details

We simulate 100 turbulent crossing times (t_C) in order to capture the full growth and saturation of magnetic energy. Our calculations utilise 128^3, 256^3, and 512^3 resolution elements, comparing results between the following two codes:

Flash: An Eulerian grid-based code which uses a finite volume scheme to solve the MHD equations (Waagan et al, 2011). These calculations utilise a grid of fixed size.

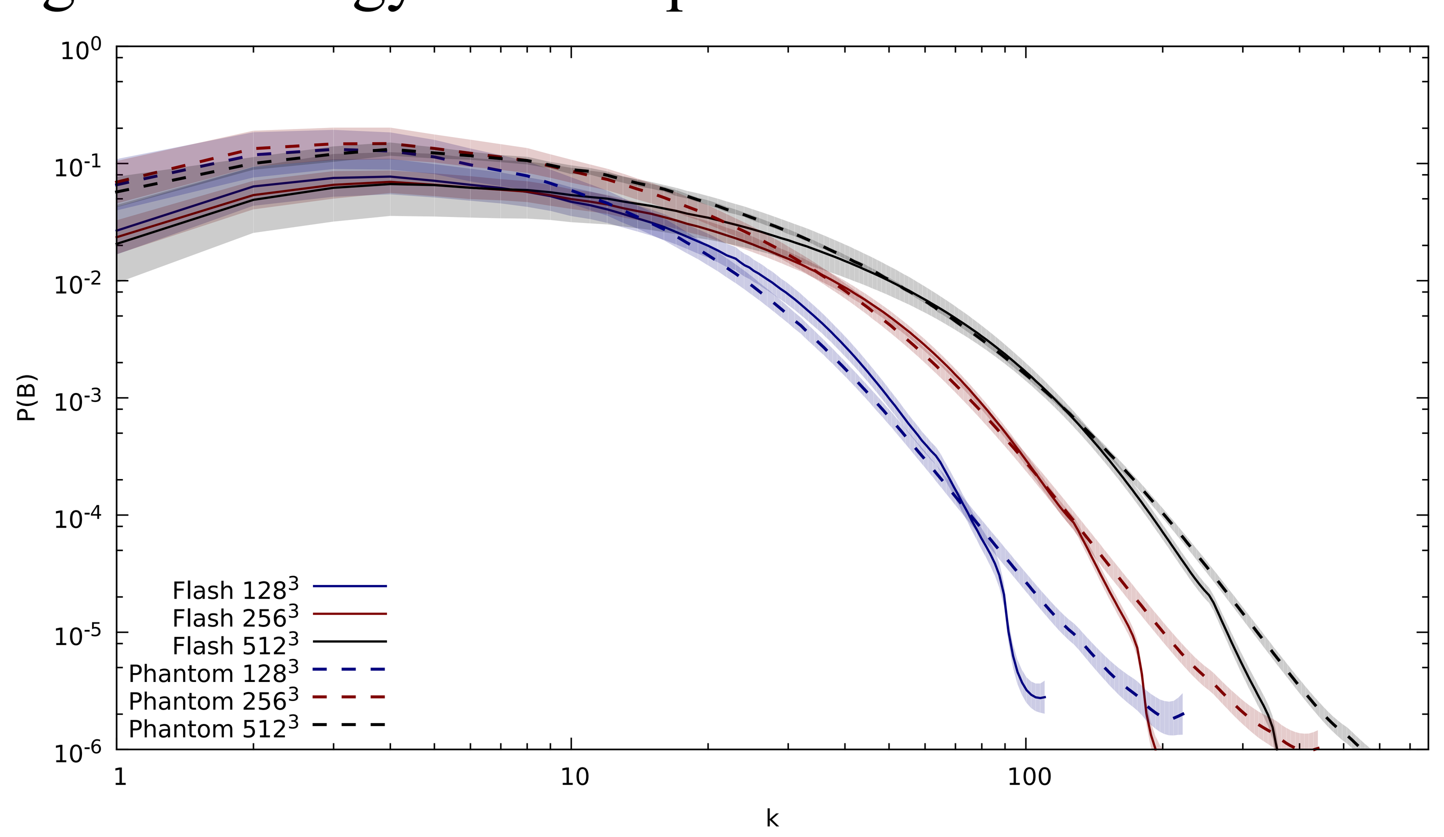
Phantom: A Lagrangian particle based code which solves the MHD equations using smoothed particle magnetohydrodynamics (SPMHD). Phantom utilises recent numerical SPMHD advancements – constrained hyperbolic/parabolic divergence cleaning to minimise the divergence of the magnetic field (Tricco & Price, 2012), and a switch to reduce the numerical dissipation of the magnetic shock capturing (Tricco & Price, 2013).

Magnetic Energy Growth



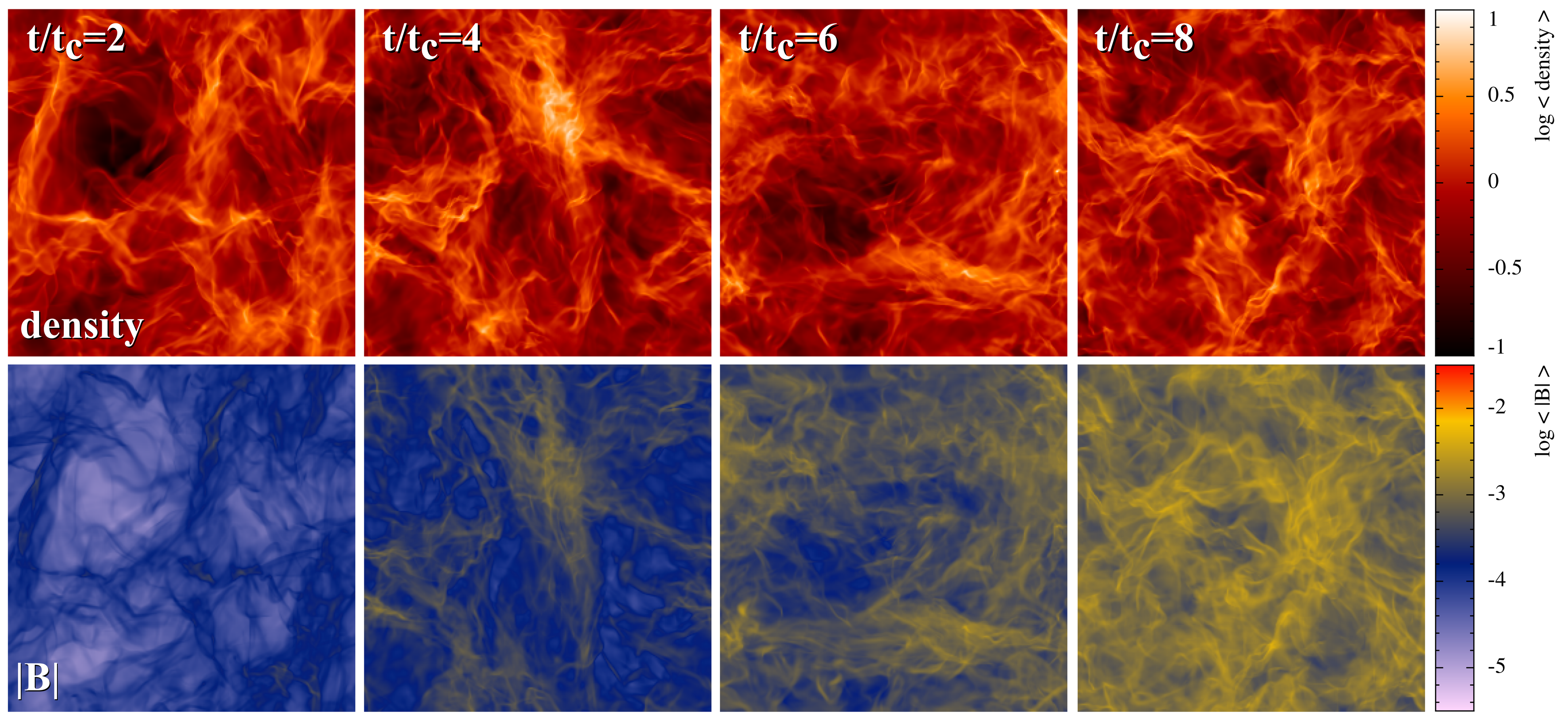
The small-scale dynamo exponentially amplifies magnetic energy through the conversion of kinetic energy. The growth rate of magnetic energy in Flash is largely insensitive to resolution, whereas Phantom exhibits a resolution dependence arising from the scaling of the numerical dissipation terms. The magnetic energy saturates at around 2-4% of the mean kinetic energy.

Magnetic Energy Power Spectra

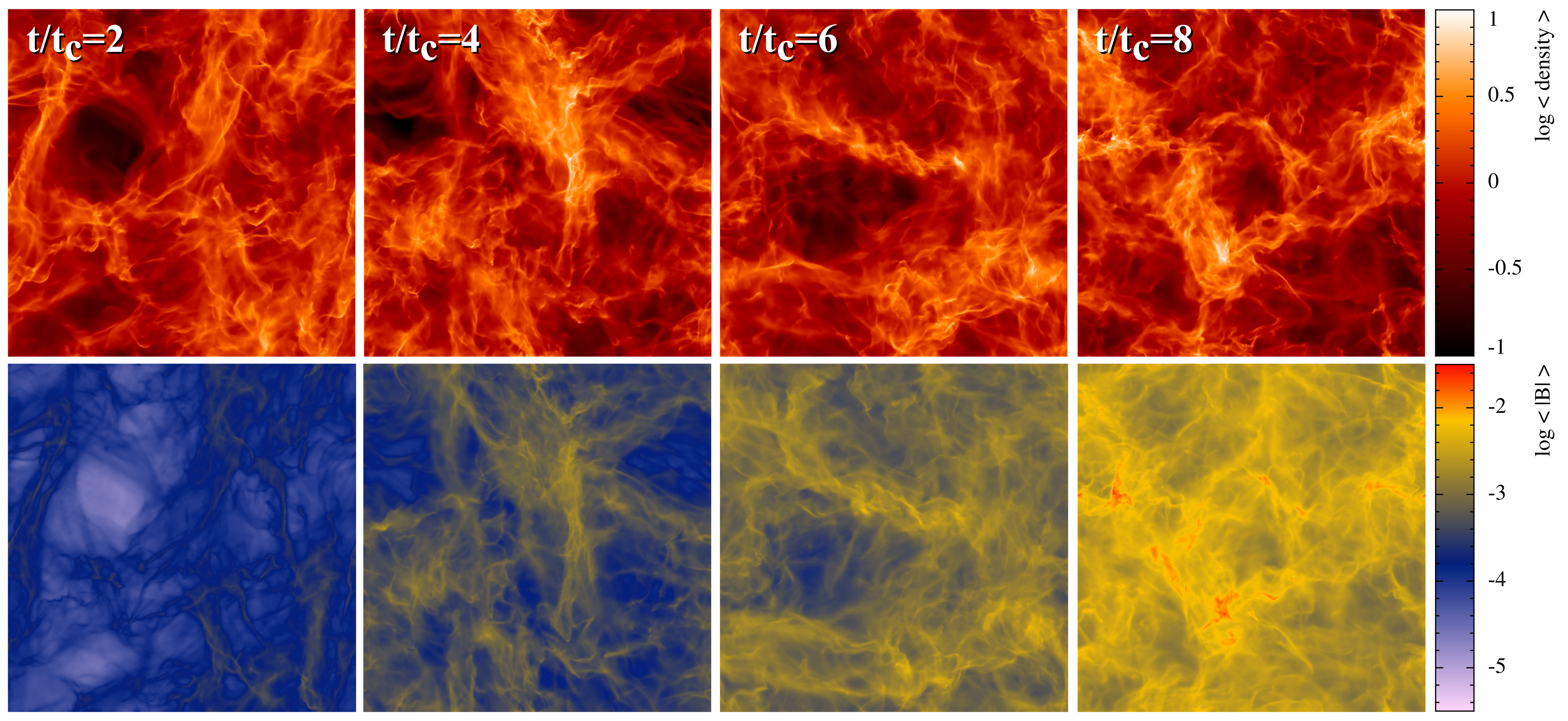


We present the time averaged power spectra of the magnetic energy once the magnetic field has saturated. The shaded regions represent the standard deviation of the time averaging. Both codes contain comparable amounts of magnetic energy at similar spatial scales, though with Phantom containing approximately 2x as much energy on large scales than Flash.

Flash (Grid)

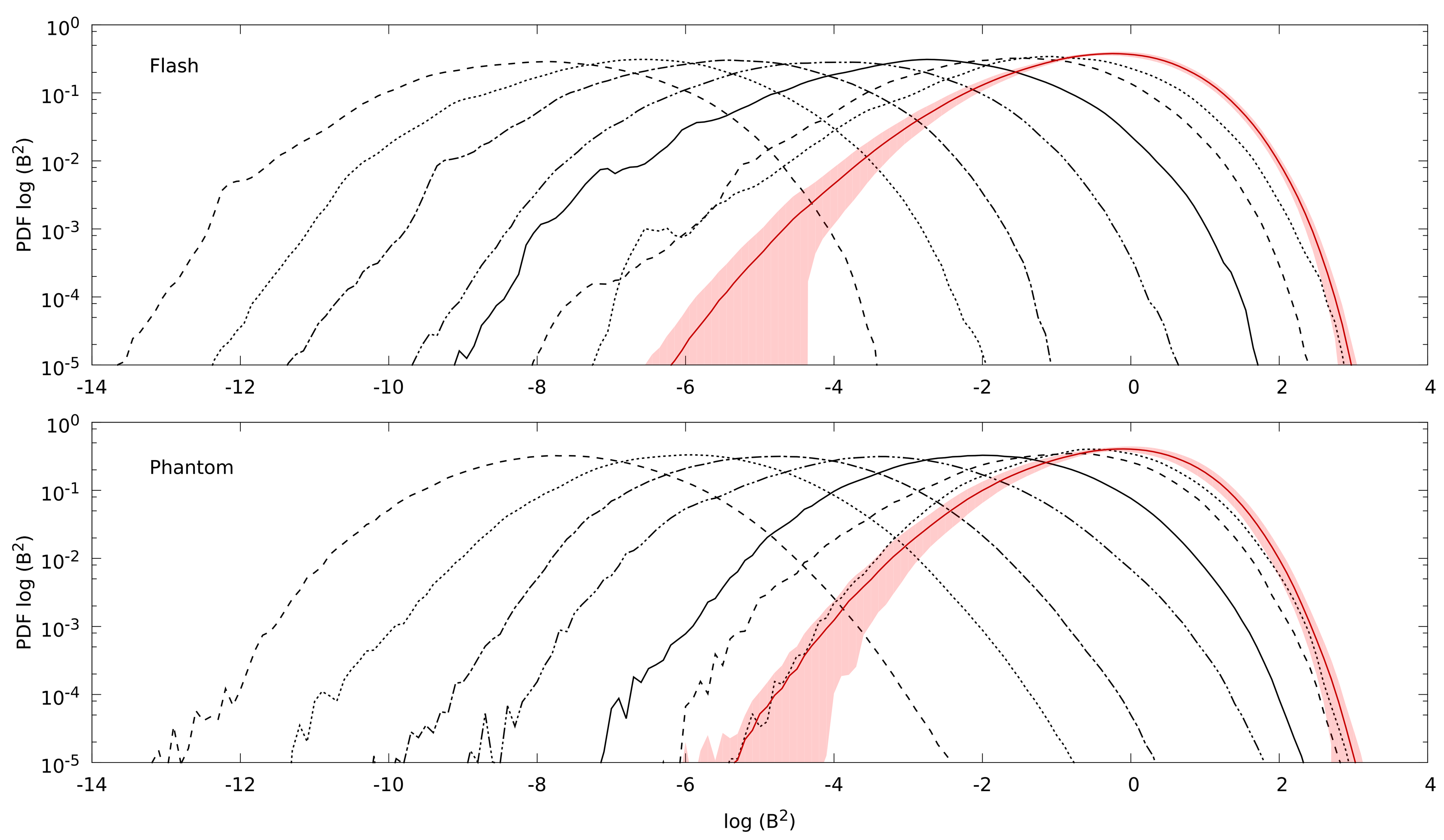


Phantom (SPMHD)



z-column integrated density and |B| for $t/t_c = 2, 4, 6, 8$ at resolutions of 256^3. The density field has similar structure in both codes at early times, but diverge at late times due to the non-linear behaviour of the turbulence. The magnetic field is strongest in the densest regions, while the mean magnetic field strength also increases with time.

Probability Distribution Function of Magnetic Field Strengths



The distribution of magnetic field strengths during the growth phase (equi-spaced every 4 t_c) and during the saturation phase (red line, shaded region representing the standard deviation of the time averaging). For both codes, the distribution of magnetic field strengths is log-normal during the growth phase, which becomes lopsided after saturation.

Conclusions

1) Both codes exhibit similar qualitative behaviour: the initially weak magnetic field is exponentially amplified over tens of turbulent crossing times, saturating when the magnetic energy is 2-4% of the kinetic energy.
2) Both codes show magnetic energy power spectra that is relatively flat at large scales, peaking around k~3-5.
3) During the growth phase, both codes produce PDFs of B^2 which are log-normal and linearly translate to higher magnetic field strengths. As the magnetic field approaches saturation, the PDFs become skewed.

We thus conclude that SPMHD can reliably simulate small-scale turbulent dynamo amplification of magnetic fields.

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