Computer Science 3711
Winter 2003

Final Exam

April 16, 2003

Instructor:
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NAME: ________________________ STUDENT ID #: ______________

- This exam will be 120 minutes long.
- This exam has 11 pages (including this cover page). There are 5 questions, most of which have multiple parts.
- Answer all questions in the space provided on this exam; if you find it necessary to continue an answer on the back of a sheet of paper, that is fine, but please make a note on the front side, *e.g.*, "answer cont'd on back".

Question		Mark
1.	24	
2.	16	
3.	20	
4.	40	
5.	20	
	120	

1. (24 marks)

a) (18 marks) Circle the letters associated with the appropriate answers in the following multiple choice questions. Note that some of these questions may have more than one answer whose letter needs to be circled.

i) (6 marks) Which of the following is true of the function $T(n) = 3^{n+c}$ where c is an integer constant greater than zero?

a) $T(n)$ is $O(2^n)$

b) $T(n)$ is $O(c^n)$

c) $T(n)$ is $\Omega(n^c)$

d) $T(n)$ is $\Theta(3^n)$

ii) (6 marks) Which of the following is true of the function $T(n) = n^3 + 6$?

a) $T(n)$ is $O(5^{\log_2 n})$

b) $T(n)$ is $\Theta(n^3)$

c) $T(n)$ is $\Omega(n^4)$

d) $T(n)$ is $\Omega(9^{\log_4 n})$

iii) (6 marks) Which of the following is true of the function $T(G) = |V| \log_2 |E|$ relative to graphs $G = (V, E)$?

a) $T(G)$ is $\Theta(|V| \log_2 |E|^2)$

b) $T(G)$ is $\Omega(|E| \log_2 |V|)$ for complete graphs

c) $T(G)$ is $O(|E| \log_2 |V|)$ for tree graphs

d) $T(G)$ is $O(|V| \log_2 \sqrt{|V|})$ for complete graphs

b) (6 marks) Give the recurrence for the time complexity of the following recursive algorithm:

```
procedure FUNKY-REC(m, n)
    if (n <= 3)
        mult = 1
        for (i = 1; i <= n; i++)
            for (j = 1; j <= m; j++)
                mult = mult * FUNKY-ITR(log2(m))
        return(mult)
    else if (n > 3)
        mult = 1
        for i = 1 to m/3 do
            l = FUNKY-REC(m, n/4)
            mult = FUNKY-ITR(m * m) * l
        return(mult)
    else
        return(m)
```

Assume that procedure `FUNKY-ITR(x)` runs in $O(x^3)$ time.

2. (16 marks)

Recall from Assignment #3 that the minimum free energy associated with a secondary structure for a given RNA sequence s is defined by the recurrence

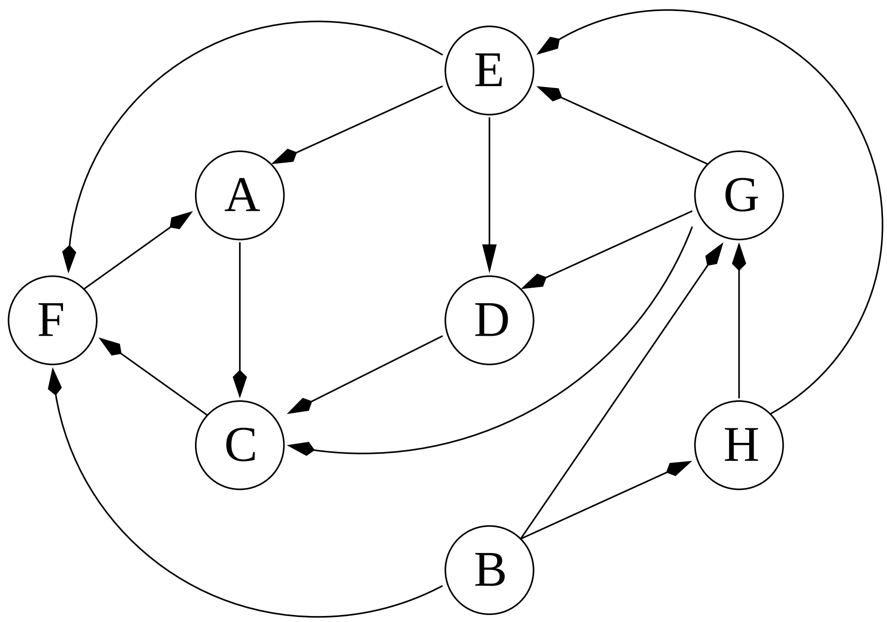
$$E[i,j] = \begin{cases} 0 & j = i \text{ or } j = i+1 \\ \min \begin{cases} E[i,j] + BFE(s(i), s(j)), \\ \min_{1 \leq k < j} E[i,k] + E[k+1,j] \end{cases} & \text{otherwise} \end{cases}$$

where $s(i)$ is the ith base in s and $BFE(x, y)$ is the base-pair free energy for RNA bases x and y. Let $BFE(G,C) = BFE(C,G) = -2$, $BFE(A,U) = BFE(U,A) = -1$, and all other BFE-entries be 1. Compute a minimum free energy secondary structure for RNA sequence $s =$ **GACAUC** using the recurrence given above – that is, fill in the dynamic programming matrix given below (showing all matrix-cell backpointers for each cell), indicate one of the backpointer-trees that gives a minimum free energy secondary structure for s, and give the secondary structure corresponding to that tree.

	1	2	3	4	5	6		
j	G	A	C	A	U	C		
							G	1
							A	2
							C	3
							A	4
							U	5
							C	6
G	A	C	A	U	C		*i*	

3. (20 marks)

a) (8 marks) Consider the following directed graph:



Give the graph at the end of the execution of the DFS algorithm, with the d- and f-values of all vertices as well as the types of all edges clearly marked. Assume that the algorithm considers vertices in alphabetical order and that each adjacency list is ordered alphabetically.

b) (2 marks) Give the topological sort for the graph in Part (a).

c) (10 marks) Run Dijkstra's algorithm on the directed graph in Table 1 (see next page) using vertex v as the source vertex. Show the d and π values and the vertices in set S after each iteration of the **while** loop by filling in the appropriate values on the spare copies of the graph given in this table. Did Dijkstra's algorithm correctly compute the shortest-path distances from y to every other vertex in this graph? If not, why not?

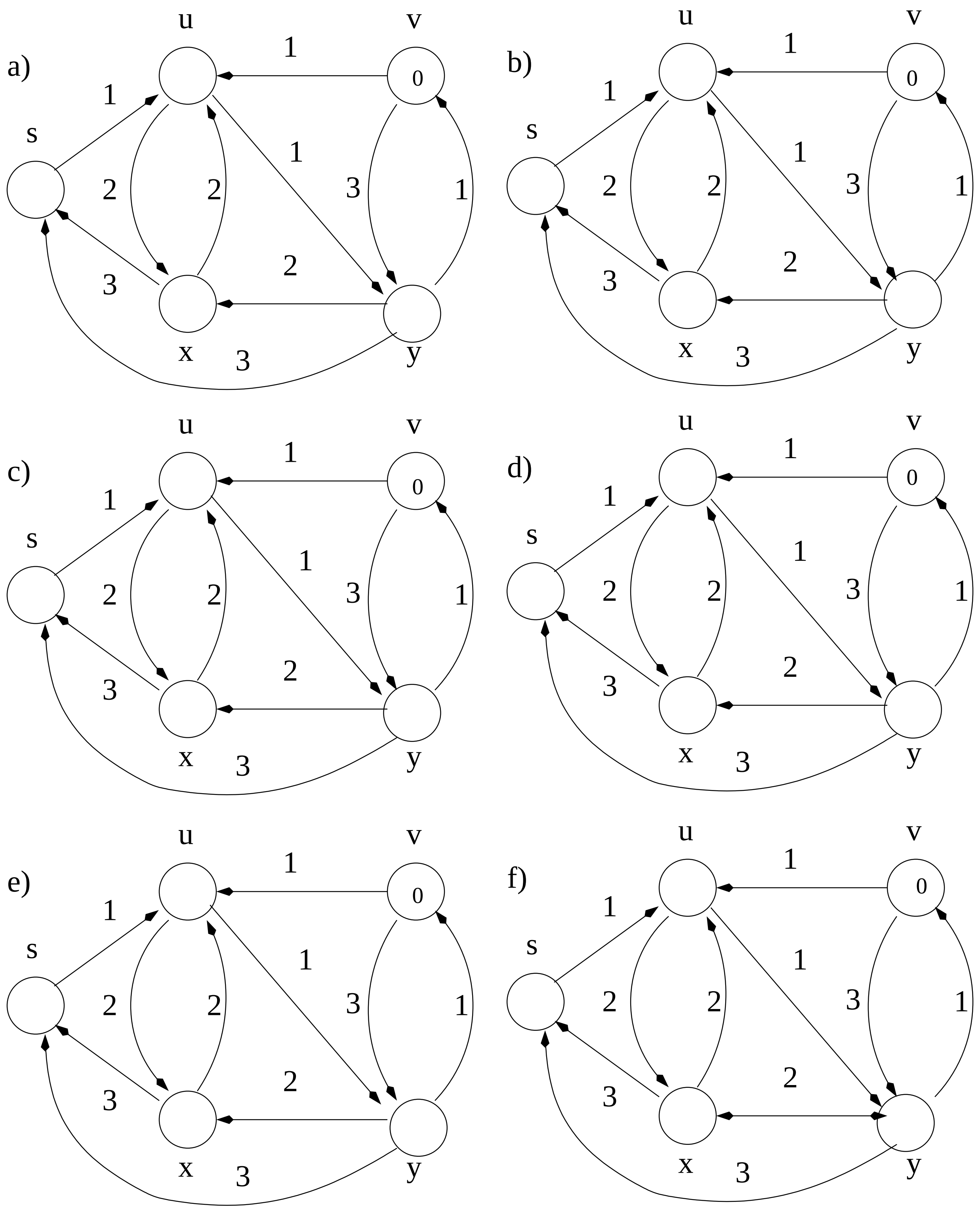


Table 1: Answer for Question #3(c).

4. (40 marks)

a) (20 marks) Consider the following problem that has recently arisen at Bob's Trucking Company: Bob has gone an extended stress leave, and in his absence, a committee of three division-heads will run the company. Each division-head is based at a particular location on the route-map serviced by Bob's Trucking Company, where this route map consists of n depots connected by a web of m roads, each of which is at most B miles long. A new committee is chosen at the end of each week. In the interests of maintaining order and sanity, it has been decided that Gilroy should always be located as far as possible from the division-heads currently on the committee.

Give pseudocode for a polynomial-time algorithm that solves the problem above – namely, given an edge-weighted undirected graph $G = (V, E, w)$ representing the route-map of Bob's Trucking Company and a selection of three vertices $x, y, z \in V$ representing the depots at which the current division-heads on the committee reside, compute one of the depots $d \in V - \{x, y, z\}$ such that $\min_{v \in \{x,y,z\}} sp(v, d)$ is the largest possible, where $sp(x, y)$ is the shortest (in terms of summed edge-weight) simple path between x and y in G. Please give the asymptotic worst-case time complexity of your algorithm.

b) (20 marks) Given an undirected graph $G = (V, E)$, the **components** of G are the maximal connected subgraphs of G – that is, each component of G is a subset $V' \subseteq V$ such that each pair of vertices in V' has a path between them in G and no vertex outside of V' is reachable by a path in G from some vertex in V'. A set of vertices $V' \subseteq V$, $|V'| = 5$, is a k-**pentacle**, $k > 0$, if all five vertices of V' are in the same component of G and for all $x, y \in V'$, $sp(x, y) \leq k$, where $sp(x, y)$ is the shortest (in terms of number of edges) simple path between x and y in G .

Give pseudocode for a polynomial-time algorithm that solves the following problem: Given an undirected graph $G = (V, E)$ and an integer $k > 0$, determine if G contains a k-pentacle. Please give the asymptotic worst-case time complexity of your algorithm.

5. (20 marks) Circle the letters associated with the appropriate answers in the the multiple choice questions in parts (i–iv). Note that some of these questions may have more than one answer whose letter needs to be circled.

Suppose we have seven decision problems A, B, C, D, E, F, and G such that $A \leq_p B$, $A \leq_e E$, $B \leq_p C$, $C \leq_p A$, $C \leq_p F$, $D \leq_p B$, $F \leq_e B$, and $G \leq_p E$, where $X \leq_p Y$ ($X \leq_e Y$) means that there is a polynomial-time (exponential-time) many-one reduction from X to Y, *i.e.*, X reduces to Y.

i) (5 marks) What is implied if we know that F is NP-hard and A is solvable in polynomial time?

a) B is not solvable in polynomial time.
b) if B is solvable in polynomial time then $P = NP$.
c) C is NP-hard.
d) D is NP-hard.
e) E is solvable in exponential time.

ii) (5 marks) What is implied if we know that A is NP-hard and F is solvable in polynomial time?

a) E is NP-hard.
b) B is NP-complete.
c) $P \neq NP$.
d) B is not solvable in polynomial time unless $P = NP$.
e) $P = NP$.

iii) (5 marks) What is implied if we know that C is NP-hard and E is solvable in polynomial time?

a) D may be solvable in polynomial time.
b) $P = NP$.
c) B is solvable in polynomial time.
d) $P \neq NP$.
e) C is solvable in exponential time.

iv) (5 marks) What is implied if we know that G is NP-hard and D is solvable in polynomial time?

a) E is solvable in polynomial time.
b) $P = NP$.
c) A is solvable in exponential time.
c) B may be solvable in polynomial time.
e) A is NP-hard.