The function of knowledge representation scheme is

a) to capture the essential features of problem domain, and

b) make that information accessible to a problem-solving procedure.

A representation scheme should:

a) Be adequate to express all of the necessary information.
b) Support efficient execution of the resulting code.
c) Provide a natural scheme for expressing the required knowledge.

The Predicate Calculus

2.0 Introduction

2.1 The Propositional Calculus

2.2 The Predicate Calculus

2.3 Using Inference Rules to Produce Predicate Calculus Expressions

The Propositional Calculus

- Propositional symbols denote propositions, i.e., statements about the world that may be either true or false.
- Legal sentences are called well-formed formulas or WFFs.
- Only expressions that are formed of legal symbols through some sequence of the above rules are well-formed formulas.

DEFINITION

PROPOSITIONAL CALCULUS SYMBOLS

The *symbols* of propositional calculus are the propositional symbols:

P, Q, R, S, …

truth symbols:

true, false

and connectives:

$\wedge, \vee, \neg, \rightarrow, \equiv$

DEFINITION

PROPOSITIONAL CALCULUS SENTENCES

Every propositional symbol and truth symbol is a sentence.

For example: true, P, Q, and R are sentences.

The *negation* of a sentence is a sentence.

For example: $\neg$ P and $\neg$ false are sentences.

The *conjunction*, or *and*, of two sentences is a sentence.

For example: $P \wedge \neg P$ is a sentence.

The *disjunction*, or *or*, of two sentences is a sentence.

For example: $P \vee \neg P$ is a sentence.

The *implication* of one sentence from another is a sentence.

For example: $P \rightarrow Q$ is a sentence.

The *equivalence* of two sentences is a sentence.

For example: $P \vee Q \equiv R$ is a sentence.

Legal sentences are also called *well-formed formulas* or *WFFs*.

DEFINITION

PROPOSITIONAL CALCULUS SEMANTICS

An *interpretation* of a set of propositions is the assignment of a truth value, either T or F, to each propositional symbol.

The symbol true is always assigned T, and the symbol false is assigned F.

The interpretation or truth value for sentences is determined by:

> The truth assignment of *negation*, $\neg$ P, where P is any propositional symbol, is F if the assignment to P is T, and T if the assignment to P is F.
>
> The truth assignment of *conjunction*, $\wedge$, is T only when both conjuncts have truth value T; otherwise it is F.
>
> The truth assignment of *disjunction*, $\vee$, is F only when both disjuncts have truth value F; otherwise it is T.
>
> The truth assignment of *implication*, $\rightarrow$, is F only when the premise or symbol before the implication is T and the truth value of the consequent or symbol after the implication is F; otherwise it is T.
>
> The truth assignment of *equivalence*, $\equiv$, is T only when both expressions have the same truth assignment for all possible interpretations; otherwise it is F.

For propositional expressions **P**, **Q** and **R**:

$\neg (\neg \mathbf{P}) \equiv \mathbf{P}$

$(\mathbf{P} \vee \mathbf{Q}) \equiv (\neg \mathbf{P} \rightarrow \mathbf{Q})$

the contrapositive law: $(\mathbf{P} \rightarrow \mathbf{Q}) \equiv (\neg \mathbf{Q} \rightarrow \neg \mathbf{P})$

de Morgan's law: $\neg (\mathbf{P} \vee \mathbf{Q}) \equiv (\neg \mathbf{P} \wedge \neg \mathbf{Q})$ and $\neg (\mathbf{P} \wedge \mathbf{Q}) \equiv (\neg \mathbf{P} \vee \neg \mathbf{Q})$

the commutative laws: $(\mathbf{P} \wedge \mathbf{Q}) \equiv (\mathbf{Q} \wedge \mathbf{P})$ and $(\mathbf{P} \vee \mathbf{Q}) \equiv (\mathbf{Q} \vee \mathbf{P})$

the associative law: $((\mathbf{P} \wedge \mathbf{Q}) \wedge \mathbf{R}) \equiv (\mathbf{P} \wedge (\mathbf{Q} \wedge \mathbf{R}))$

the associative law: $((\mathbf{P} \vee \mathbf{Q}) \vee \mathbf{R}) \equiv (\mathbf{P} \vee (\mathbf{Q} \vee \mathbf{R}))$

the distributive law: $\mathbf{P} \vee (\mathbf{Q} \wedge \mathbf{R}) \equiv (\mathbf{P} \vee \mathbf{Q}) \wedge (\mathbf{P} \vee \mathbf{R})$

the distributive law: $\mathbf{P} \wedge (\mathbf{Q} \vee \mathbf{R}) \equiv (\mathbf{P} \wedge \mathbf{Q}) \vee (\mathbf{P} \wedge \mathbf{R})$

Figure 2.1: Truth table for the operator $\wedge$.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Figure 2.2: Truth table demonstrating the equivalence of P, Q and related.

P	Q	$\neg P$	$\neg P \vee Q$	$P \Rightarrow Q$	$(\neg P \vee Q) = (P \Rightarrow Q)$
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

The Predicate Calculus

- It is a basic representation language.
- Its advantages include a well-defined formal semantics and sound and complete inference rules.
- It provides the way to access the components of an individual proposition.
- It allows expressions to contain variables which may refer to classes of entities.

DEFINITION

PREDICATE CALCULUS SYMBOLS

The alphabet that makes up the symbols of the predicate calculus consists of:

1. The set of letters, both upper- and lowercase, of the English alphabet.
2. The set of digits, 0, 1, …, 9.
3. The underscore, _.

Symbols in the predicate calculus begin with a letter and are followed by any sequence of these legal characters.

Legitimate characters in the alphabet of predicate calculus symbols include

a R 6 9 p _ z

Examples of characters not in the alphabet include

% @ / & " "

Legitimate predicate calculus symbols include

George fire3 tom_and_jerry bill XXXX friends_of

Examples of strings that are not legal symbols are

3jack "no blanks allowed" ab%cd ***71 duck!!!

DEFINITION

SYMBOLS and TERMS

Predicate calculus symbols include:

1. *Truth symbols* true and false (these are reserved symbols).
2. *Constant symbols* are symbol expressions having the first character lowercase.
3. *Variable symbols* are symbol expressions beginning with an uppercase character.
4. *Function symbols* are symbol expressions having the first character lowercase. Functions have an attached arity indicating the number of elements of the domain mapped onto each element of the range.

A *function expression* consists of a function constant of arity n, followed by n terms, $t_1, t_2, \ldots, t_n$, enclosed in parentheses and separated by commas.

A predicate calculus *term* is either a constant, variable, or function expression.

DEFINITION

PREDICATES and ATOMIC SENTENCES

Predicate symbols are symbols beginning with a lowercase letter.

Predicates have an associated positive integer referred to as the *arity* or "argument number" for the predicate. Predicates with the same name but different arities are considered distinct.

An atomic sentence is a predicate constant of arity n, followed by n terms, t_1, t_2, ..., t_n, enclosed in parentheses and separated by commas.

The truth values, true and false, are also atomic sentences.

DEFINITION

PREDICATE CALCULUS SENTENCES

Every atomic sentence is a sentence.

1. If s is a sentence, then so is its negation, $\neg s$.
2. If s_1 and s_2 are sentences, then so is their conjunction, $s_1 \wedge s_2$.
3. If s_1 and s_2 are sentences, then so is their disjunction, $s_1 \vee s_2$.
4. If s_1 and s_2 are sentences, then so is their implication, $s_1 \rightarrow s_2$.
5. If s_1 and s_2 are sentences, then so is their equivalence, $s_1 \equiv s_2$.
6. If X is a variable and s a sentence, then $\forall X\ s$ is a sentence.
7. If X is a variable and s a sentence, then $\exists X\ s$ is a sentence.

verify_sentence algorithm

```
function verify_sentence(expression);
begin
  case
    expression is an atomic sentence: return SUCCESS;
    expression is of the form Q X s, where Q is either ∀ or ∃, X is a variable,
          and s is an expression;
      if verify_sentence(s) returns SUCCESS
      then return SUCCESS
      else return FAIL;
    expression is of the form ¬ s:
      if verify_sentence(s) returns SUCCESS
      then return SUCCESS
      else return FAIL;
    expression is of the form s1 op s2, where op is a binary logical operator:
      if verify_sentence(s1) returns SUCCESS and
        verify_sentence(s2) returns SUCCESS
      then return SUCCESS
      else return FAIL;
    otherwise: return FAIL
  end
end.
```

DEFINITION

INTERPRETATION

Let the domain D be a nonempty set.

An *interpretation* over D is an assignment of the entities of D to each of the constant, variable, predicate, and function symbols of a predicate calculus expression, such that:

1. Each constant is assigned an element of D.
2. Each variable is assigned to a nonempty subset of D; these are the allowable substitutions for that variable.
3. Each function f of arity m is defined on m arguments of D and defines a mapping from D^m into D.
4. Each predicate p of arity n is defined on n arguments from D and defines a mapping from D^n into {T, F}.

- Predicate calculus semantics provide a formal basis for determining the truth value of well-formed expressions.
- Quantifications introduce problems in computation:

a) Exhaustive testing of all substitutions to a universally quantified variable is computationally impossible, therefore the predicate calculus is said to be undecidable.

b) Evaluating the truth of an expression containing an existentially quantified variable may be no easier than evaluating the truth of expressions containing universally quantified variables.

- First-order predicate calculus allows quantified variables to refer to objects in the domain of discourse, and not to predicate or functions.
- Almost any grammatically correct English sentence may be represented in first-order predicate calculus.
- The limitation of the predicate calculus is that it is difficult to represent possibility, time, and belief.

DEFINITION

TRUTH VALUE OF PREDICATE CALCULUS EXPRESSIONS

Assume an expression E and an interpretation I for E over a nonempty domain D. The truth value for E is determined by:

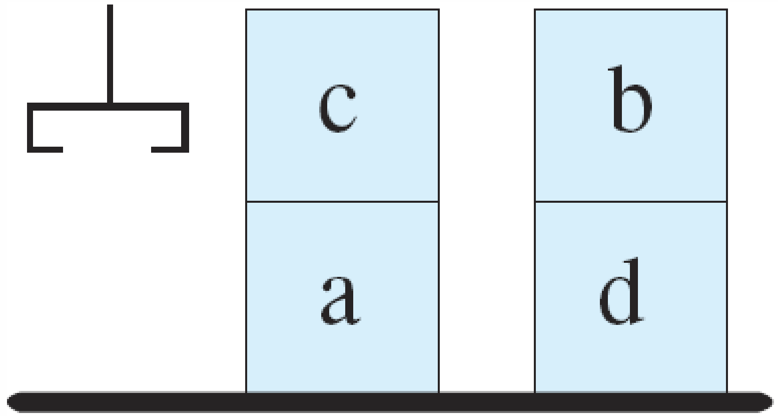
1. The value of a constant is the element of D it is assigned to by I.
2. The value of a variable is the set of elements of D it is assigned to by I.
3. The value of a function expression is that element of D obtained by evaluating the function for the parameter values assigned by the interpretation.
4. The value of truth symbol "true" is T and "false" is F.
5. The value of an atomic sentence is either T or F, as determined by the interpretation I.
6. The value of the negation of a sentence is T if the value of the sentence is F and is F if the value of the sentence is T.
7. The value of the conjunction of two sentences is T if the value of both sentences is T and is F otherwise.

8.–10. The truth value of expressions using $\vee$, $\rightarrow$, and $\equiv$ is determined from the value of their operands as defined in Section 2.1.2.

Finally, for a variable X and a sentence S containing X:

11. The value of $\forall$ X S is T if S is T for all assignments to X under I, and it is F otherwise.
12. The value of $\exists$ X S is T if there is an assignment to X in the interpretation under which S is T; otherwise it is F.

Figure 2.3: A blocks world with its predicate calculate description.



on(c,a)
on(b,d)
ontable(a)
ontable(d)
clear(b)
clear(c)
hand_empty

DEFINITION

SATISFY, MODEL, VALID, INCONSISTENT

For a predicate calculus expression X and an interpretation I:

If X has a value of T under I and a particular variable assignment, then I is said to *satisfy* X.

If I satisfies X for all variable assignments, then I is a *model* of X.

X is *satisfiable* if and only if there exist an interpretation and variable assignment that satisfy it; otherwise, it is *unsatisfiable*.

A set of expressions is *satisfiable* if and only if there exist an interpretation and variable assignment that satisfy every element.

If a set of expressions is not satisfiable, it is said to be *inconsistent*.

If X has a value T for all possible interpretations, X is said to be *valid*.

DEFINITION

PROOF PROCEDURE

A *proof procedure* is a combination of an inference rule and an algorithm for applying that rule to a set of logical expressions to generate new sentences.

We present proof procedures for the *resolution* inference rule in Chapter 12.

DEFINITION

LOGICALLY FOLLOWS, SOUND, and COMPLETE

A predicate calculus expression X *logically follows* from a set S of predicate calculus expressions if every interpretation and variable assignment that satisfies S also satisfies X.

An inference rule is *sound* if every predicate calculus expression produced by the rule from a set S of predicate calculus expressions also logically follows from S.

An inference rule is *complete* if, given a set S of predicate calculus expressions, the rule can infer every expression that logically follows from S.

DEFINITION

MODUS PONENS, MODUS TOLLENS, AND ELIMINATION, AND INTRODUCTION, and UNIVERSAL INSTANTIATION

If the sentences P and $P \rightarrow Q$ are known to be true, then *modus ponens* lets us infer Q.

Under the inference rule *modus tollens*, if $P \rightarrow Q$ is known to be true and Q is known to be false, we can infer $\neg$ P.

And elimination allows us to infer the truth of either of the conjuncts from the truth of a conjunctive sentence. For instance, $P \wedge Q$ lets us conclude P and Q are true.

And introduction lets us infer the truth of a conjunction from the truth of its conjuncts. For instance, if P and Q are true, then $P \wedge Q$ is true.

Universal instantiation states that if any universally quantified variable in a true sentence is replaced by any appropriate term from the domain, the result is a true sentence. Thus, if a is from the domain of X, $\forall$ X p(X) lets us infer p(a).

Unification

- It is an algorithm for determining the substitutions needed to make two predicate calculus expressions match.
- A variable cannot be unified with a term containing that variable. The test for it is called the occurs check.

e.g., X cannot unify with F(X, b)

DEFINITION

MOST GENERAL UNIFIER (mgu)

If s is any unifier of expressions E, and g is the most general unifier of that set of expressions, then for s applied to E there exists another unifier s′ such that Es = Egs′, where Es and Egs′ are the composition of unifiers applied to the expression E.

```
function unify(E1, E2);
  begin
    case
      both E1 and E2 are constants or the empty list:                    %recursion stops
        if E1 = E2 then return {}
          else return FAIL;
      E1 is a variable:
        if E1 occurs in E2 then return FAIL
          else return {E2/E1};
      E2 is a variable:
        if E2 occurs in E1 then return FAIL
          else return {E1/E2}
      either E1 or E2 are empty then return FAIL        %the lists are of different sizes
      otherwise:                                              %both E1 and E2 are lists
        begin
            HE1 := first element of E1;
            HE2 := first element of E2;
            SUBS1 := unify(HE1,HE2);
            if SUBS1 : = FAIL then return FAIL;
            TE1 := apply(SUBS1, rest of E1);
            TE2 : = apply (SUBS1, rest of E2);
            SUBS2 : = unify(TE1, TE2);
            if SUBS2 = FAIL then return FAIL;
                else return composition(SUBS1,SUBS2)
        end
  end                                                                   %end case
end
```

EXAMPLE: FIND MGU FOR THE FOLLOWING PAIRS OF TERMS:

1. f(g(X), W) and f(Y, V)

   The mgu is {Y=g(X), V=W}

2. f(g(X), W) and f(X, W)

   No mgu.

3. f(a, Y, g(X, Y, Z), t(Y, Z)) and f(a, W, g(b, V, a), t(c, a))

   The mgu is {V=c, W=c, X=b, Y=c, Z=a}

4. p(X, Y, t(a, b, c)) and p(g(a, Y), f(b), t(Z, b, c))

   The mgu is {X=g(a, f(b)), Y=f(b), Z=a}

Figure 2.5: Further steps in the unification of (parents X (father X) (mother bill)) and (parents bill (father bill) Y).

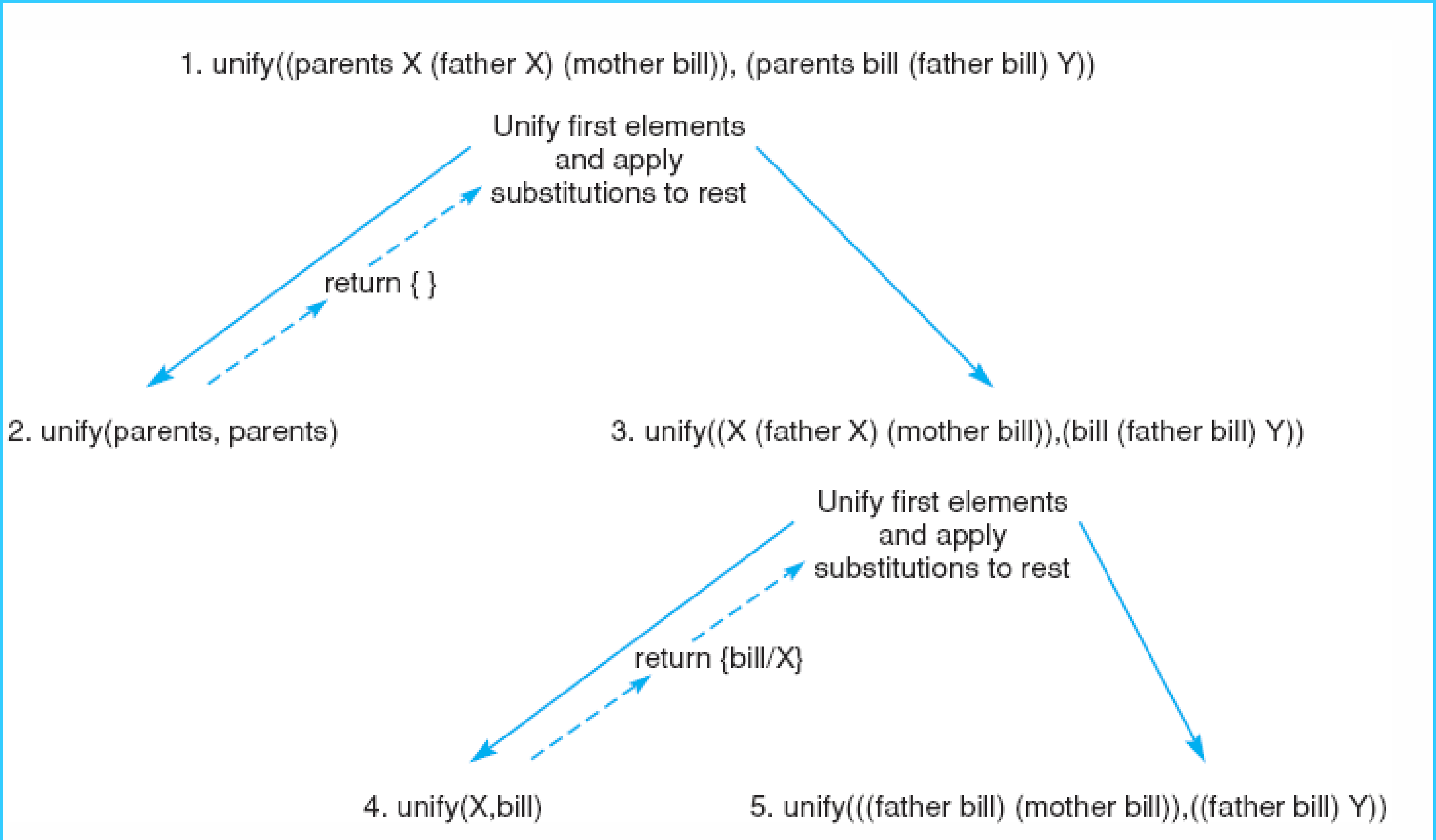
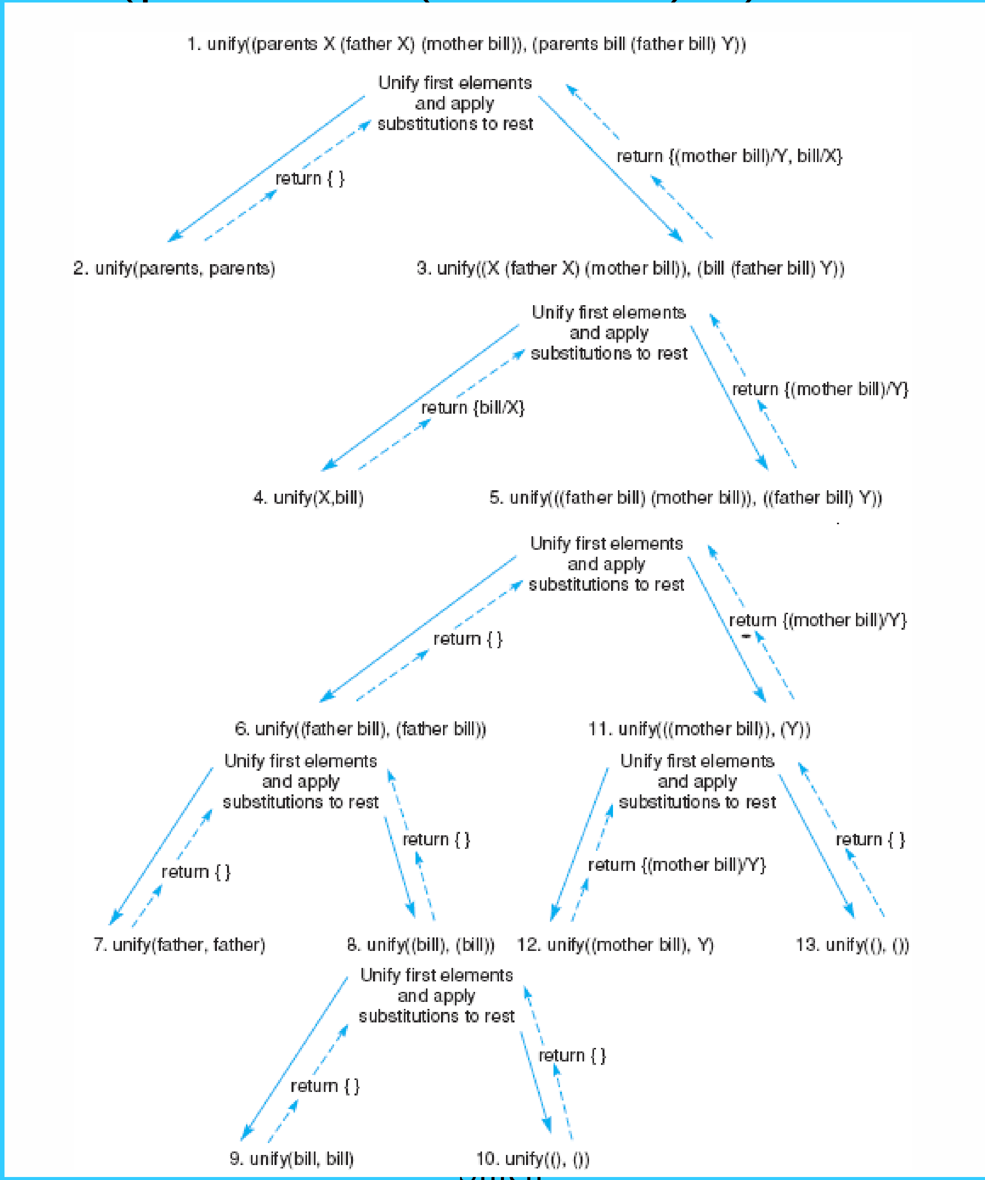


Figure 2.6: Final trace of the unification of (parents X (father X) (mother bill)) and (parents bill (father bill) Y).



A Logic-Based Financial Advisor

- Adequacy:
  - At least $5000 in the bank for each dependent
  - A steady income: at least $15,000/year + $4,000/dependent
- Investment:
  - Saving account
  - Stocks
  - blend

1. $savings\_account(inadequate) \rightarrow investment(savings).$
2. $savings\_account(adequate) \wedge income(adequate) \rightarrow investment(stocks).$
3. $savings\_account(adequate) \wedge income(inadequate)$
   $\rightarrow investment(combination).$
4. $\forall\ amount\_saved(X) \wedge \exists\ Y\ (dependents(Y) \wedge$
   $greater(X, minsavings(Y))) \rightarrow savings\_account(adequate).$
5. $\forall\ X\ amount\_saved(X) \wedge \exists\ Y\ (dependents(Y) \wedge$
   $\neg\ greater(X, minsavings(Y))) \rightarrow savings\_account(inadequate).$
6. $\forall\ X\ earnings(X, steady) \wedge \exists\ Y\ (dependents\ (Y) \wedge$
   $greater(X, minincome(Y))) \rightarrow income(adequate).$
7. $\forall\ X\ earnings(X, steady) \wedge \exists\ Y\ (dependents(Y) \wedge$
   $\neg\ greater(X, minincome(Y))) \rightarrow income(inadequate).$
8. $\forall\ X\ earnings(X, unsteady) \rightarrow income(inadequate).$
9. $amount\_saved(22000).$
10. $earnings(25000, steady).$
11. $dependents(3).$