Computer Science 3600 (Winter 2023):
Handout:
Computational Problems

Vertex Cover (VC)

Input: An undirected graph $G = (V, E)$ and an integer $k > 0$.
Question: Is there a vertex cover of G of size at most k, *i.e*, is there a subset $V' \subseteq V$ such that $|V'| \leq k$ and for all edges $(u, v) \in E$, at least one of u and v is in V'?

Vertex Cover Cost (VC-C)

Input: An undirected graph $G = (V, E)$.
Output: The size of the smallest vertex cover of G.

Vertex Cover Example (VC-E)

Input: An undirected graph $G = (V, E)$.
Output: One of the smallest vertex covers of G.

Clique

Input: An undirected graph $G = (V, E)$ and an integer $k > 0$.
Question: Is there a clique in G of size at least k, *i.e.*, is there a subset $V' \subseteq V$, $|V'| \geq k$, such that for all $u, v \in V'$, $(u, v) \in E$?

Subset sum

Input: A set $S \subset \mathcal{N}$ of integers and an integer $k \geq 0$.
Question: Is there a subset S' of S whose elements sum to k?

Steiner tree in graphs (STG)

Input: An undirected graph $G = (V, E)$, a set $V' \subseteq V$, and an integer $k > 0$.
Question: Is there a tree in G that connects all vertices in V' and contains at most k edges?