The Kelvin-Helmholtz Instability and Smoothed Particle Hydrodynamics

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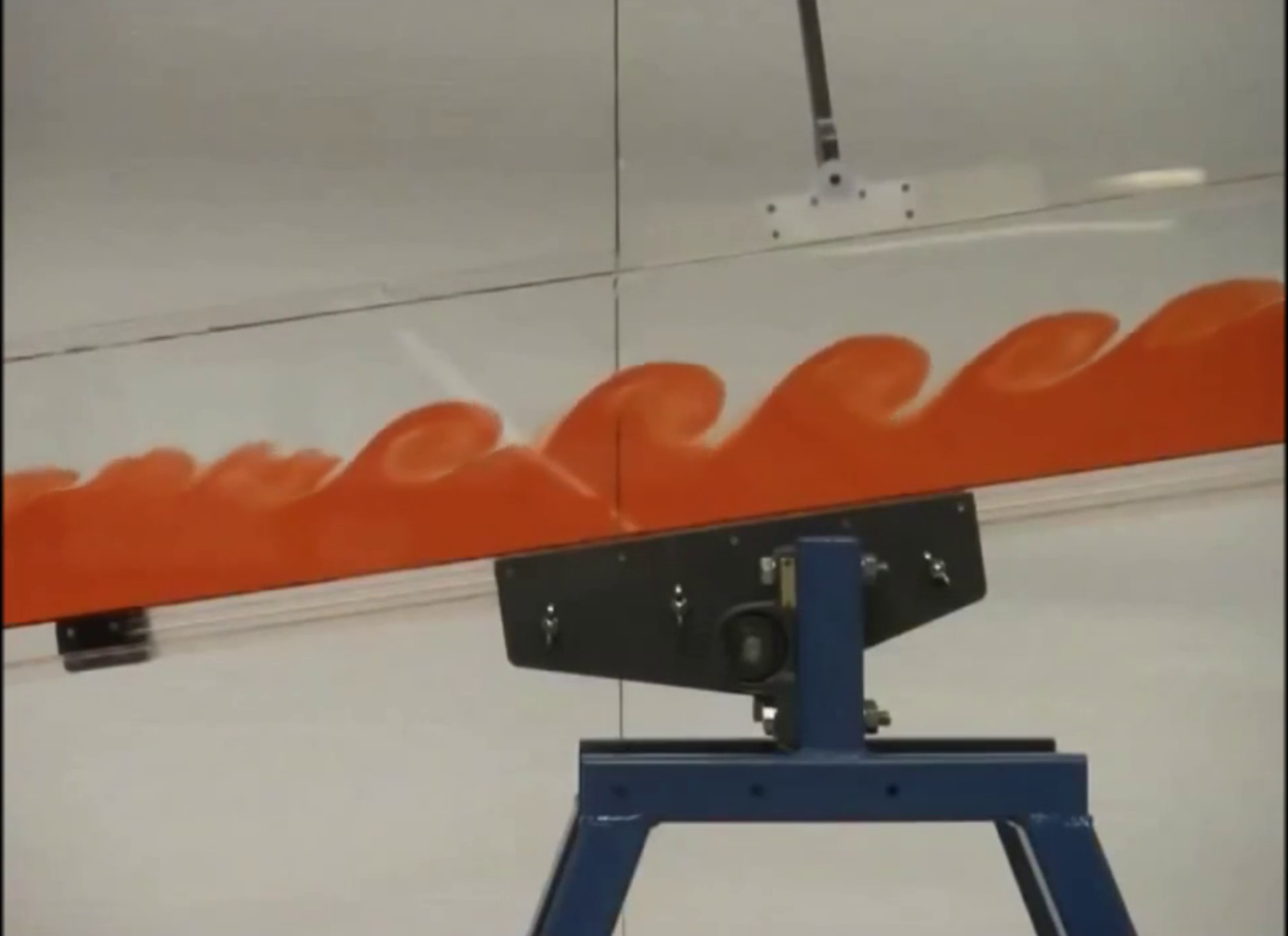

The Kelvin-Helmholtz Instability in SPH

Fundamental differences between SPH and grid methods

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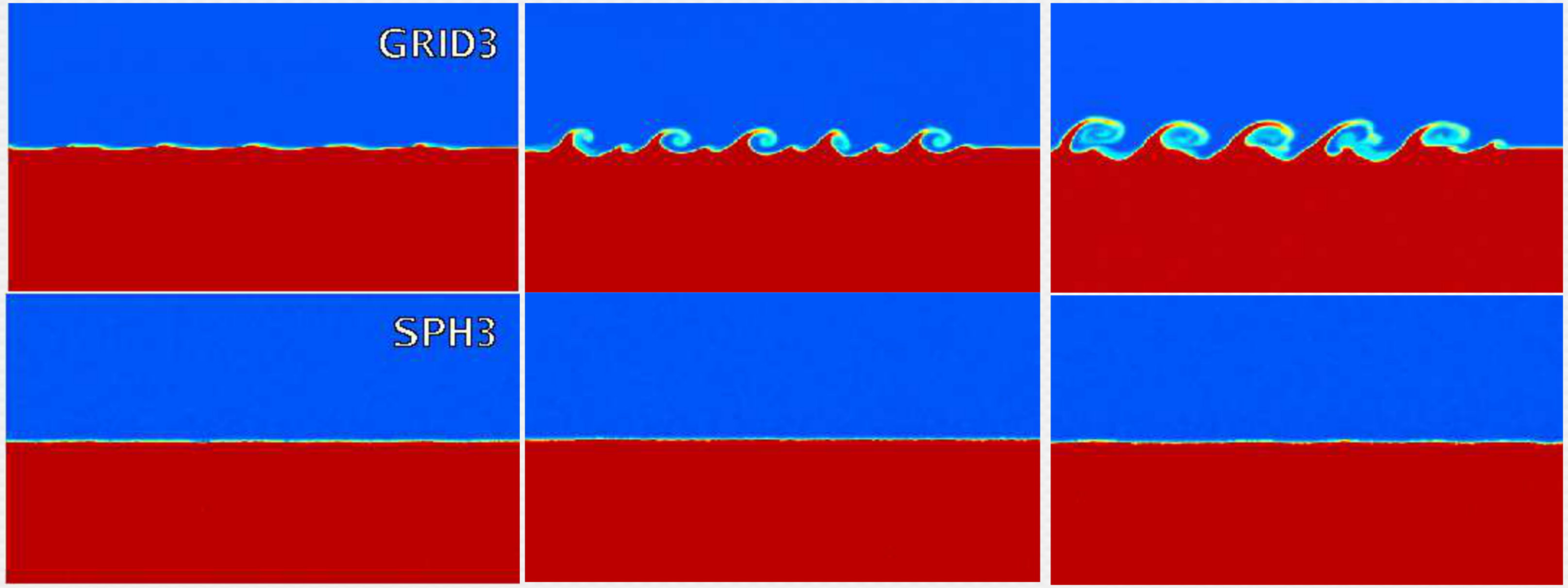
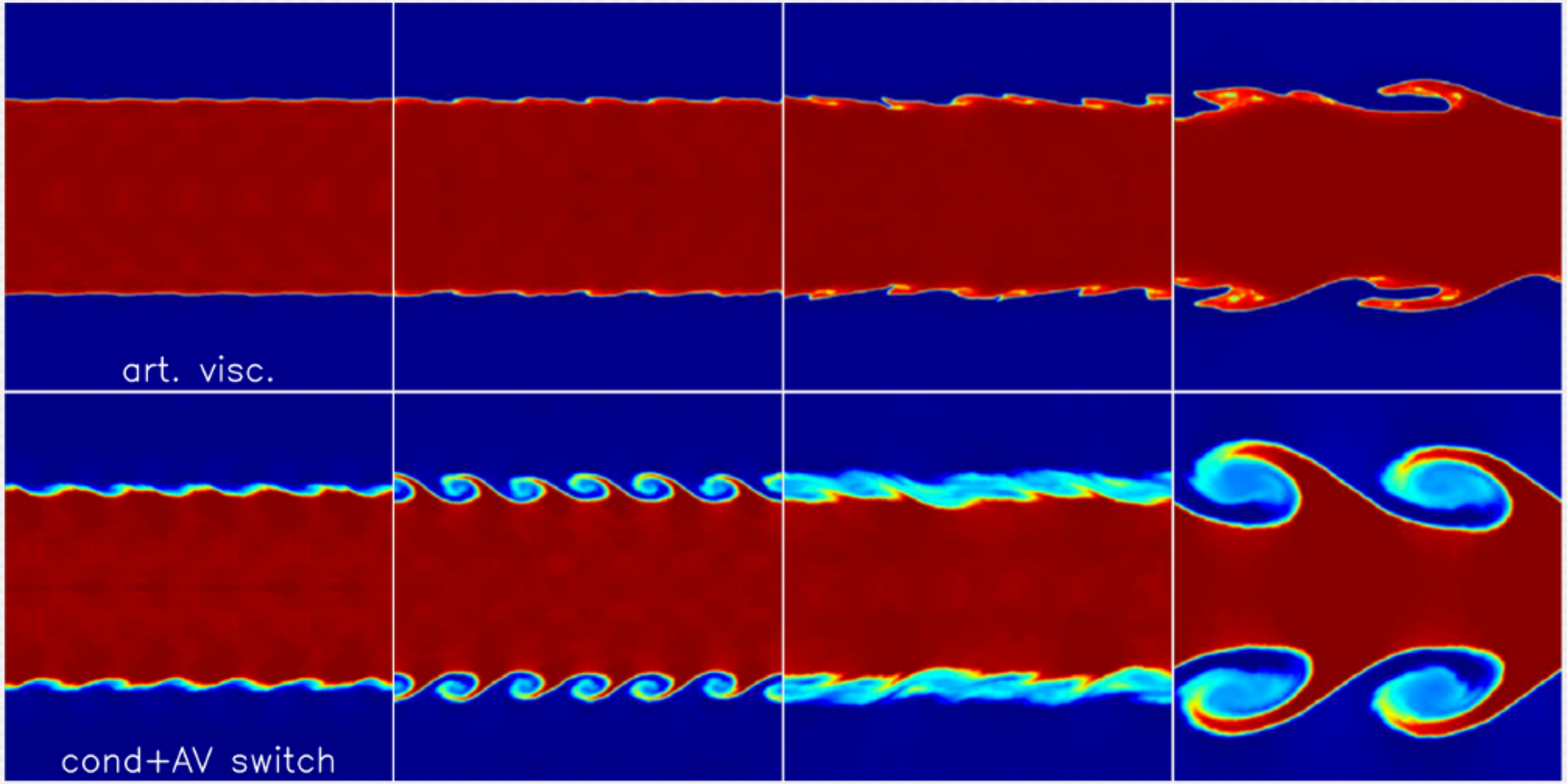
between the two main techniques for simulating fluids. While grid codes are able to resolve and treat dynamical instabilities and mixing, these processes are poorly or not at all resolved by the current SPH techniques. We show that the reason for this is that SPH, at

Agertz et al (2007)



The Kelvin-Helmholtz Instability in SPH

- Discontinuities require numerical treatment (e.g., Riemann solver)
- In SPH, include an **artificial conductivity**



Price (2008)

The Kelvin-Helmholtz Instability in SPH

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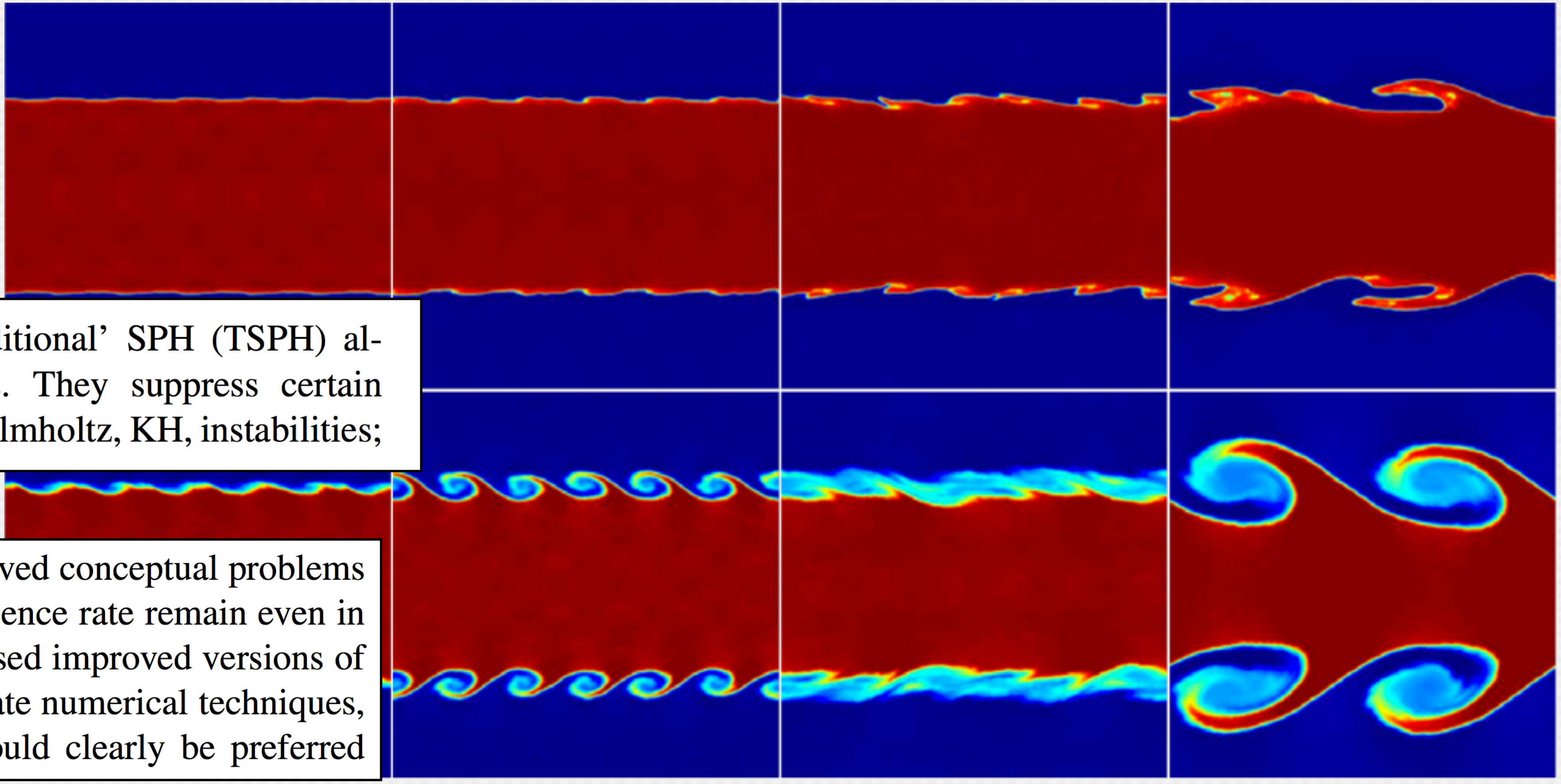


> However, it is well known that ‘traditional’ SPH (TSPH) algorithms have a number of problems. They suppress certain fluid-mixing instabilities (e.g. Kelvin–Helmholtz, KH, instabilities;

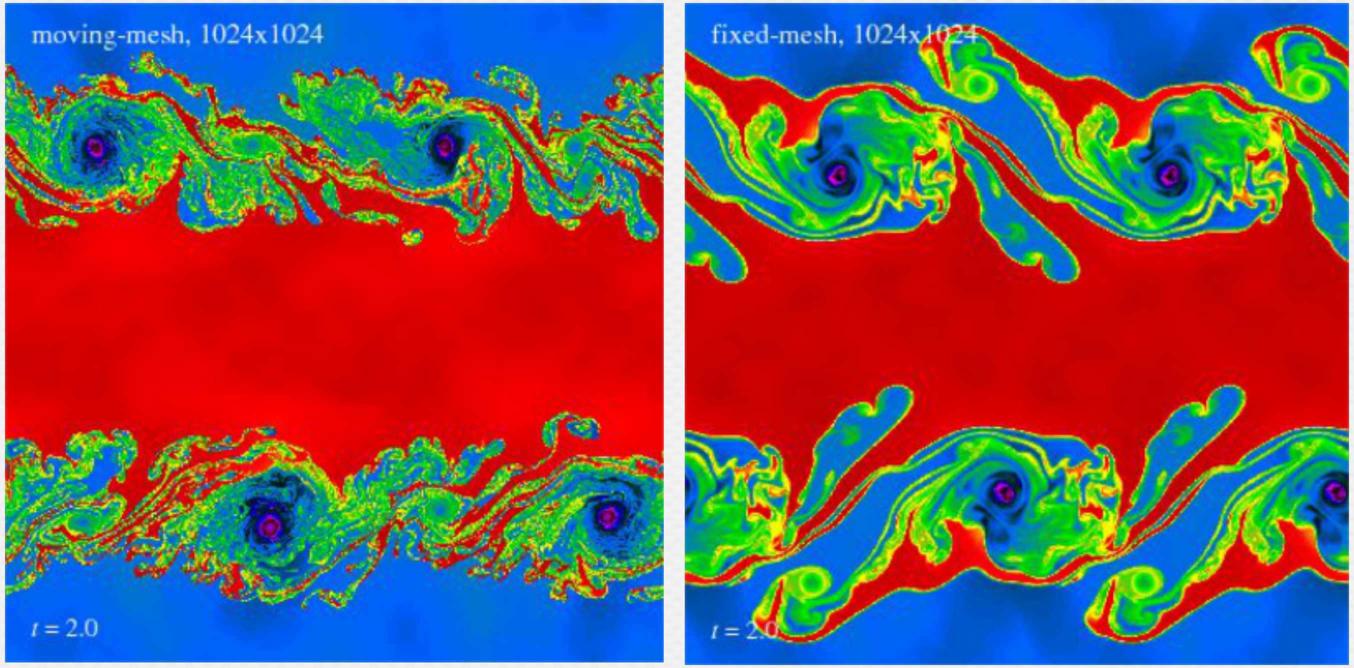
Hopkins (2015)

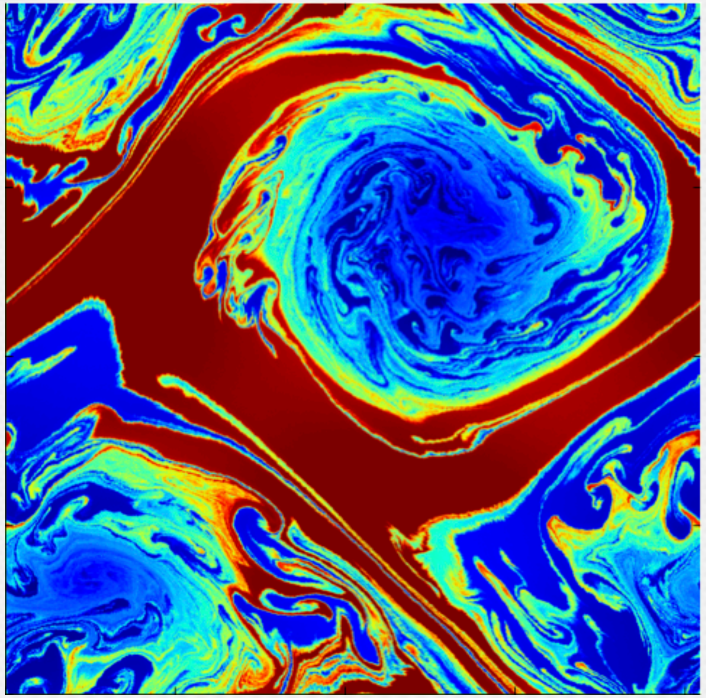
> be incorrect. As we have discussed, unsolved conceptual problems with the accuracy of SPH and its convergence rate remain even in the most recent incarnations of the proposed improved versions of SPH. We therefore think that more accurate numerical techniques, such as our moving-mesh approach, should clearly be preferred

Hayward et al (2014)

Price (2008)

Voronoi Moving Mesh / Meshless Finite Volume





a resolution of 1024^2. The moving-mesh preserves much more fine detail in the flow. This is because contact discontinuities between different phases can be advected with large speeds without being necessarily mixed. We think this is a very interesting difference, which makes the moving-mesh code particularly attractive for the study of multi-phase media.

Springel (2010)

complicated contact discontinuities. In the late non-linear phases, it is truly remarkable how much fine-structure is captured by the MFV runs, given the relatively low resolution used. In these stages,

(1024^2), showing the same character and the exceptional degree of resolved sub-structure and small-scale modes. Very similar results are obtained with moving-mesh methods (see Springel 2011, fig. 8).

Hopkins (2015)

Mo’ Mixing, Mo’ Problems

- Robertson et al (2010), McNally et al (2012), Lecoanet et al (2016) introduce KH tests with **well-posed initial conditions, demonstrating convergence**

> (e.g. Springel 2010; Hopkins 2015). Presumedly, more small-scale structure implies less numerical dissipation, and therefore greater accuracy. We find in the current paper that this intuition can, in some cases, lead to false conclusions. Mocz et al. (2015) show

Lecoanet et al (2016)

Mo' Mixing, Mo' Problems

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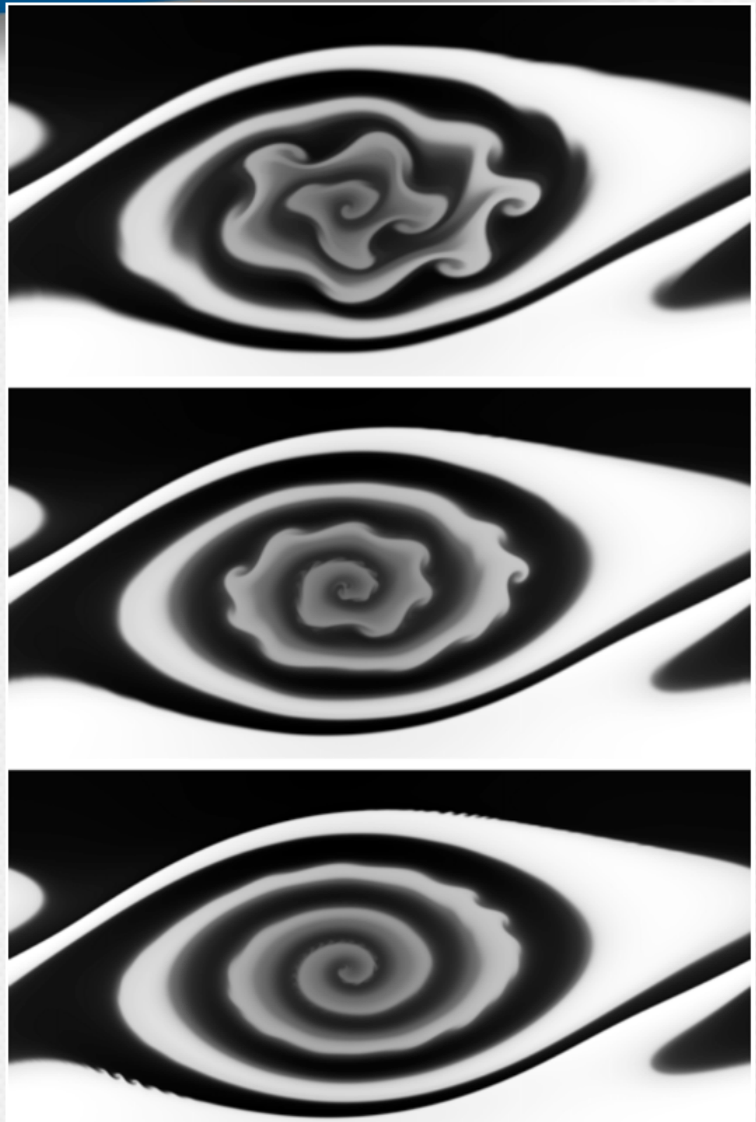
> Not all new instabilities seen as resolution is increased when solving the discretized Euler equations are physically real. New numerical instabilities can reveal themselves as resolution is increased, as the flow can enter into new regimes where it is more sensitive to the inevitable numerical noise in a method.

> the exact details of the setup used. However, we can show that for our problem that secondary instabilities that do develop are of a purely numerical origin. This strongly suggests that the secondary billows seen in Springel (2011) are a numerical artifact, so the observation that a fixed grid codes does not develop these on the same problem does not imply that the fixed grid code is too diffusive to support the modes.

McNally et al (2012)



The KH Tests of Lecoanet et al (2016)

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\nabla\cdot\boldsymbol{v},$$

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\frac{\nabla P}{\rho} - \frac{1}{\rho}\nabla\cdot\boldsymbol{\Pi},$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{P}{\rho}\nabla\cdot\boldsymbol{v} + \nabla\cdot(\boldsymbol{v}\cdot\boldsymbol{\Pi}) + \frac{1}{\rho}\nabla\cdot(\chi\rho\nabla T)$$

$$\frac{\mathrm{d}c}{\mathrm{d}t} = \frac{1}{\rho}\nabla\cdot(\nu_{\mathrm{c}}\rho\nabla c)$$

$$\Pi^{ij} = \nu\rho\left(\frac{\partial v^i}{\partial x^j} + \frac{\partial v^j}{\partial x^i} - \frac{2}{3}\frac{\partial v^k}{\partial x^k}\delta^{ij}\right)$$

- **2D** tests with smooth initial conditions
- 1:2 aspect ratio with periodic boundaries
- Introduce a **passive scalar colour field** to quantify and measure degree of mixing
- Include **Navier-Stokes viscosity, physical thermal conductivity**, (also **colour diffusion**)
  - dissipation is resolution independent!
- Lecoanet et al (2016) show **convergence in the non-linear regime** between Athena (finite volume) and Dedalus (spectral)

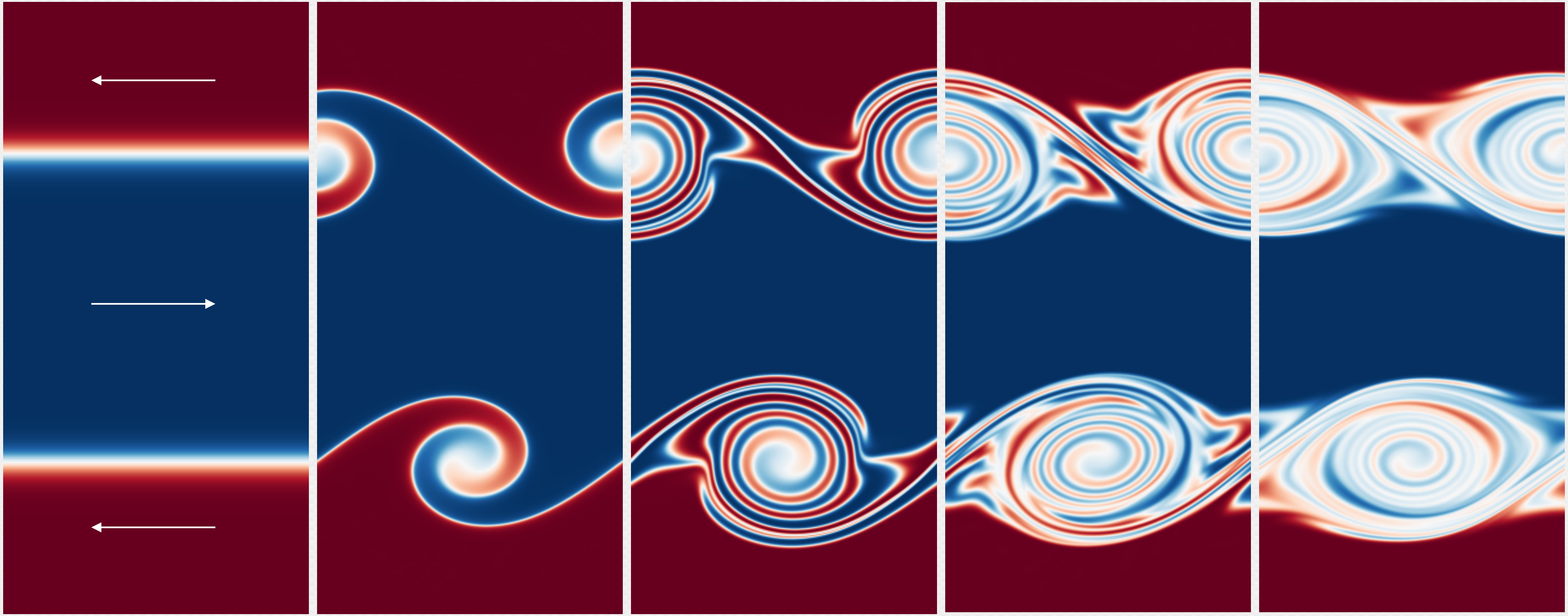
SPH Simulations

- I am using the **Reynolds Number = 10^5**, unstratified (uniform density) Kelvin-Helmholtz test of Lecoanet et al (2016)

- **Goal**: Obtain convergence of SPH results towards reference n_x = 2048 Dedalus calculation (spectral code)

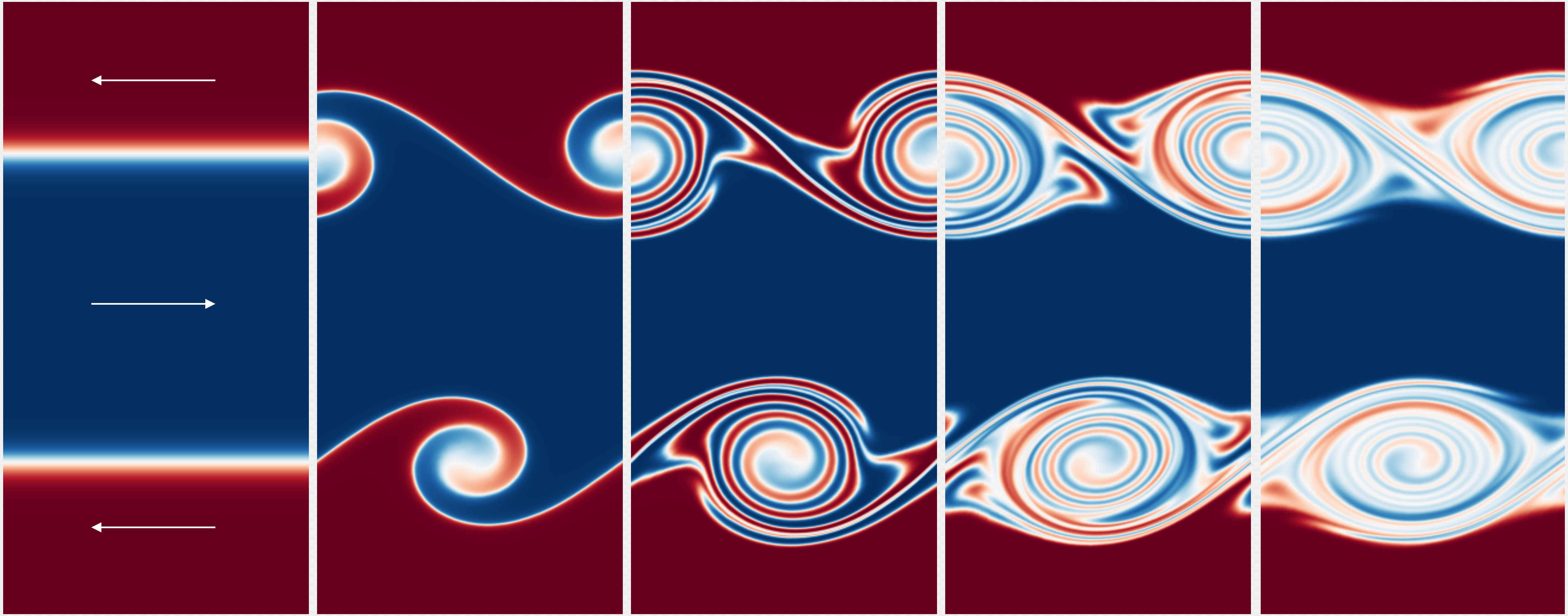
- **Resolution**: n_x = 256, 512, 1024, 2048 particles (~10 million; 70k cpu-hours)
- **SPH Implementation**: Standard, out of the box
- **Dissipation Implementation**:
  - Two first derivatives for Navier-Stokes viscosity
  - Direct second derivative for thermal conduction and colour diffusion

Results of Lecoanet et al (2016)



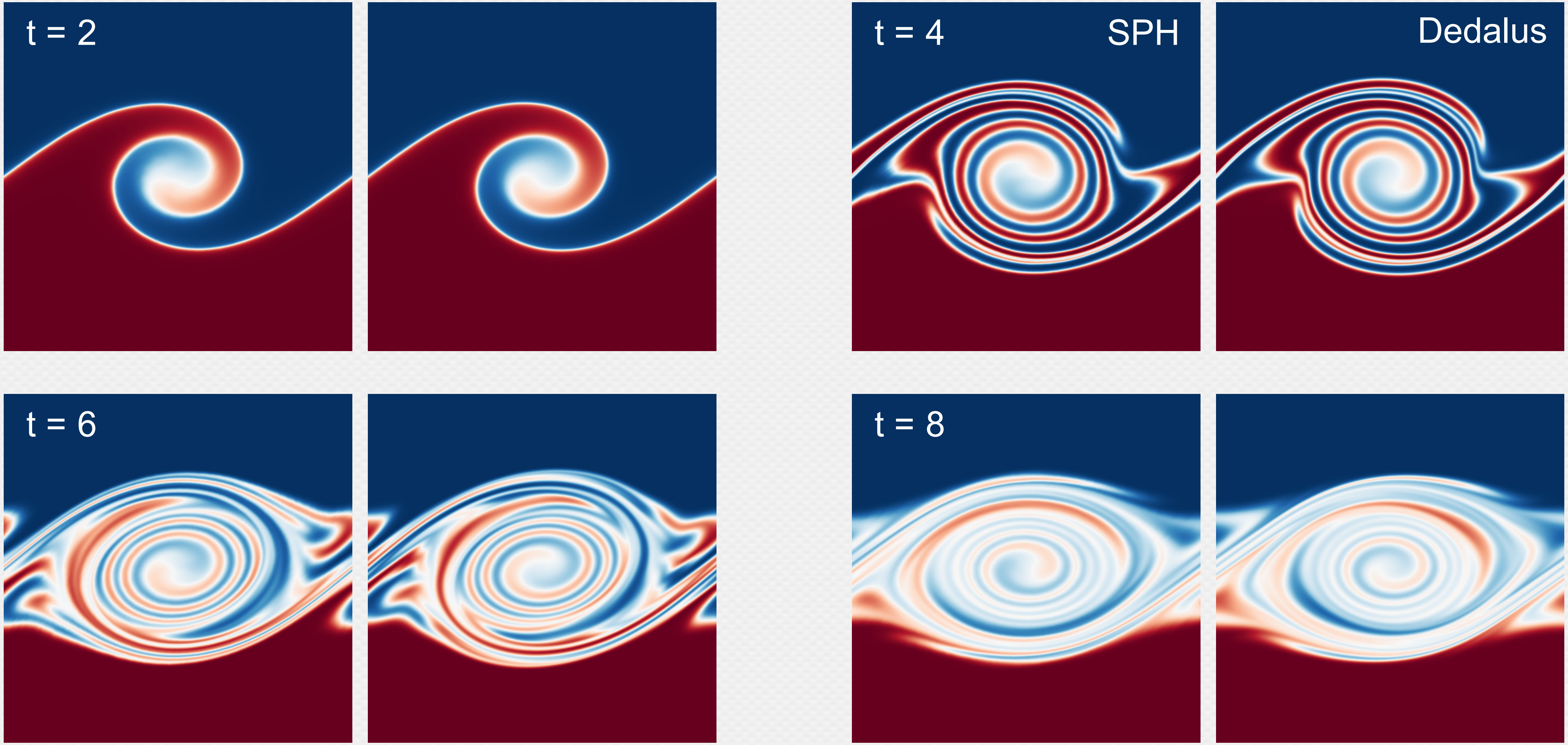
t = 0 | t = 2 | t = 4 | t = 6 | t = 8

SPH Results



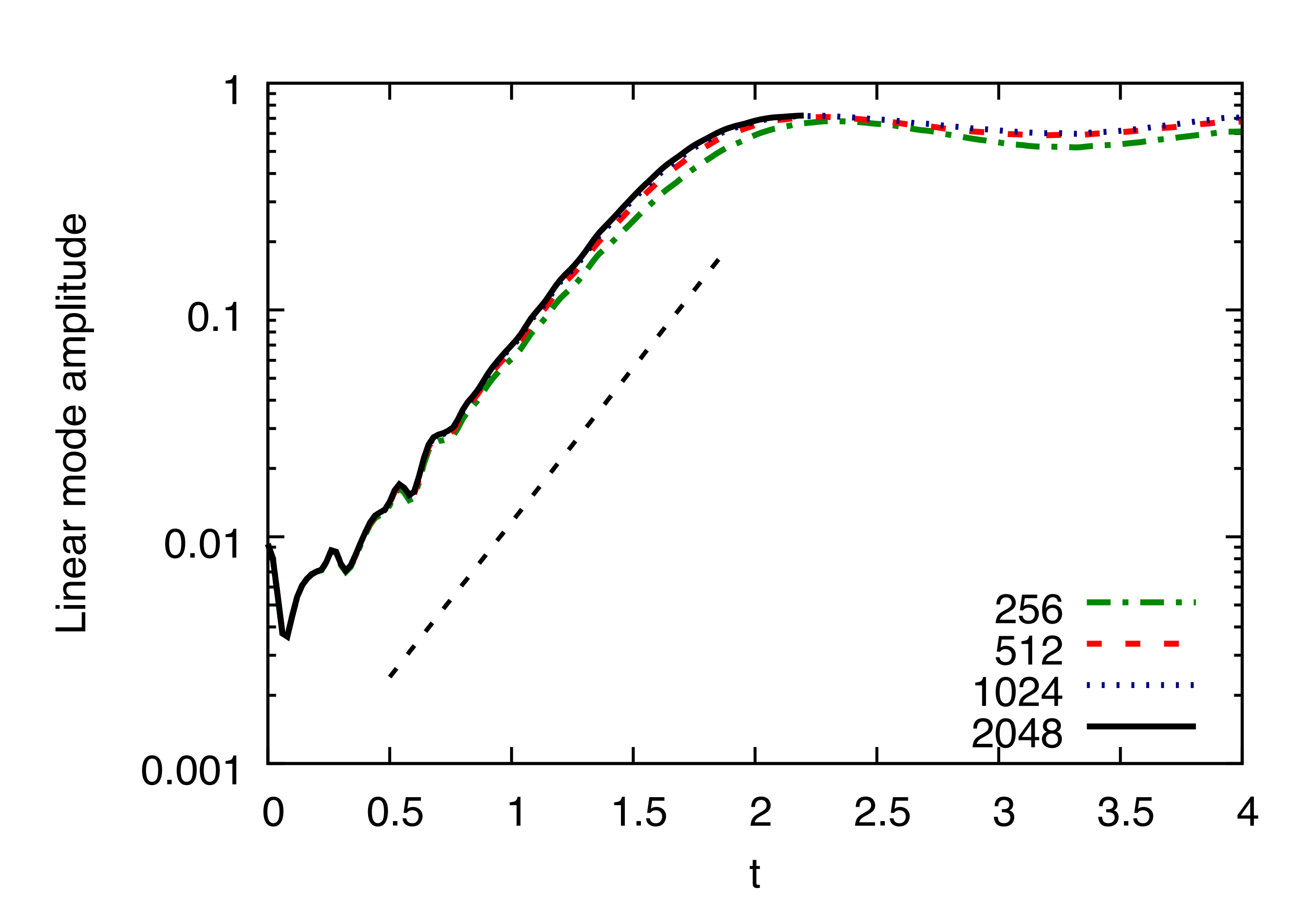
t = 0 t = 2 t = 4 t = 6 t = 8

Qualitative Comparison



Linear Mode Amplitude

- Growth rate of the dominant seeded mode in the linear regime
- Converges to analytic expectation even at modest resolution of n_x=256



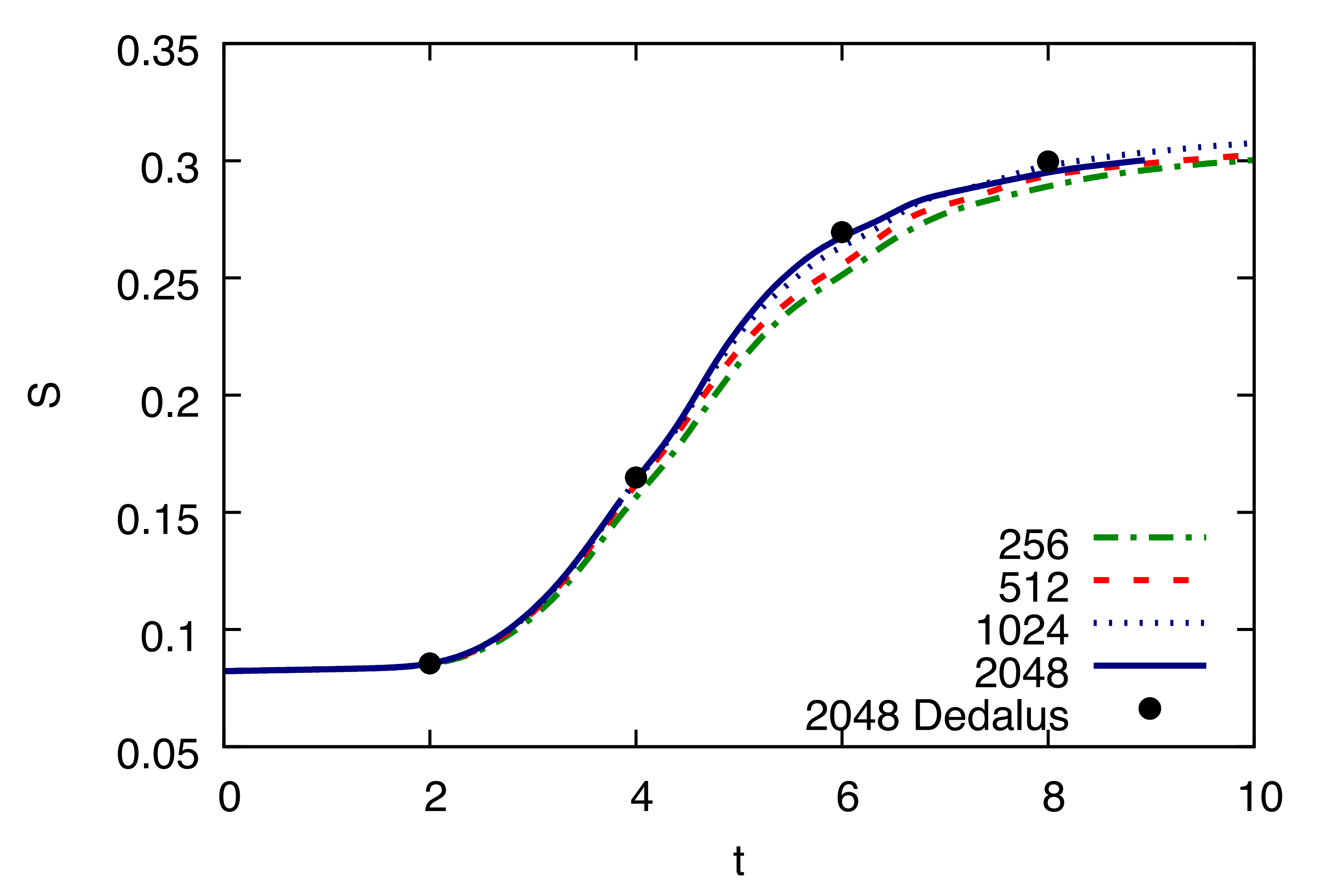
Colour Entropy

- Define entropy for colour

$$s \equiv -c \ln c$$

- and total colour entropy

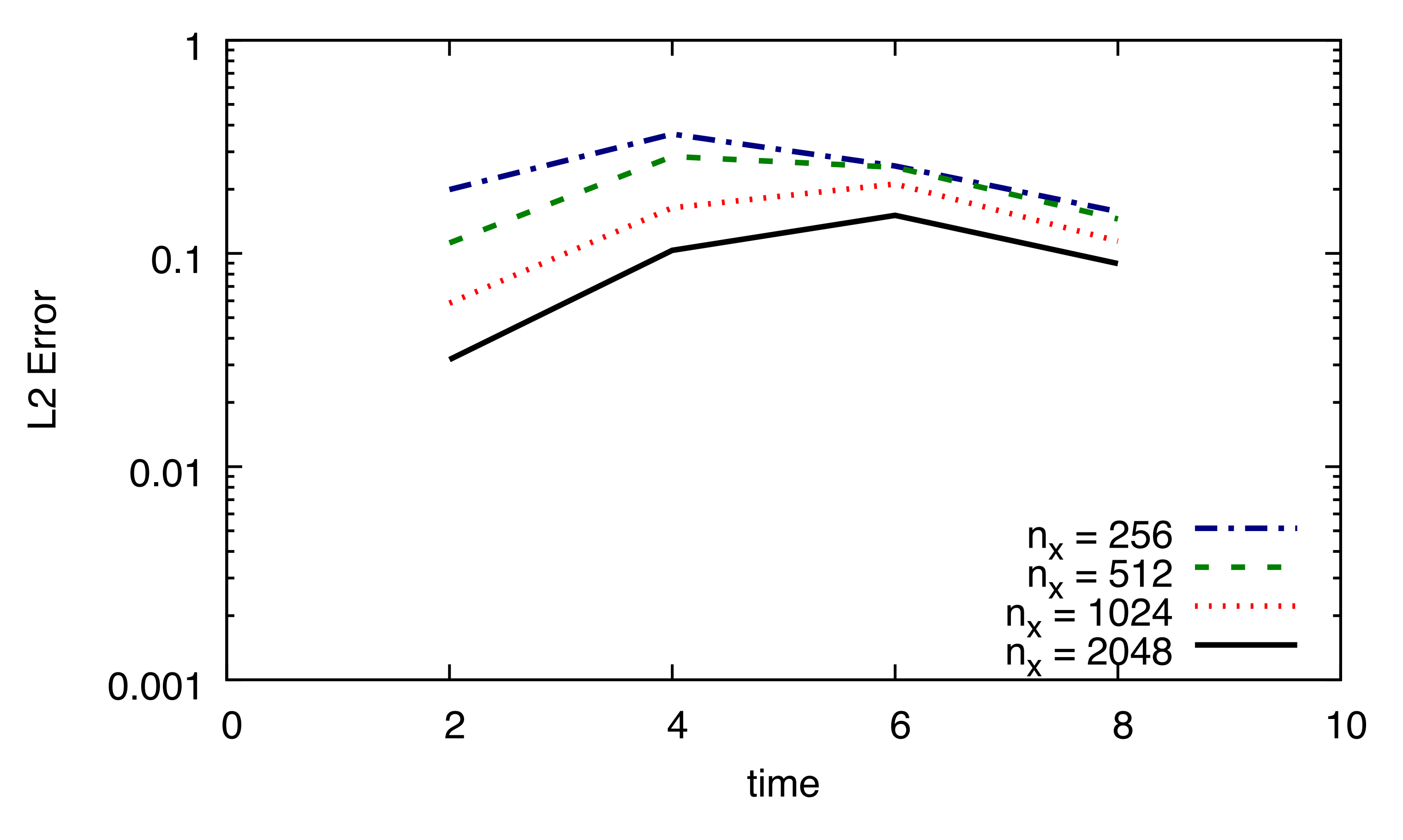
$$S \equiv \int \rho \, s \, \mathrm{d}V$$



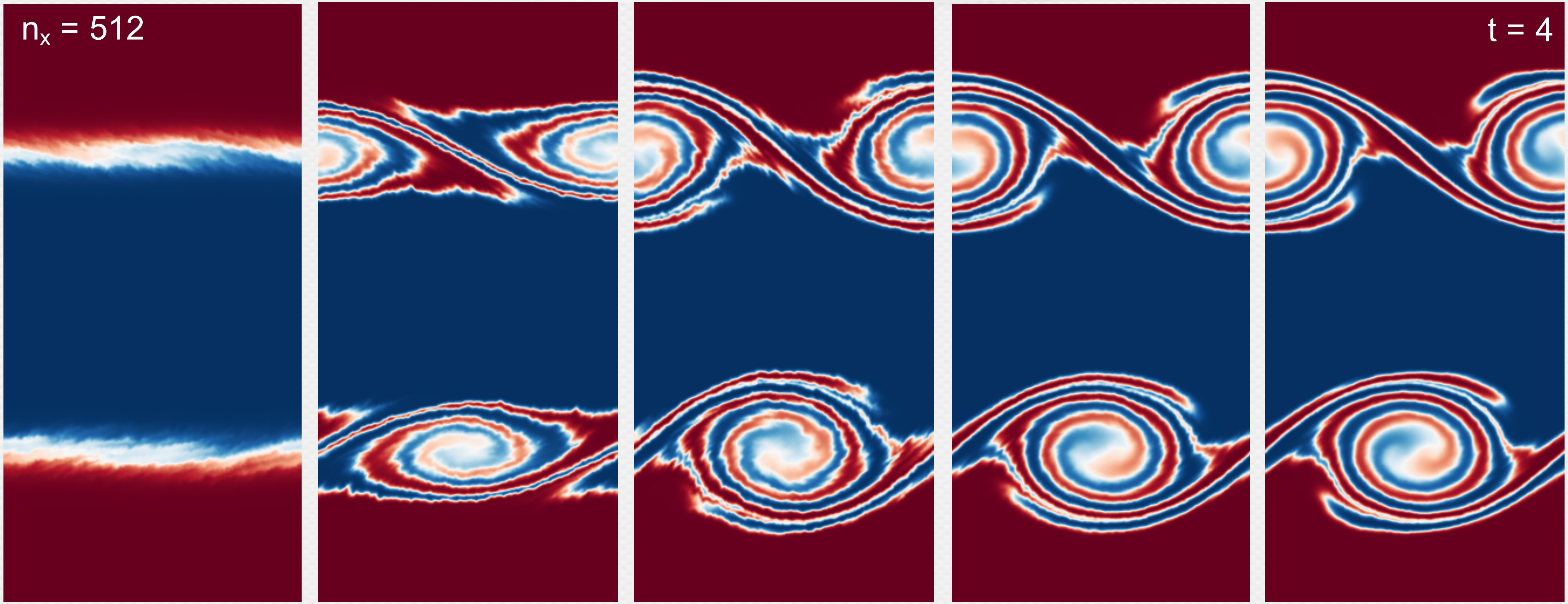
L2 Error

$$L_2(c_\mathrm{X} - c_\mathrm{Y}) = \left[\int \mathrm{d}V \, (c_\mathrm{X} - c_\mathrm{Y})^2 \right]^{1/2}$$

- Interpolate particles to grid
- Difference taken wrt Dedalus 2048 calculation
- By comparison, Athena has max error of 10^{-3} to 10^{-2} for Athena 1024 and 2048



Quality of Smoothing Kernel Matters



Cubic Spline | Quartic | Quintic | Sextic | Septic

- Required to resolve the initial small velocity perturbation (Mach ~ 0.0025)

Summary

- Rumours of the "fundamental flaws" of SPH have been greatly exaggerated
- SPH exhibits quantitative, numerical convergence of the Kelvin-Helmholtz instability
- Requires no alternative SPH formulations, modifications or hacks; only a high order kernel to resolve the amplitude of the initial velocity perturbation