

Systolic Machine

Example Linear Systolic Machine
Calculates point-wise sum of products

$$\text{ie. } W = W_3, W_2, W_1, W_0$$

$$X = X_3, X_2, X_1, X_0$$

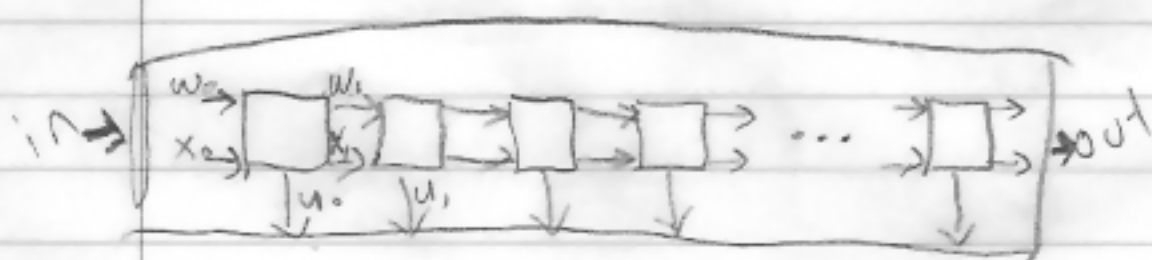
We want to calculate

$$W_0 X_0$$

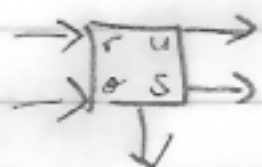
$$W_0 X_0 + W_1 X_1$$

$$W_0 X_0 + W_1 X_1 + W_2 X_2$$

In general, $\sum_{i=0}^k W_i X_i$, for k .



Where a module



Def'n of a Module

$$\left. \begin{aligned} u(t+1) &= r(t) \\ s(t+1) &= p(t) \end{aligned} \right\} \text{"flow through"}$$

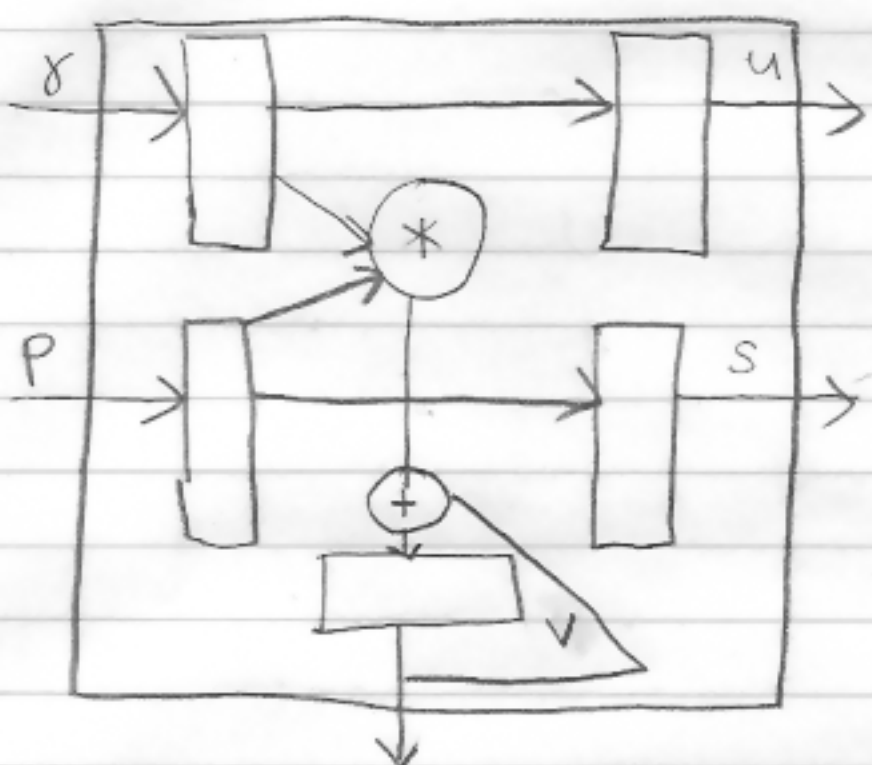
$$o(t+1) = u(t) + r(t) * p(t)$$

Difference between Systolic Array and Data Pipeline,

① In Data Pipeline the stages are different. But in systolic array, all the "stages" (modules) are same.

- ③ In Data Pipelining, stages do not have memory but systolic array modules have internal memory.

Circuit Design of a Module

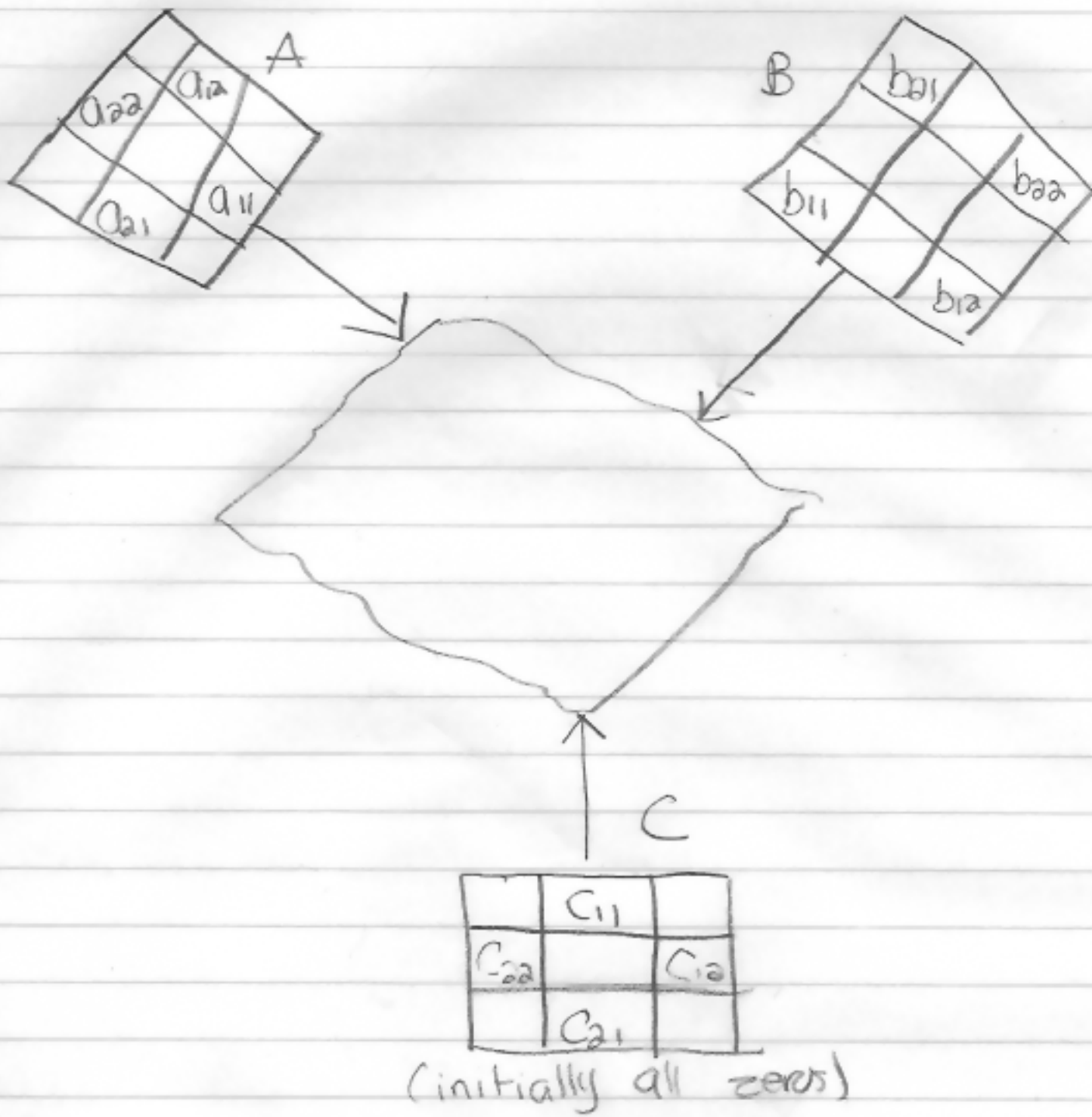


Matrix Multiplication

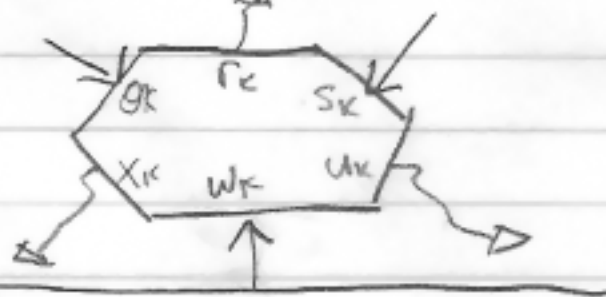
$$\text{Let } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$\text{Obtain } C = A * B = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$\begin{aligned} \text{Where } c_{11} &= a_{11}b_{11} + a_{12}b_{21} \\ c_{21} &= a_{21}b_{11} + a_{22}b_{21} \\ c_{12} &= a_{11}b_{12} + a_{12}b_{22} \\ c_{22} &= a_{21}b_{12} + a_{22}b_{22} \end{aligned}$$



Each Cell



$$\left. \begin{aligned} x_k &= s_{k-1} \\ v_k &= y_{k-1} \end{aligned} \right\} \text{Flow through}$$

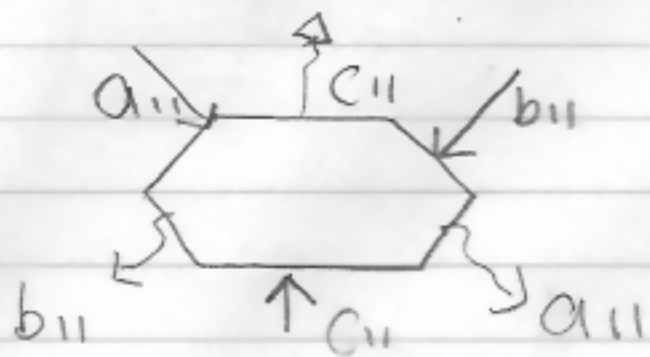
$$r_k = w_{k-1} + y_{k-1} * s_{k-1}$$



Example 2

Note that $w_k = \gamma_{k-1}$

At Time t_2



$$c_{11} = \hat{c}_{11} + a_{11} b_{11}$$

At time t_3

