The Small-Scale Turbulent Dynamo in Smoothed Particle Magnetohydrodynamics

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MHD codes in Astrophysics

- 3 broad classes of hydrodynamics codes used in astrophysics:
  - Grid: (*e.g., Pluto, Athena, Ramses, Flash*)
  - SPH: (*e.g., Gadget, Gasoline, Phantom*)
  - Moving Mesh: (*e.g., Arepo, Gizmo*)

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".. magnetic fields may be included without difficulty.."

Gingold & Monaghan (1977)

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\nabla\cdot\mathbf{v}$$

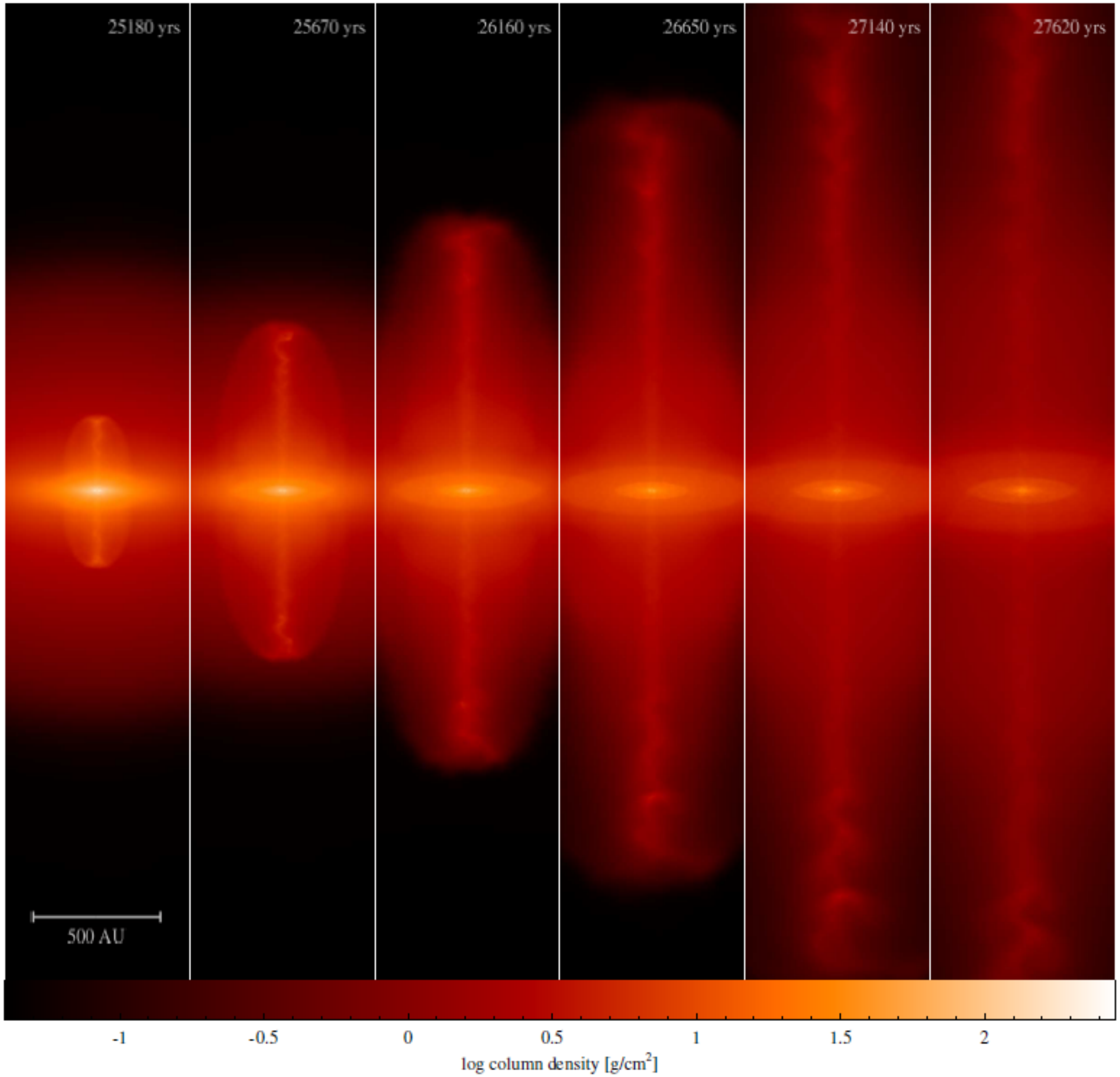
$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{1}{\rho}\nabla\left(P + \frac{B^2}{2\mu_0}\right) + \frac{1}{\mu_0\rho}\nabla\cdot\mathbf{BB}$$

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = -\mathbf{B}\left(\nabla\cdot\mathbf{v}\right) + \left(\mathbf{B}\cdot\nabla\right)\mathbf{v}$$

$$\nabla\cdot\mathbf{B} = 0$$

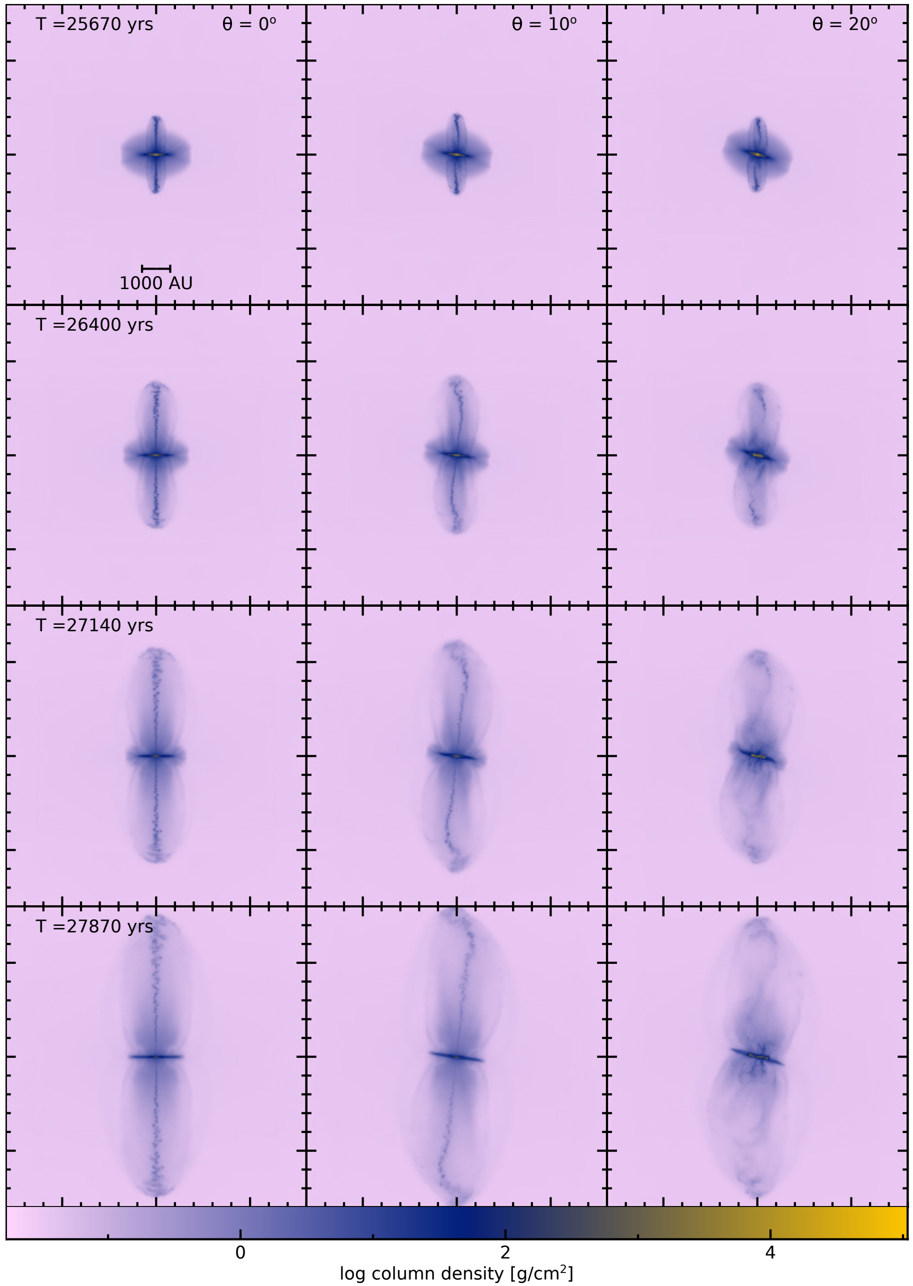
“Collimated Jets from the First Core”

Price, Tricco & Bate (2012)



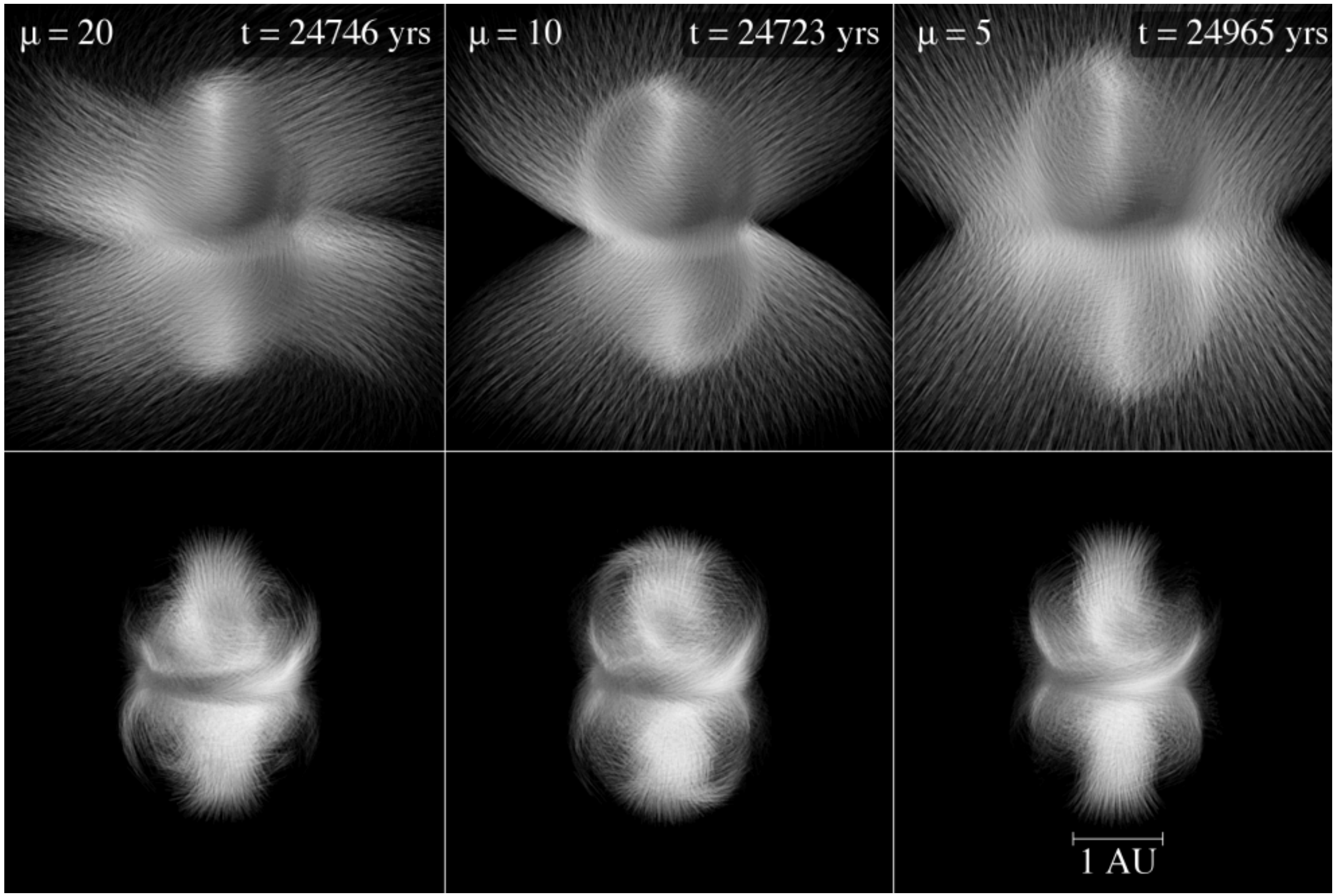
“Protostellar Outflows with Misaligned Magnetic Field and Rotation Axes”

Lewis, Bate & Price (2015)



“Collapse of a Molecular Cloud Core to Stellar Densities”

Bate, Tricco & Price (2014)



Constrained Hyperbolic Divergence Cleaning

- Couple scalar field ψ to magnetic field (Dedner et al 2002):

$$\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)_{\psi} = -\nabla\psi \qquad\qquad \frac{\mathrm{d}\psi}{\mathrm{d}t} = -c_h^2 \nabla\cdot\mathbf{B} - \frac{\psi}{\tau}$$

- Produces damped "divergence" waves:

$$\frac{\partial^2(\nabla\cdot\mathbf{B})}{\partial t^2} - c_h^2\nabla^2(\nabla\cdot\mathbf{B}) + \frac{1}{\tau}\frac{\partial(\nabla\cdot\mathbf{B})}{\partial t} = 0$$

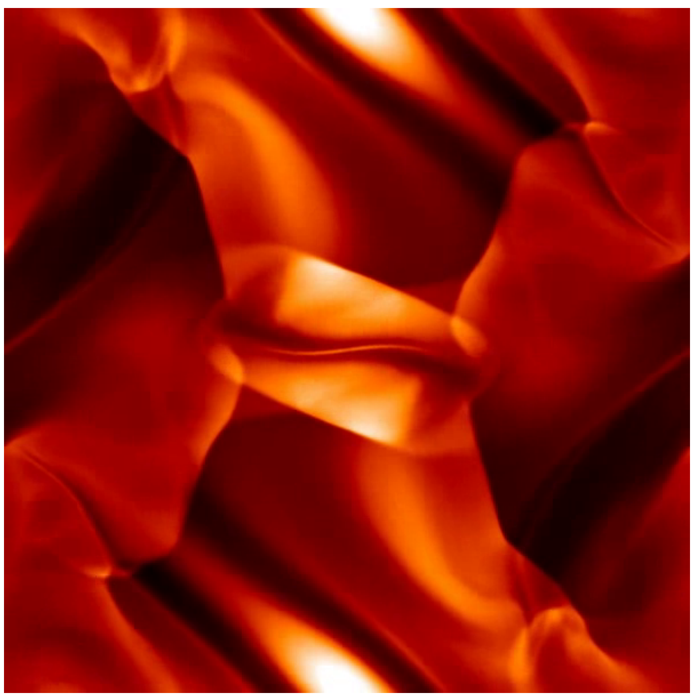
- Implementation in SPMHD:
  - Define energy of the ψ field
  - Construct cleaning equations from discretised Lagrangian,
  - Retains conservation properties inherent to SPH (i.e., *constrained*)

$$e_{\psi} \equiv \frac{\psi^2}{\mu_0 \rho c_h^2} \qquad\qquad L = \int \left(\tfrac{1}{2}\rho\mathbf{v}^2 - \rho u - \frac{\mathbf{B}^2}{2\mu_0} - \frac{\psi^2}{2\mu_0 c_h^2}\right)\mathrm{d}V$$

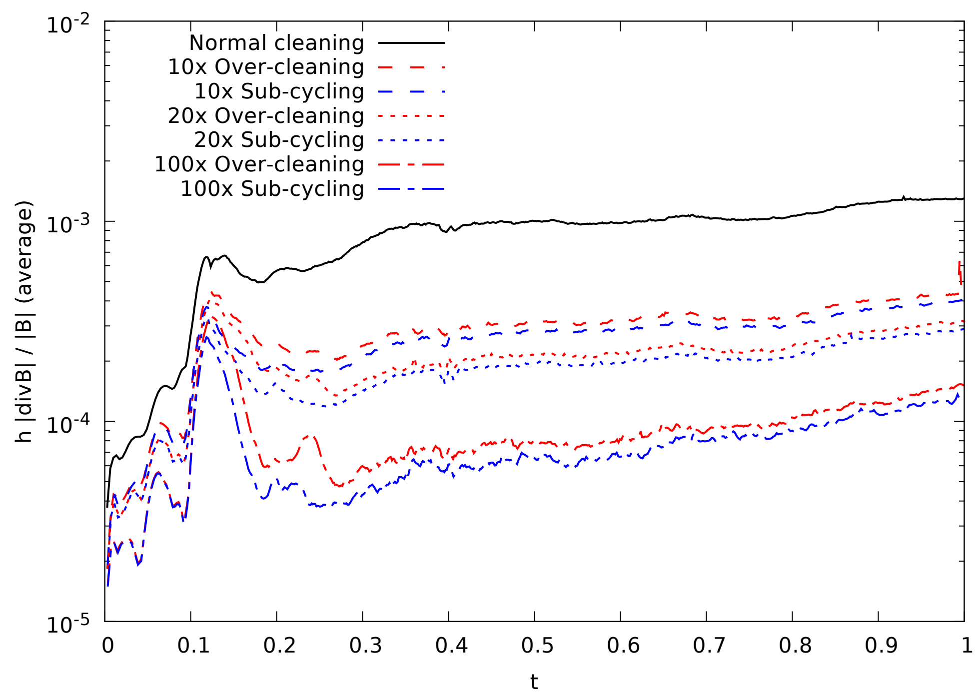
Tricco & Price (2012)

Enhancing the Divergence Cleaning

- Over-cleaning:
  - Explicitly increase cleaning speed: c_h -> 10x c_h, 20x c_h, etc
- Sub-cycling:
  - Evolve cleaning equations in isolation for a number of substeps



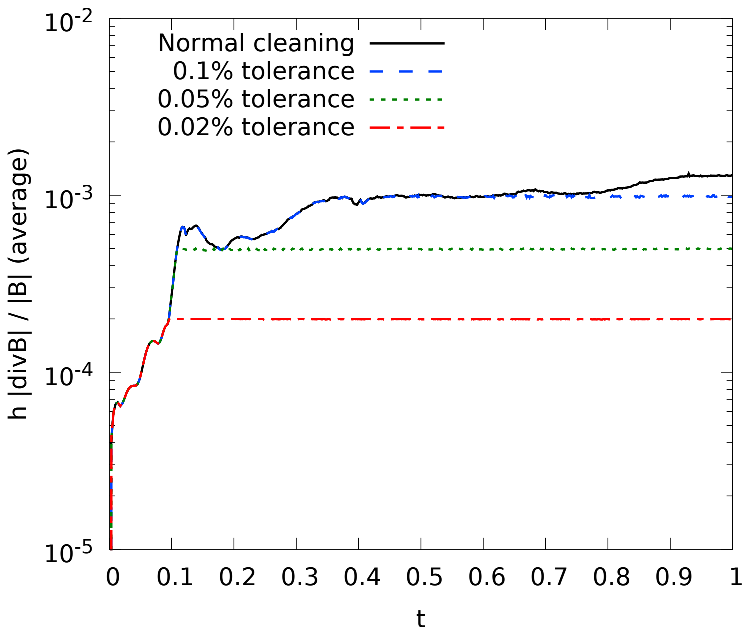
Orszag-Tang vortex



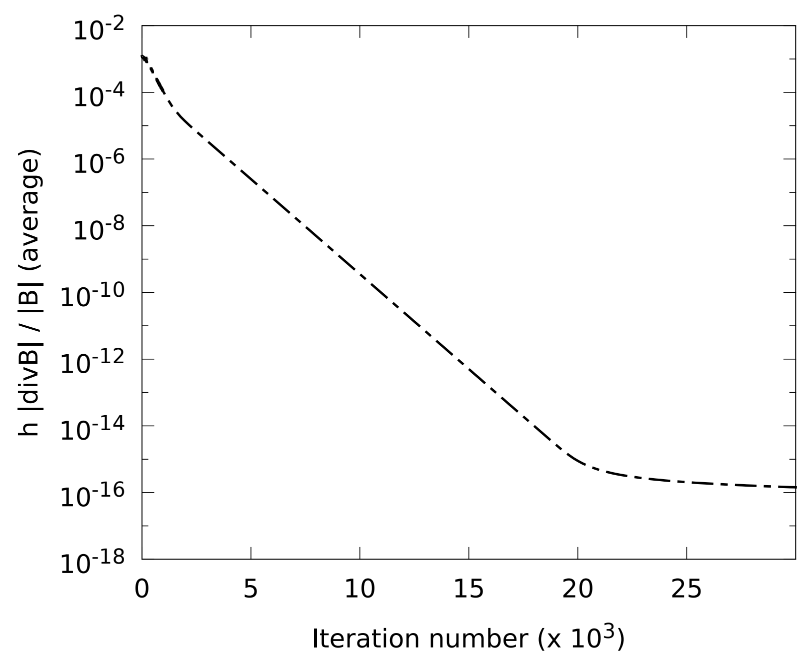
Tricco (2015) *(PhD Thesis, available on arXiv)*

Enhancing the Divergence Cleaning

- Can set a tolerance on divB using substeps:



- Possible to achieve $\nabla \cdot \mathbf{B} = 0$ in SPMHD! ... but expensive.



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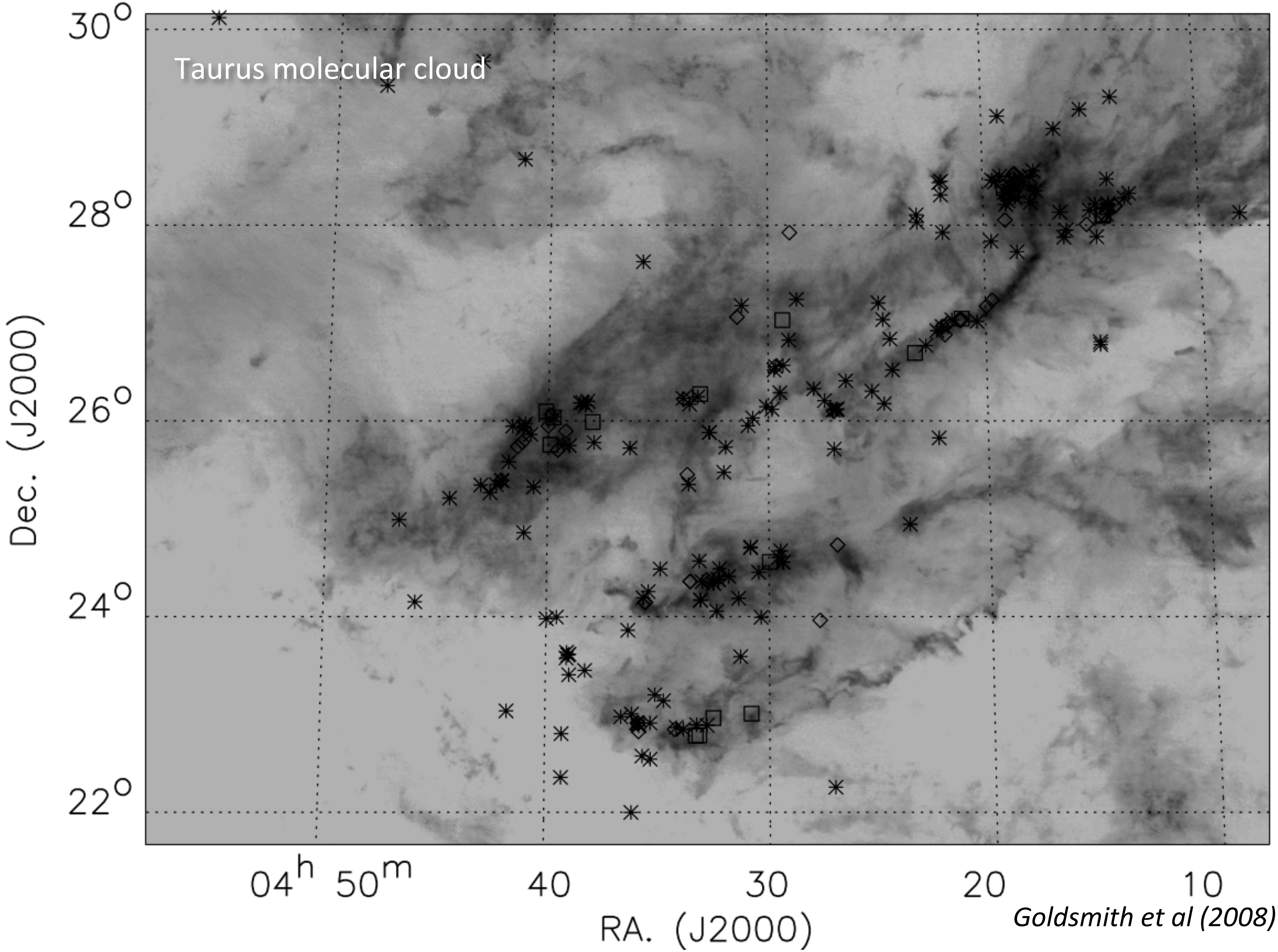
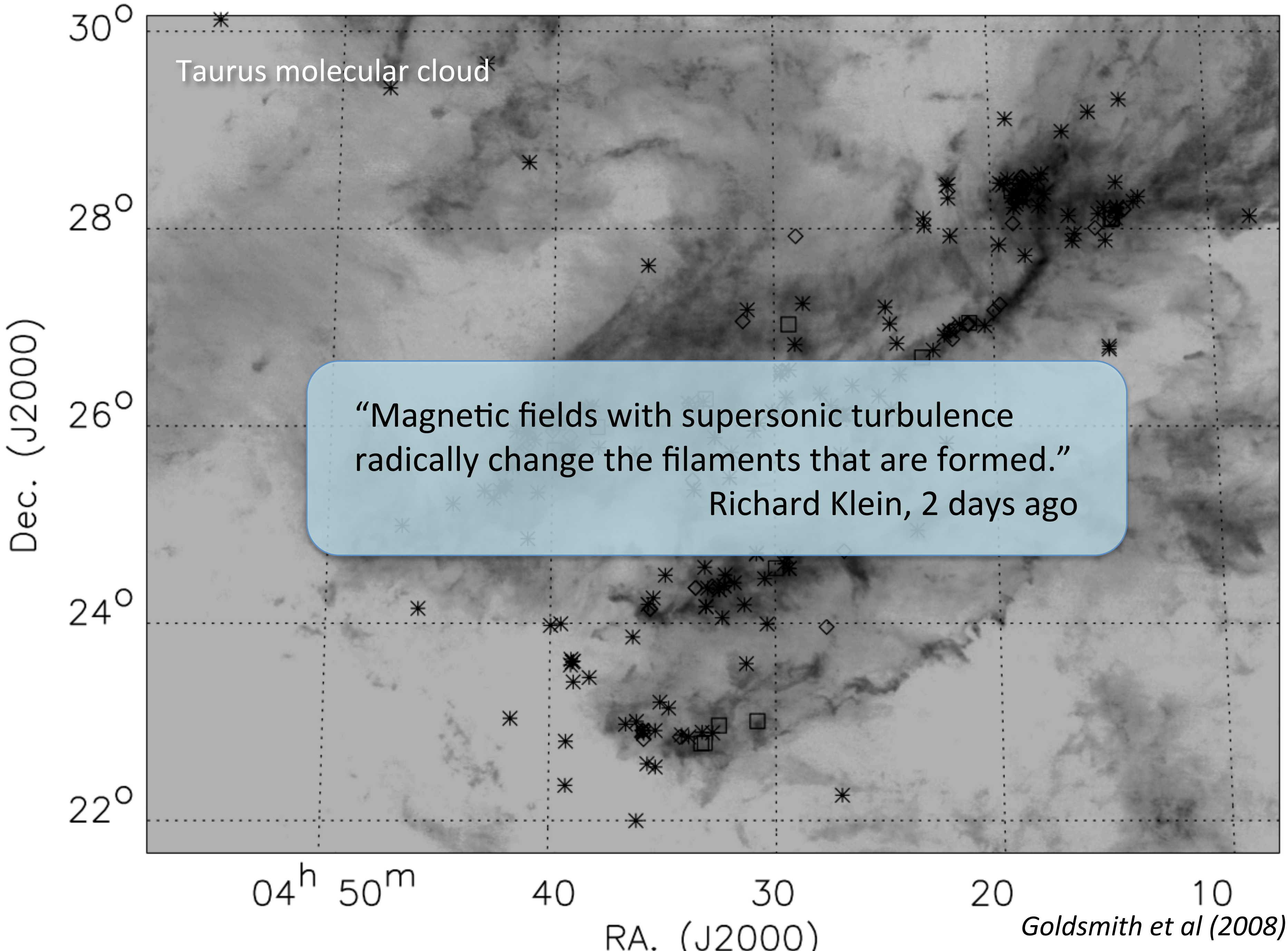


Goldsmith et al (2008)



Goldsmith et al (2008)

Magnetised `Turbulence-in-a-box’

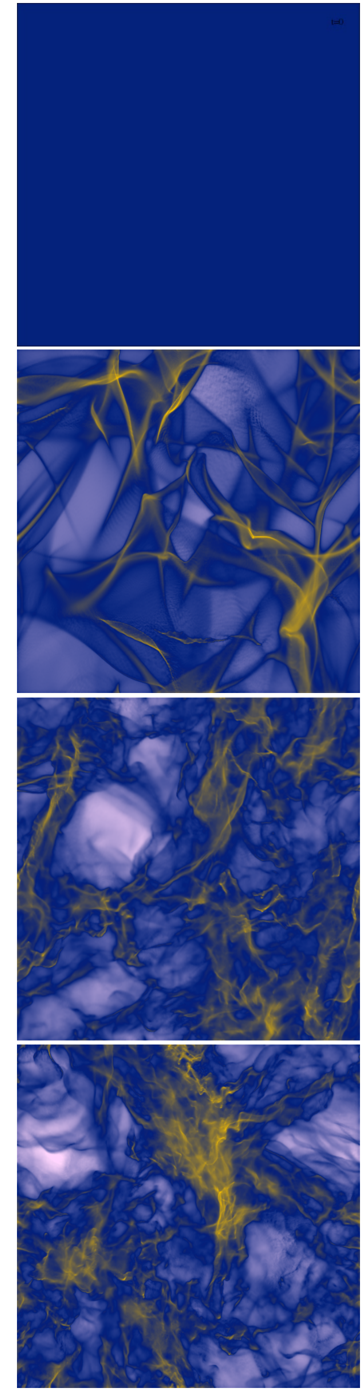
Initial conditions:

- Uniform density
- Zero velocity
- Uniform vertical magnetic field
- Magnetic energy 10^{11} weaker than mean kinetic energy (in turbulent quasi-steady state)
- Initial plasma $\beta = 10^{10}$

Simulation conditions:

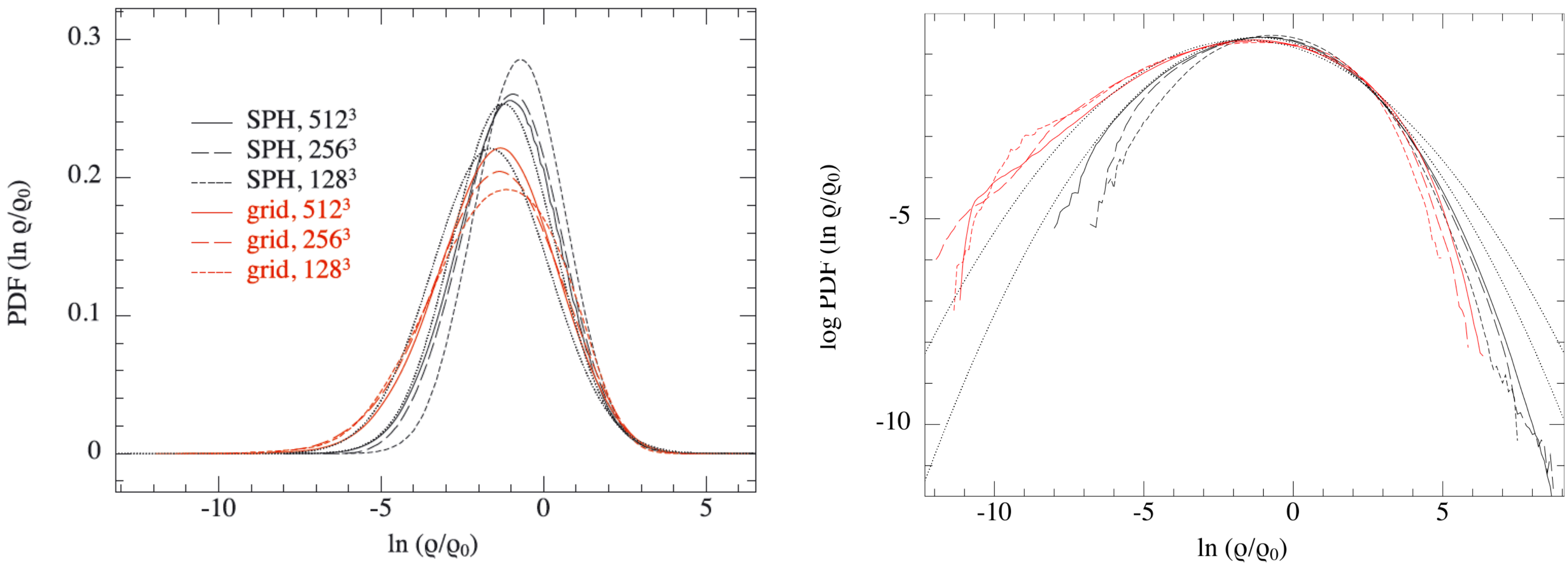
- Ornstein-Uhlenbeck stochastic solenoidal driving force at large scales (k=1-3)
- Mach 10 turbulence
- Periodic boundary conditions
- Isothermal equation of state

Run same set of simulations with Flash to compare results.



Purely Hydrodynamic Turbulence

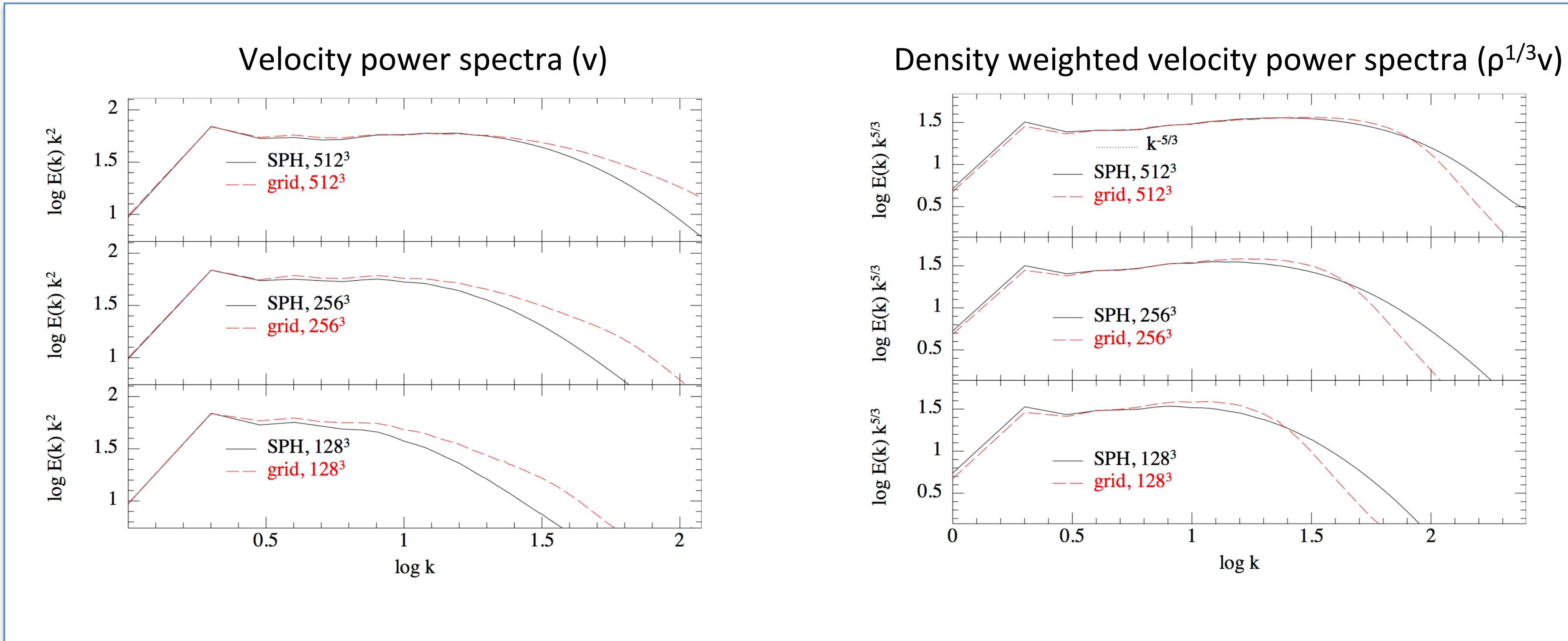
Price & Federrath (2010)



- Same data in both plots; left has linear scaling, right has log scaling
- Grid better at sampling low density, SPH better at high densities

Purely Hydrodynamic Turbulence

Price & Federrath (2010)



- Grid better at sampling volumetric statistics;
  SPH better at sampling density weighted statistics

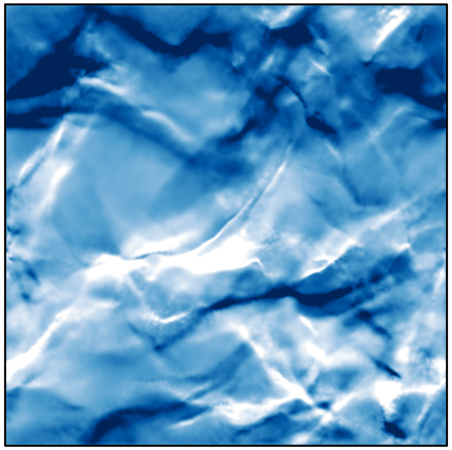
Reducing numerical dissipation

- Capture magnetic discontinuities with artificial resistivity:

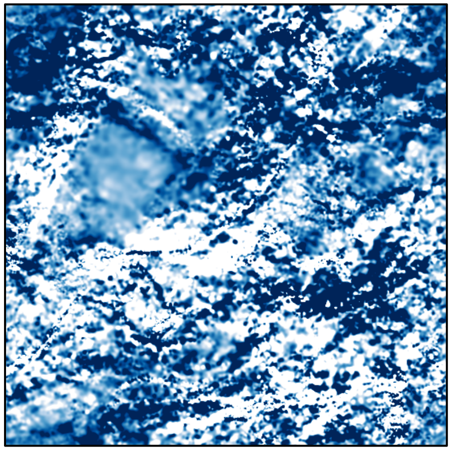
AR ~ $\eta_{\rm AR}\nabla^2\mathbf{B}$ where $\eta_{\rm AR} \sim \alpha_{\rm B} v_{\rm sig} h$

- Price & Monaghan (2005) switch: $\alpha_{\rm B} \propto \nabla\times\mathbf{B}$
- Tricco & Price (2013) switch: $\alpha_{\rm B} \propto \nabla\mathbf{B}/|B|$
  - Measure relative size of jump in magnetic field at the resolution scale
  - Can detect shocks during magnetic field amplification

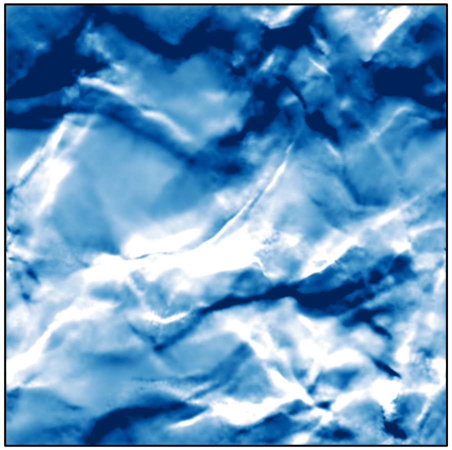
Column integrated magnetic field strength



No switch; fixed $\alpha_{\rm B}$ = 1



Previous switch

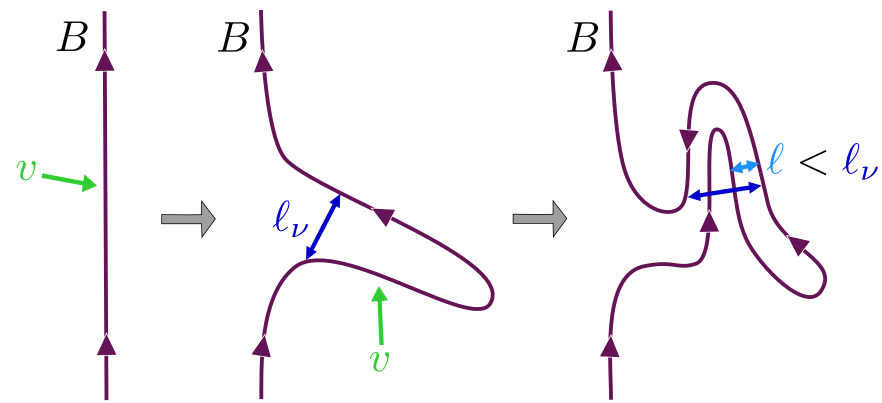


New switch

Tricco & Price (2013)

Small-scale Turbulent Dynamo

- Stretches, twists and winds the magnetic field
- Converts kinetic energy to magnetic energy on the smallest scale (dissipation scale)



(Schober et al, 2015)

- 3 phases:
  - Exponential amplification of magnetic energy
  - Slow linear amplification once magnetic energy saturated on smallest scale
  - Saturation of magnetic energy on all spatial scales

Magnetic Energy Amplification

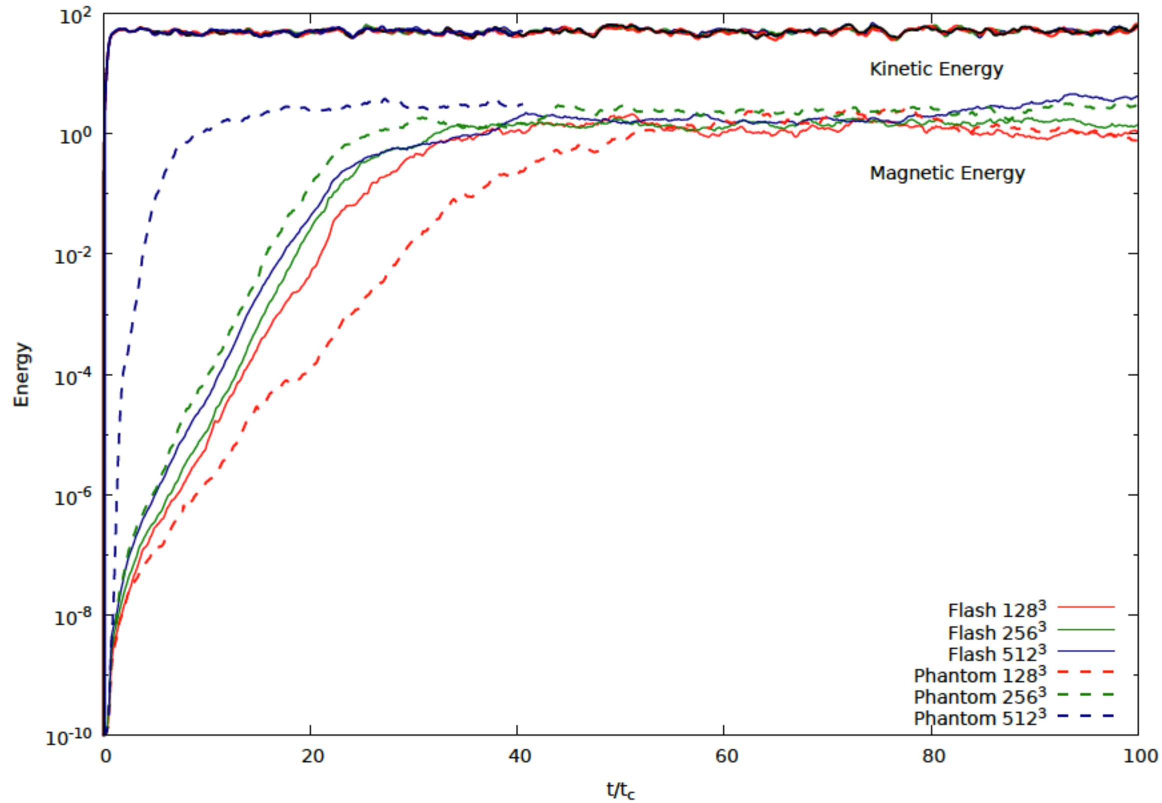


Figure 1. Growth and saturation of the magnetic energy for FLASH and PHANTOM at resolutions of 128^3, 256^3, and 512^3. The top lines are the kinetic energy for the six calculations. FLASH has similar growth rates across the resolutions simulated, while PHANTOM exhibits faster growth rates with increasing resolution. This resolution dependence is a consequence of the artificial dissipation terms. Both codes saturate the magnetic energy at similar levels.

Tricco, Price & Federrath (submitted)

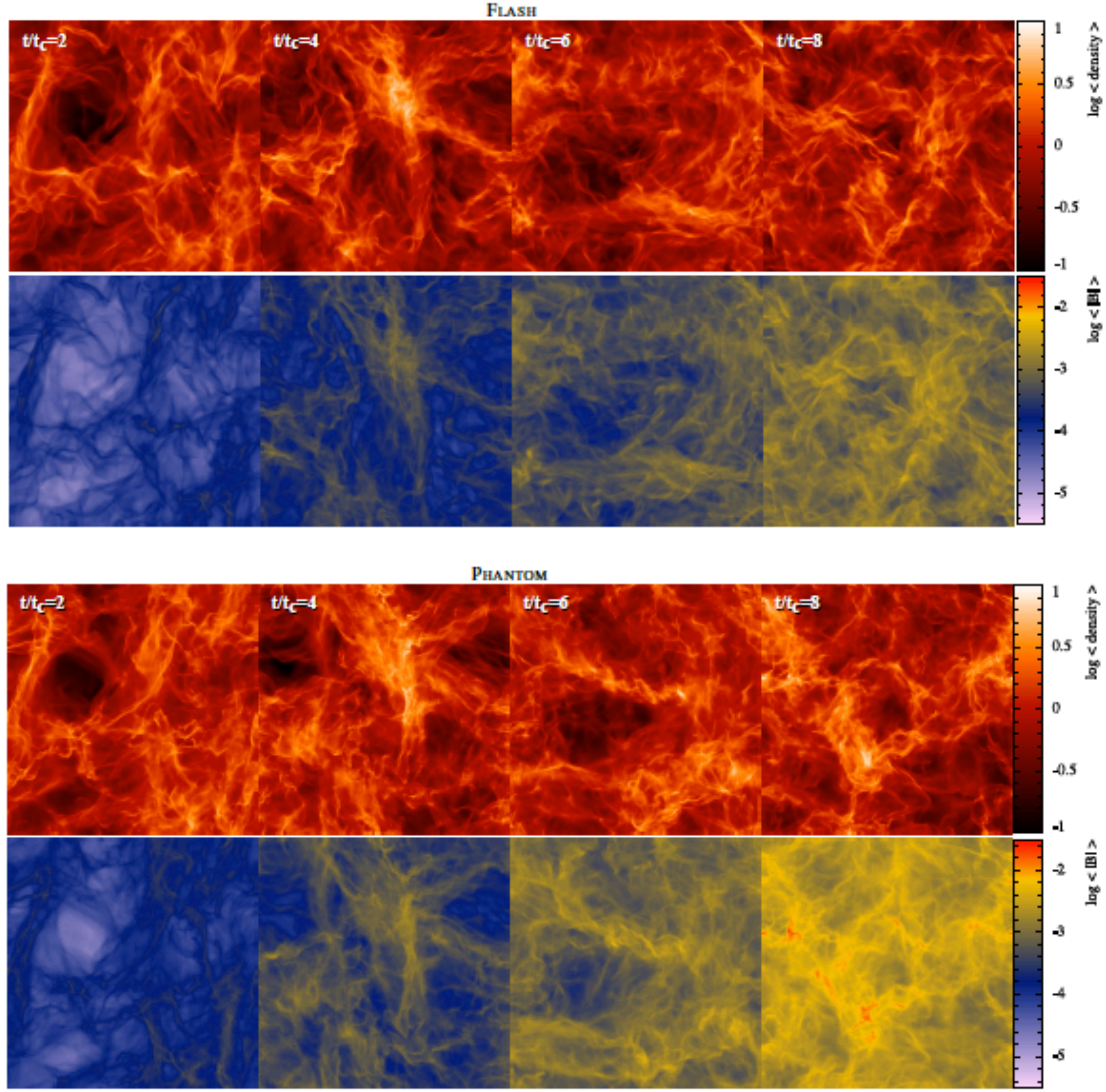
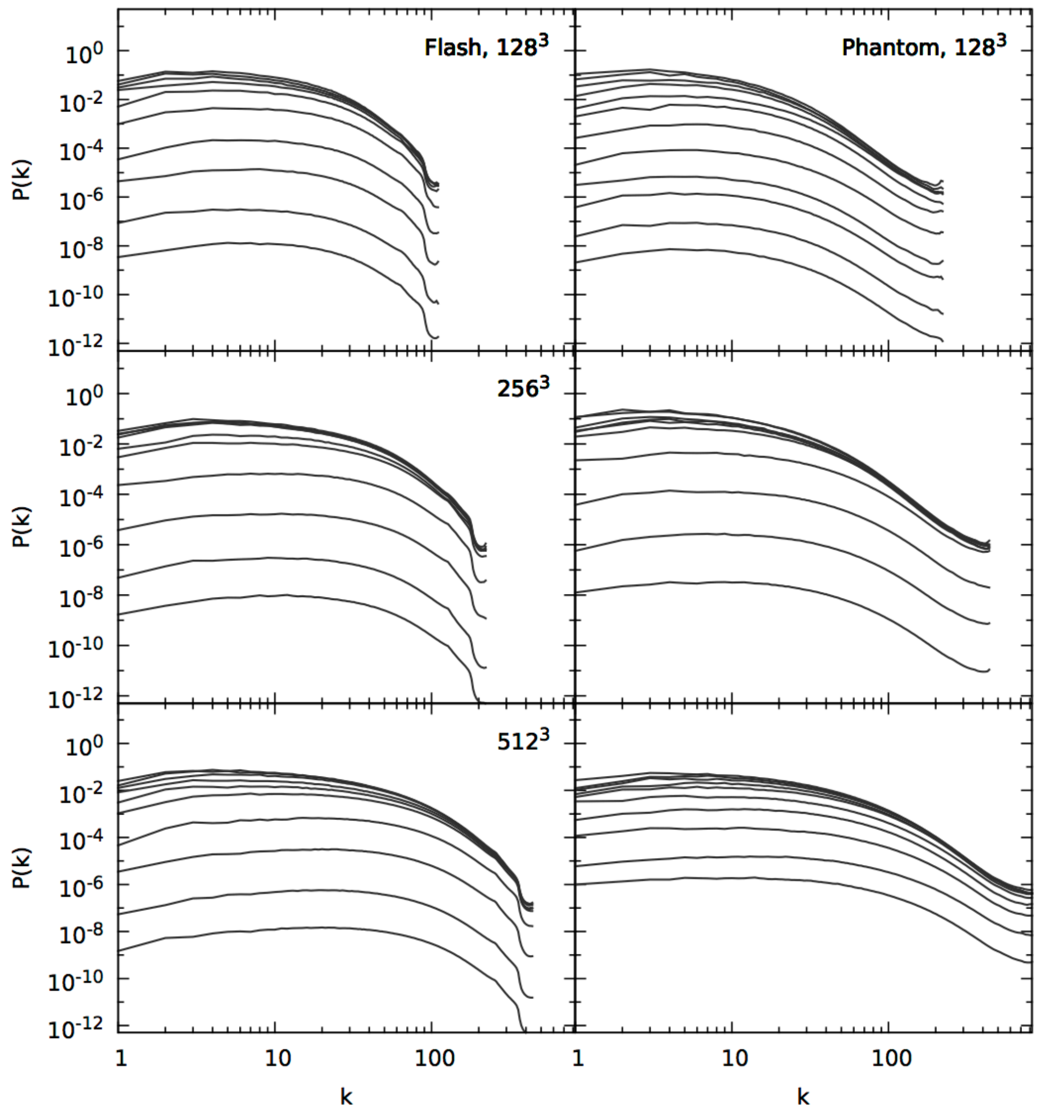


Figure 3. z-column integrated ρ and $|B|$, defined $< B >= \int |B| dz / \int dz$, for Flash (top) and Phantom (bottom) at resolutions of 256^3 for $t/t_c = 2, 4, 6, 8$. The density field has similar structure in both codes at early times, but diverge at late times due to the non-linear behaviour of the turbulence. The magnetic field is strongest in the densest regions, while the mean magnetic field strength also increases with time.

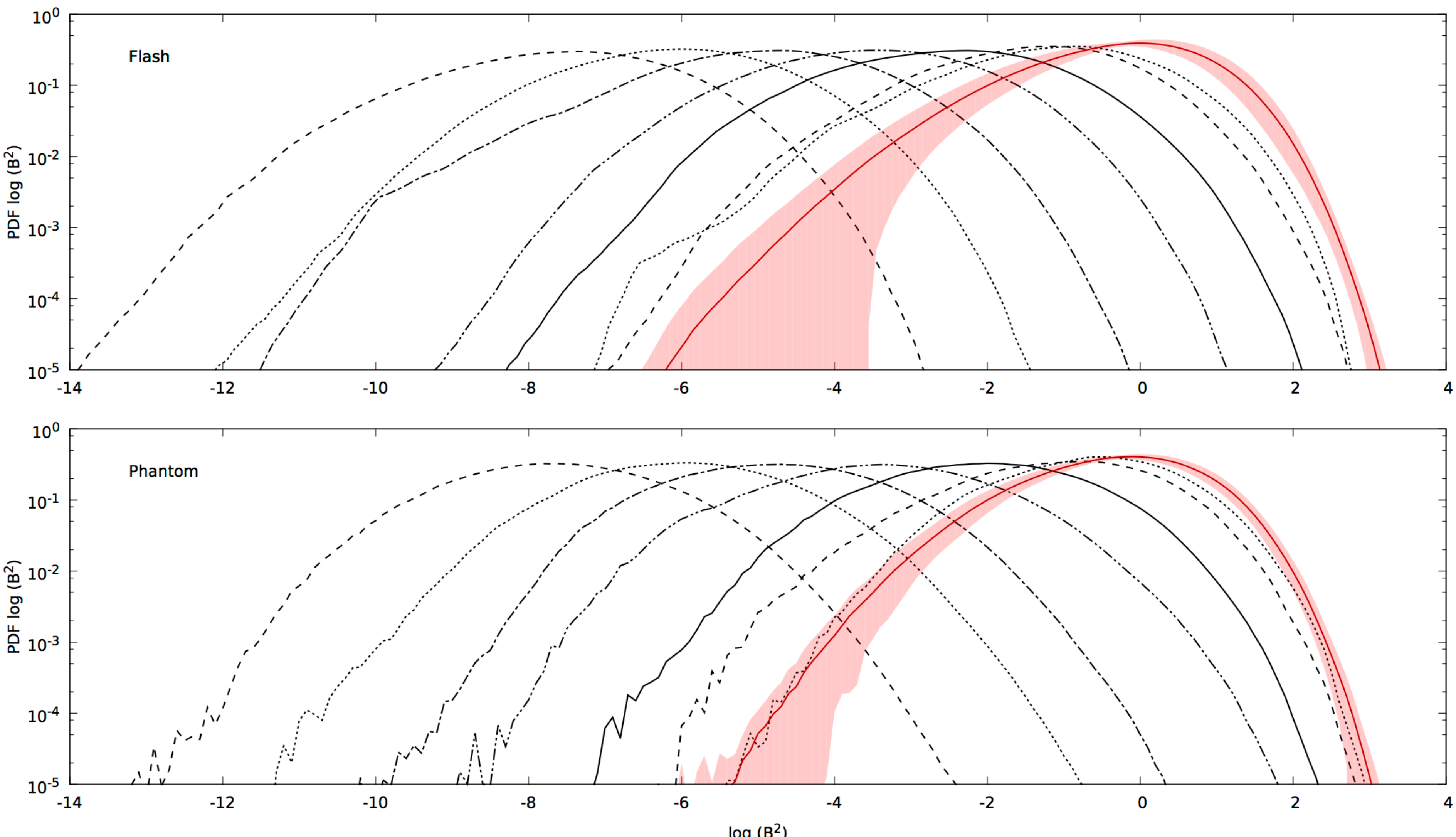
Tricco, Price & Federrath (submitted)

Magnetic Energy Power Spectra



Tricco, Price & Federrath (submitted)

Distribution of Field Strengths



Tricco, Price & Federrath (submitted)

Conclusions

- Significant improvements have been made to SPMHD:
  - "Constrained" hyperbolic divergence cleaning
  - Artificial resistivity switch to reduce numerical dissipation
- See clear signatures that SPMHD can simulate the small-scale dynamo:
  - Qualitative behaviour – steady, exponential growth of magnetic energy
  - Uniform growth of the power spectrum of the magnetic energy during exponential growth phase; saturation on smallest scales first; slow saturation on large scales
  - Normal distribution of magnetic field strengths during growth phase; lopsided distribution after saturation
  - Saturation level of magnetic energy consistent between grid and SPMHD