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Practical Strategy of Acquaintance Immunization without Contact Tracing

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***Abstract*—Preventing the spread of contagious diseases will bring many benefits to human's life and world economy. Pathogens spread among individuals through the contact network. Various immunization strategies have been proposed over the past two decades. Most of them assume that the full picture of the contact network is known and its topology will not change overtime. In this paper, we propose a localized immunization strategy without global knowledge of the network. In this strategy, a random nominator and co-nominator are obtained at first, and they nominate a nominee jointly to be vaccinated. To test the effectiveness of the proposed strategy, we conducted the experiments on both synthetic and real-world networks and used the epidemic threshold, degree exponent, and degree distribution to measure the effectiveness of the strategies. The proposed strategy improves the epidemic threshold of three different networks by a factor 2~3, and the degree exponent by 35%~80%.**

***Index Terms*—Immunization strategy, complex networks, epidemic spreading**

I. Introduction

Contagious diseases, such as COVID-19, bring a large amount of damage to human's life and world economy. Globally, as of September 8th, 2020, there have been 27,205,275 confirmed cases of COVID-19, including 890,392 deaths, reported to WHO [1]. This disease has the characteristic of long incubation period. The median incubation period is reported to be 5.1 days and some rare cases can take up to 14 days or more to develop symptoms [2], which makes it an even harder work for medical experts and staff to stop the disease from spreading.

Vaccines can be efficiently used to impede the spread of contagious disease, but usually there is no or not enough vaccine resources at the beginning of the breakout of a pandemic, especially when a new contagious disease, such as COVID-19, emerges for the first time. Thus, an efficient and practical immunization strategy is necessary.

Many researchers have studied how to perform a vaccination optimally on a network to stop epidemics [3]–[6]. Recent studies focus on how to identify the influential spreaders in the network [7]–[12]. However, these previous models either need full knowledge of the network structure, which is difficult to achieve in most cases, or can only work with static networks. In real-world scenarios, the contact network among people varies over time.

As we know, it is the latent contact network between individuals that accounts for most of the disease spread. It is observed that most social networks show a long-tail degree distribution, implying that hubs exist in these networks. In network epidemics, hubs can not only accelerate the spreading speed but also reduce the epidemic threshold, so that even a weakly infectious pathogen can cause a pandemic.

With limited vaccine and medical resources, which is particularly the case with the current COVID-19 pandemic or at the beginning of the vaccine's availability in the future, we aim to find an immunization strategy to allocate these limited vaccine and medical resources to reduce the pandemic as much as possible. It is known that an efficient strategy is to allocate limited resources to the hubs of the network [13]. This strategy not only immunizes individuals but also removes hubs and dismantles the network, reducing the remaining network's capability in spreading viruses. However, as with the current COVID-19 pandemic, the detailed social contact network of individuals is unknown, making the task of finding hubs difficult.

To solve this problem, we study the selective immunization or node removal in networks without the full knowledge of networks. The idea of selective immunization is to find and immunize the hubs, fragment the contact network and make it more difficult for the pathogen to reach other nodes in the network [14].

This paper proposes a localized immunization strategy without global information of the network, called Joint Nomination (JN) strategy. This strategy randomly selects a fraction of nodes in the network as nominators and lets each nominator nominate one of its neighbours, called co-nominator, and then selects a node from the common neighbours of the nominator

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and co-nominator to be immunized. Simulations show that our strategy has a better performance in disrupting the contact network than random immunization and acquaintance immunization [14]. Though we perform the simulation on static networks, our model can also work on dynamic networks.

II. Related work

In recent years, network dismantling has been studied extensively [15]–[17]. It seeks and removes a minimal set of nodes, leaving the network broken into small size components. This technique can improve immunization effect with limited immunization resources, but it needs full knowledge of the network and can be only used to static network.

Some researchers have focused on identifying influential spreaders or vital nodes and immunizing these nodes to improve immunization efficiency [10], [11]. It tends to calculate the spreading ranks for all the nodes in the network, which is not very efficient when facing a severe pathogen. It also requires full knowledge of the network and cannot work when the network changes frequently.

The Friendship Paradox (FP) states that our friends have more friends than ourselves on average [18]. This rule is valid in both random networks and scale-free networks. The more heterogeneous the network is, the more observable the FP is. Researchers in [14] introduce a selective immunization strategy called acquaintance immunization, which can be explained by the FP. This strategy decreases the desired vaccination threshold more effectively than random immunization does. It assumes that every node in the network only knows who its neighbours are, and randomly selects a node from its neighbours to be immunized. Acquaintance immunization is a localized immunization strategy and easy to implement in practice.

In real-world scenarios, one not only knows who his/her neighbours are but also has some knowledge about these neighbours, such as who is more active. An improvement of acquaintance immunization is proposed to incorporate this information [19]. This immunization strategy has a higher capability of disrupting the network, thus, a better immunization performance than the acquaintance immunization strategy.

A distributed network immunization strategy is proposed to cope with the network with communities [20]. Instead of using degree as the metric of measuring nodes' importance, [21] applies a computed score to measure the importance of nodes. They assume that some nodes surrounded by lower-degree neighbours are more important than those nodes with higher-degree neighbours. They use this assumption to calculate a node's score based on the node's degree and neighbours, so it is also a localized immunization strategy. However, this method needs several iterations and complicated calculation of scores for nodes, making it impractical in real cases.

III. The model

The selective immunization protocol has an advantage over the random immunization: the selective immunization can alter the topology of the contact network. The objective of the immunization protocol is to disrupt the network as much as possible. This problem can be converted into the problem of how to remove a fraction of nodes from the network, making the remaining network as homogeneous as possible, then reducing the variance of node degrees. In this section, we propose a localized immunization strategy called Joint Nomination (JN) strategy.

A. *Friend nomination strategy*

The Friend Nomination (FN) strategy is an efficient way to find nodes with relatively high degrees, being used in acquaintance immunization [14]. It is based on the Friendship Paradox (FP) saying that the average degree of a node's neighbors is higher than the average degree of a randomly chosen node. The FN strategy is localized and easy to be implemented in real cases. It consists of three steps. First, it picks a fraction, f, of nodes in the network randomly and calls these nodes *nominators*. Second, each node in the nominator set randomly nominates a node from its neighbours, generating a new set of nodes, *nominees*. The set of nominees has the same size as the set of nominators. The last step is to remove nominees from the network.

Figure 1(a) shows the process of Random Selection (RS) immunization which chooses a fraction of nodes from a network randomly and vaccinates them. An illustration of the FN strategy is shown in Figure 1(b). Instead of immunizing the randomly selected node 1, node 11 and node 19, the FN treats them as nominators and asks each of them to nominate one of its neighbours randomly as nominees (node 2, node 15 and node 20). Then these nominees will be immunized.

Consider removing one node from the network with the FN strategy. In this scenario, only one nominator needs to be selected and used to nominate one nominee. $p(k)$ denotes the probability that a node with degree k is nominated, j denotes the degree of the picked nominator, N and E denote the total number of nodes and the total number of edges, respectively. The probability of the nominator with degree j connecting with the nominee with degree k is $kj/(2E-1)$. Assuming the nominator is connected with the nominee, because the nominee from the neighbours of the nominator is selected uniformly at random, the probability that this nominee is nominated is $1/j$. For a specific node with degree k, if any node except itself in the network is selected in the first step, there is an opportunity that this node with degree k can be nominated. The probability that any node is picked uniformly at random from the network is $1/N$, we have

$$\begin{aligned} p(k) &= \sum_{N-1} \frac{1}{N} \left(\frac{kj}{2E-1} \times \frac{1}{j} \right) \\ &= \frac{N-1}{N} \times \frac{k}{2E-1}. \end{aligned} \tag{1}$$

Then consider removing a fraction f of nodes from the network. In this scenario, Nf nodes need to be removed. $p_f(k)$ denotes the probability that a node with degree k is chosen, then

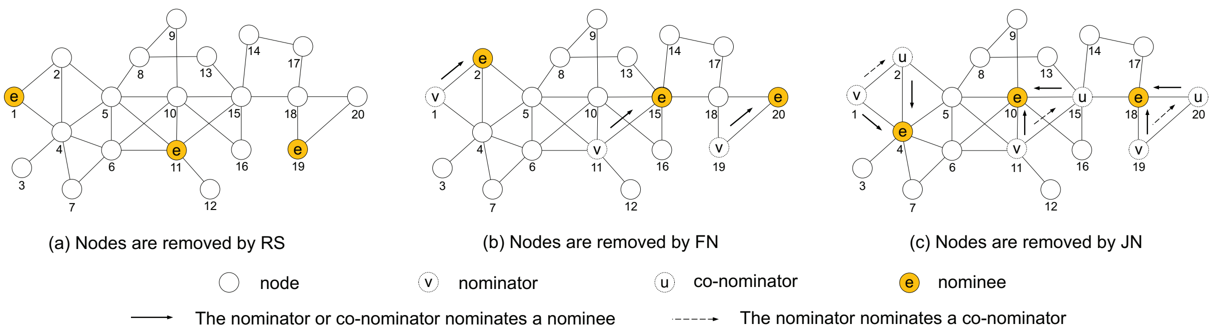

Fig. 1. Illustration of the immunization strategies

$$\begin{aligned} p_f(k) &= 1-(1-p(k))^{Nf} \\ &\approx 1-e^{-p(k)Nf} \\ &= 1-e^{-\frac{(N-1)kf}{2E-1}}. \end{aligned} \tag{2}$$

From this equation, the probability that a node is nominated with the FN is determined by its degree, k. The relation between them is a positive correlation. With a larger degree, a node is more probably nominated. This explains why the FN can reduce the heterogeneity of the network.

B. Joint nomination strategy

Inspired by acquaintance immunization, we propose a new localized network immunization strategy: Joint Nomination (JN). With the JN, some number of nodes from the network are selected as nominators randomly, and for each nominator, one *co-nominator* is chosen from its neighbours. A node is then selected from the common neighbours of a nominator and its co-nominator and this node is removed from the network. Specifically, we select a fraction, f, of individuals randomly at first and call them nominators. For each nominator, a co-nominator node is chosen. The co-nominator is selected from the neighbour nodes of the nominator randomly, called the co-nominator candidates. Then we choose one node as a nominee from the common neighbours of the nominator and its co-nominator randomly. The last step is to remove the nominee. If the nominator and its co-nominator do not have common neighbours, another co-nominator is chosen from the co-nominator candidates, continuing until a nominee is selected. If no nominee can be found, we randomly select a co-nominator candidate as a nominee to immunize.

As shown in Figure 1(c), node 1, node 11 and node 19 are selected as nominators randomly. Next, each of them nominates a co-nominator from its neighbours randomly. Here, node 2, node 15 and node 20 are co-nominators. Finally, each pair of nominator and co-nominator chooses a nominee from their common neighbours. In this figure, node 4, node 10 and node 18 will be nominated to be immunized. All of these steps are localized and random, without any knowledge of the network. The algorithm of the proposed JN strategy is briefly summarized in Algorithm 1.

Algorithm 1 Joint Nomination Strategy (JN)

Input:
Network, $\mathbf{G}$
Total number of nodes in the network, $\mathbf{N}$
Fraction of nodes to be removed, f
Output:
The remaining network after node removal, $\mathbf{G}_r$
Define a set $\mathbf{R}$ to store nodes to remove
Select $\mathbf{N}f$ nodes from the network randomly to $\mathbf{S}_v$
for each v in $\mathbf{S}_v$ **do**
  Obtain neighbours of v, $N(v)$
  Shuffle $N(v)$
  while $N(v)$ is not $\varnothing$ **do**
    Select a node from $N(v)$ randomly, u
    Obtain neighbours of u, $N(u)$
    if $N(v) \cap N(u)$ is not $\varnothing$ **then**
      Select a node from $N(v) \cap N(u)$ randomly, e
      Break
    else
      Remove u from $N(v)$
      Assign u to e
    end if
  end while
  Add e to $\mathbf{R}$
end for
$\mathbf{G}_r = \mathbf{G} - \mathbf{R}$
return $\mathbf{G}_r$

IV. Numerical simulations

In this section, we experimentally validate the effectiveness of the JN strategy on the task of reducing the heterogeneity of scale-free networks. The experiments are performed on an Ubuntu 18.04.3 LTS system with Lenovo ThinkStation, Xeon 24 cores, 64 GB RAM and a clock speed of 3.2 GHz.

A. Experiment settings

1) Datasets: We conduct numerical simulations on three datasets: one synthetic network, one real network and one induced network. Some statistical properties of them are shown in Table I.

TABLE I
PROPERTIES OF NETWORK DATASETS

Network	Nodes	Edges	Average degree	Clustering coefficient	Diameter
SYN-BAc	8,000	31,980	8	0.42	8
G-LastFM	7,624	27,806	7.3	0.22	15
G-Col	7,822	46,607	11.92	0.64	6

SYN-BAc [22]: We generate this synthetic network using the Holme and Kim algorithm. It is based on Barabasi-Albert preferential attachment model with an extra step that each random edge is followed by a chance of making an edge to one of its neighbors. This model can generate a network with a power-law degree distribution and tunable clustering coefficient. We set the $n = 8,000$, $m = 4$ and $p = 0.9$ respectively. The generated network contains 8,000 nodes and 32,000 edges. The average degree of this network is 8. The average clustering coefficient of this network is 0.42.

G-LastFM [23]: LastFM social network is a real network dataset collected from the public API in March 2020. Each node in this network represents a user. There is an edge between two nodes if they follow each other. This network contains 7,624 nodes and 27,806 edges. The average degree and the average clustering coefficient of this network are 7.3 and 0.22, respectively.

G-Col [24]: Scientific collaboration network is a real network dataset based on the arXiv preprint archive's high energy physics theory category covering the period from January 1993 to April 2003. Each node in this network represents an author. There is an edge between two nodes if they co-authored at least one paper. This network contains 23,133 nodes and 93,439 edges. We get an induced ego subgraph from this network by setting radius as 3. This induced network has 7,822 nodes and 46,607 edges. We conduct our experiments on this induced network. The average clustering coefficient of this network is 0.64.

2) Metrics of interest: We use two metrics to evaluate the performance of the proposed strategy: *epidemic threshold* and *degree exponent*. We also observe how many nodes need to be removed to achieve a specific epidemic threshold and the degree distributions of the remaining network after removing 20% nodes with different strategies.

Epidemic threshold [25]: How a virus propagates in a real network is determined by two factors: the spreading rate of the virus and the epidemic threshold of the network. The spreading rate of a virus depends on the biological characteristics of the virus. The epidemic threshold of a network reflects the capability of the network resisting a virus. We assume that the spreading rate of a virus is constant. If the epidemic threshold of a network exceeds the spreading rate of a virus, the virus will die out. Otherwise, it will spread and lead to an epidemic on the network. Thus, increasing the epidemic threshold of a network can improve the network's capability to impede a virus. The previous work [25] indicates that with an SIS model, the epidemic threshold τ for a network is

$$\tau = \frac{1}{\lambda_{\max}}, \tag{3}$$

where $\lambda_{\max}$ is the largest eigenvalue of the network's adjacency matrix. We use this formula to calculate the epidemic thresholds of networks.

Degree exponent [26]: Networks with a power-law degree distribution, $P(k) \propto k^{-\gamma}$, where k is the node degree and γ is the degree exponent, have a relatively small number of hubs which are nodes with a huge number of edges. The degree exponent, γ, reflects the fraction of hubs in the network. The larger the degree exponent for the networks with the same scale, the more homogeneous the network is and the smaller fraction of hubs exists in the network. Epidemic threshold vanishes in these scale-free networks, which means we have to vaccinate all the nodes in the networks to stop the epidemic. In order to bring the epidemic threshold back, we simulate the immunization by removing a fraction f of nodes in the network. We set f to the range of $[0\%, 20\%]$ with an interval of 2% and track the changes of degree exponent γ and degree distribution of the network.

3) Baselines: The baselines are the FN and the RS. We compare the JN with the FN and the RS in terms of the capability of increasing the epidemic threshold and the degree exponent of the network. We apply these three strategies to the network datasets mentioned above and observe the epidemic thresholds and degree exponents of the remaining networks for each f.

B. Simulation results

1) Epidemic threshold: We perform our simulation for 50 times and calculate the arithmetic mean of epidemic threshold. Figure 2 shows the simulation results of how epidemic threshold of a network changes with the increase of reduction proportion by different strategies. The two nomination strategies outperform the random selection strategy, improving the epidemic threshold by a factor 2~3, with the JN providing further improvement over the FN. The RS can hardly increase the epidemic threshold, which means it only protects the immunized individuals without much influence on the network structure property. The two nomination strategies can not only protect the immunized individuals but also bring an extra benefit of making the network more difficult for virus to spread. Our proposed JN performs better than FN in all three network datasets.

Figure 3 shows the efficiency of different strategies in increasing epidemic threshold. To achieve a certain epidemic threshold, the proposed JN always needs to remove the

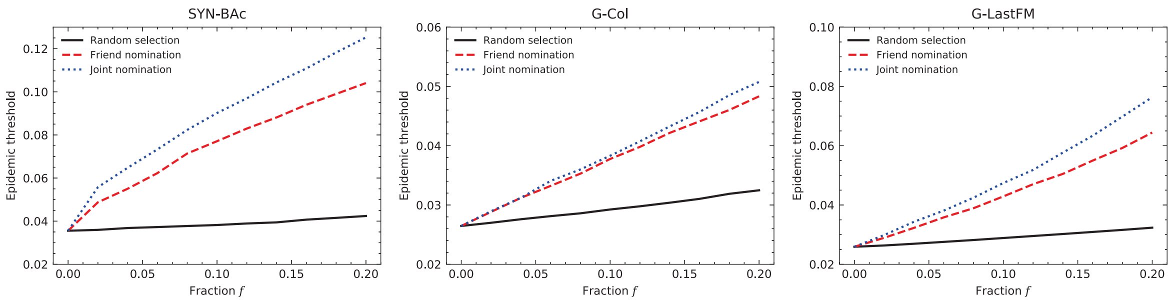

Fig. 2. The trend of epidemic threshold with different immunization strategies. Epidemic threshold for the random selection (solid, black line), friend nomination (red, dashed line) and join nomination (blue, dotted line) for the SYN-BAc (left), G-Col (center) and G-LastFM (right) networks. The nomination strategies outperform the random selection strategy, improving the epidemic threshold by a factor 2~3, with the JN providing further improvement over the FN.

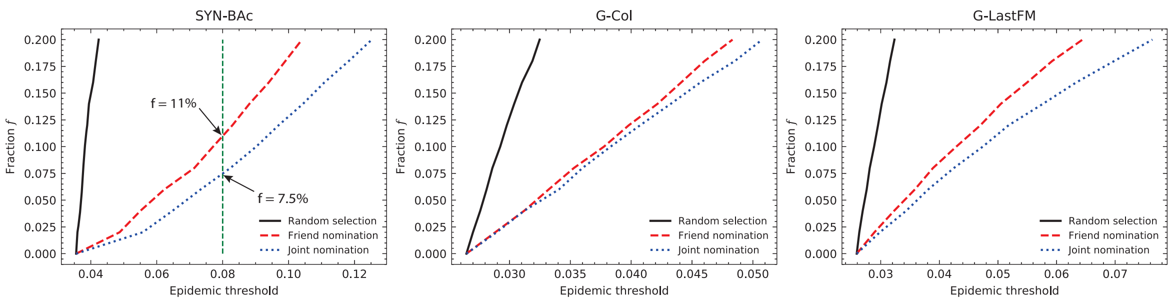

Fig. 3. The efficiency of different strategies in increasing epidemic threshold. Fraction f of nodes needed to be removed to achieve a higher epidemic threshold for the random selection (solid, black line), friend nomination (red, dashed line) and join nomination (blue, dotted line) for the SYN-BAc (left), G-Col (middle) and G-LastFM (right) networks. In the SYN-BAc network, if we aim to improve the epidemic threshold from 0.035 to 0.08, we have to immunize 11% of nodes with the FN, while only immunize 7.5% of nodes with the JN, saving 32% immunization resources.

smallest fraction of nodes, consuming the least immunization resources. In the SYN-BAc network, in order to improve the epidemic threshold from 0.035 to 0.08, we need to immunize 11% of nodes with the FN, while only immunize 7.5% of nodes with the JN, obtaining a 32% efficiency gain.

2) Degree exponent: We perform our simulation for 100 times and calculate the arithmetic mean of degree exponent. Figure 4 shows the simulation results of how the degree exponent of a network changes with the increase of reduction proportion.

The RS strategy does not change the degree exponent obviously in all the three datasets, which implies that the RS cannot reduce the heterogeneity of the network. The FN strategy contains two random processes. First, it randomly selects a certain number of nodes in the network and puts them in set 1, and then for each selected node in set 1, it selects one of its neighbours randomly and puts these nodes in set 2. Finally, it removes all nodes in set 2. By the FP theory, this method has a greater opportunity to find hubs in the network. Removing them can make the network more homogeneous and raise the degree exponent obviously. This matches with our simulation. In the SYN-BAc network, the degree exponent rises from about 3.06 to 4.8.

The JN strategy is the most effective immunization strategy among them in terms of making the network more homogeneous and less conductive when the global information of the network is unknown. For example, in the SYN-BAc network, the degree exponent goes from about 3.06 to 5.5 with the JN, almost 80% rise, better than 57% with the FN.

3) Changes of the degree distribution: The degree distribution, p_k, provides the probability that a randomly selected node in the network has degree k. Figure 5 shows the degree distributions of the remaining network after removing 20% nodes with the FN and the JN. Though both the FN and the JN can improve the probabilities of low-degree nodes and decrease that of high-degree nodes, making the network more homogeneous and less conductive, the proposed JN always performs better than the FN in all the three datasets. As we can see, some of the nodes with much higher degrees disappear, making the red dashed line and the blue dashed line discontinue in the range of high degrees. For example, the SYN-BAc network goes from a maximum of 512 to 32, almost 94% reduction. This means the network becomes quite homogeneous without large hubs.

4) Summary of the simulation results: The simulation results also show that the proposed JN strategy can improve the homogeneity of the network more efficiently than the RS and FN strategies. This can be very helpful when the JN is applied to practical immunization because the network with homogeneous degree distribution is more difficult for virus spread

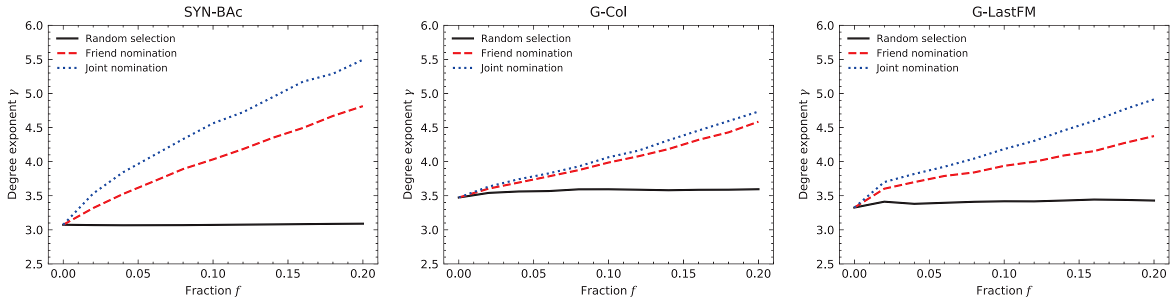

Fig. 4. The trend of degree exponent with different immunization strategies. Degree exponent for the random selection (solid, black line), friend nomination (red, dashed line) and joint nomination (blue, dotted line) for the SYN-BAc (left), G-Col (middle) and G-LastFM (right) networks. The nomination strategies outperform the random selection strategy, improving the degree exponent by a factor 35%~80%, with the JN providing further improvement over the FN.

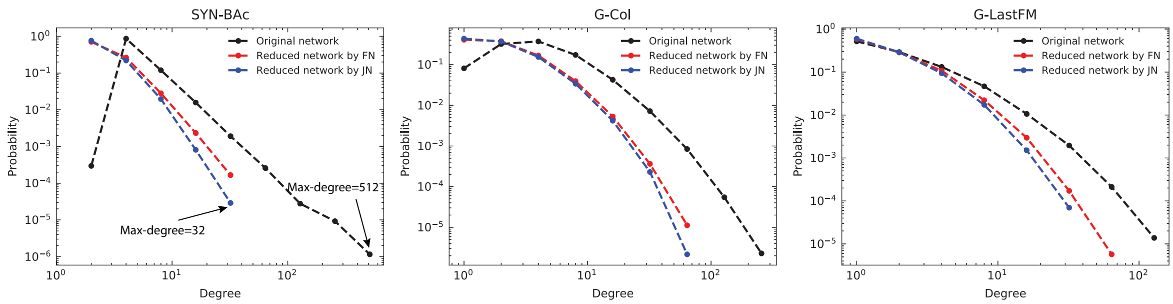

Fig. 5. Degree distributions of the remaining network after removing 20% nodes with the FN and the JN. We use log-log plot and logarithmic binning. The bin size is multiples of 2. The two nomination strategies improve the probabilities of low-degree nodes and decrease that of high-degree nodes, with the JN providing better performance over the FN. The maximum degree of SYN-BAc network goes from 512 to 32, almost 94% reduction.

than the network with power-law degree distribution. With the limited immunization resources, the JN immunization strategy will vaccinate the individuals efficiently while reducing the network's ability in spreading viruses.

V. Conclusion and Future Work

In this paper, we develop a localized immunization strategy without the global knowledge of the network. The proposed JN immunization strategy first selects a set of nodes as nominators randomly from the network. Each nominator obtains a co-nominator from its neighbours. Then each pair of nominator and co-nominator nominates a nominee from their common neighbours. Finally, the nominees are removed from the network.

To investigate the effectiveness and efficiency of the proposed strategy, we conduct experiments on both synthetic and real-world networks. We use the epidemic threshold, degree exponent, and degree distribution as metrics. The simulation results show that the proposed JN immunization strategy can raise the homogeneity of the network on a large scale, which can increase the epidemic threshold and make it more difficult for viruses to spread. Immunizing 20% nodes of the networks with the JN, the degree exponent is improved by 35%~80%, the epidemic threshold by 2~3 times, and the maximum degree is reduced by up to 94%. Like the FN strategy, the whole process of the JN strategy is random and localized, but it outperforms the FN in all the tests.

This research has opened a number of avenues for further research. We will extend the JN incorporating the biased information during the nomination process to develop a more effective localized immunization strategy. In addition, when we consider demographic information such as age and pre-conditions, we can also incorporate a "fitness" attribute to the network structure to modulate the nomination processes. Last, we would enhance joint nomination to adapt the variation of clustering coefficient in different networks or different parts of the same network.

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