COMP 4752
Computational Intelligence

Lecture 12

Two-Player Games

Mini-Max / Alpha-Beta Search

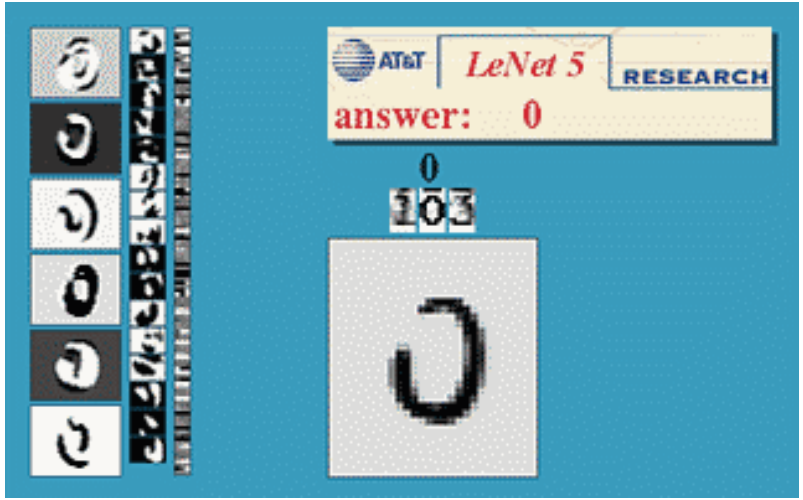
Multi-Player Games

- So far we have only looked at problems with a single agent (single agent search)
- Heuristic search can also be applied to environments (and games) with multiple agents (or players)
- Games can have a number of properties, similar to single agent environments

Fully vs. Partially Observable

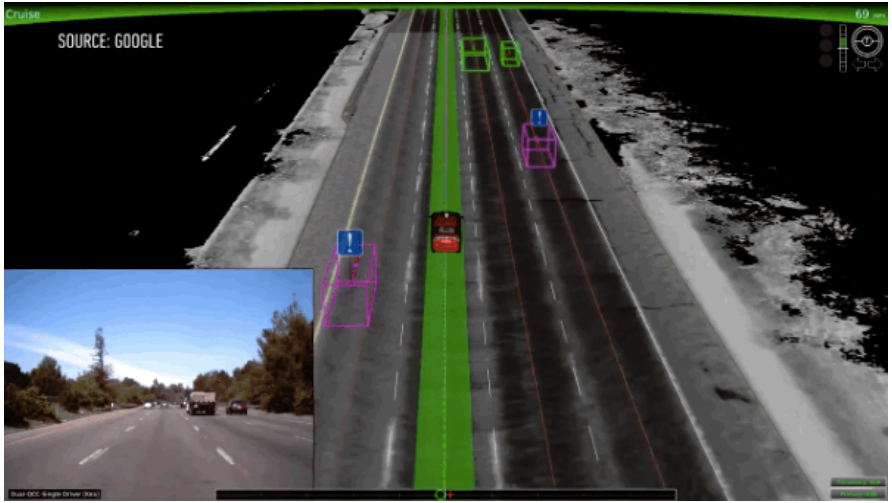
Deterministic vs. Stochastic

Episodic vs. Sequential



Dynamic vs. Static

Discrete vs. Continuous



Game Properties: Players

- How many players are in the game?
- Examples of Game Players:
  - Chess, Go: 2 Players
  - Baseball: 18 Players, 2 Teams
  - Starcraft: 1v1, 2v2, 8 Person FFA
  - Poker: 10 People at a table

Game Properties: Payoffs

- What does each player hope to achieve?
- Game Payoff Examples:
  - Prisoner's Dilemma: Maximize Reward
  - Poker: Maximize Profits
  - Chess: Win the game
- Zero Sum Game:
  - Each player's gain or loss of payoff/utility is equally balanced by the utility of the other players
  - Win / Lose games are zero sum

Games in this Course

- Two-Player
- Zero Sum
- Alternating Move
- Fully Observable (Perfect Information)
- Deterministic
- Discrete

The Chess-Playing Computer

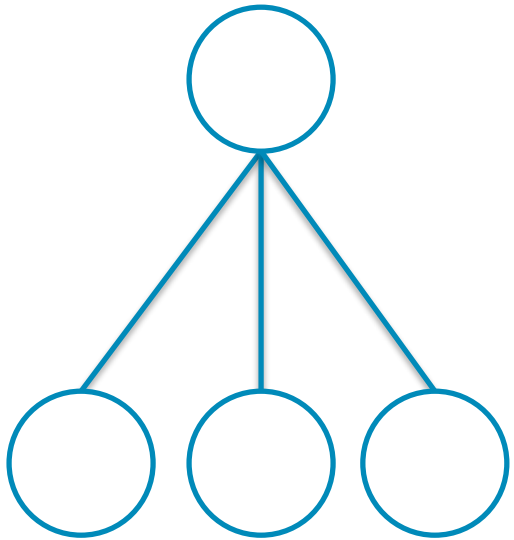
Game-Playing Computer

- How can we design an algorithm to play a two-player alternating move game?

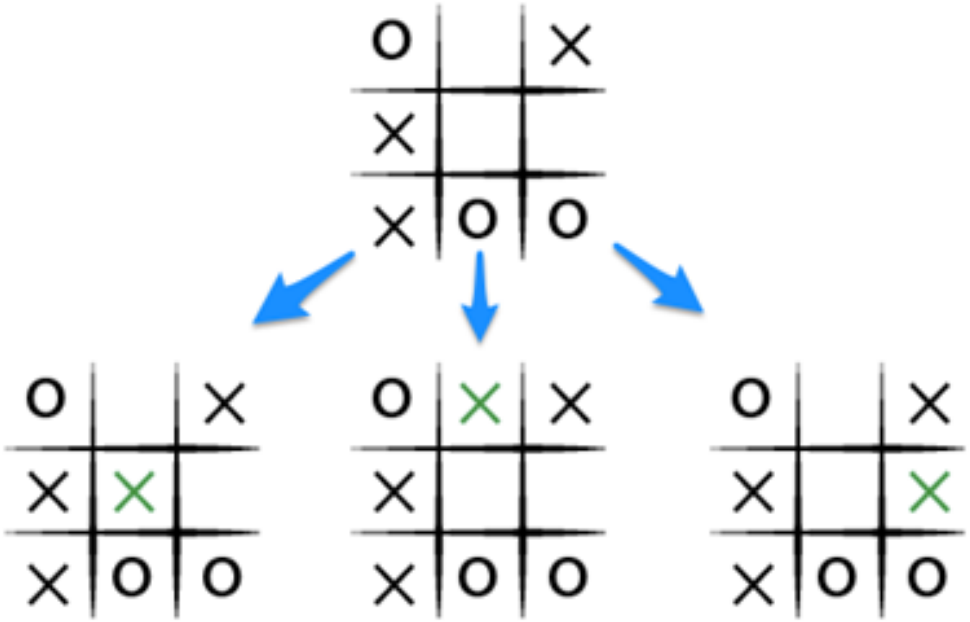
1. Analysis / Strategy / Tactics
2. Thousands of If-Then Statements
3. Look-Ahead and Evaluate

Look-Ahead and Evaluate

- Generate a list of actions from a given state
- Evaluate those actions based on features of the resulting states
- Do the action which has the highest evaluation



Tic-Tac-Toe Example



Game-Playing Computer

- How can we design an algorithm to play a two-player alternating move game?

1. Analysis / Strategy / Tactics
2. Thousands of If-Then Statements
3. Look-Ahead and Evaluate
4. Search the Entire Game Tree

How big is the game tree?

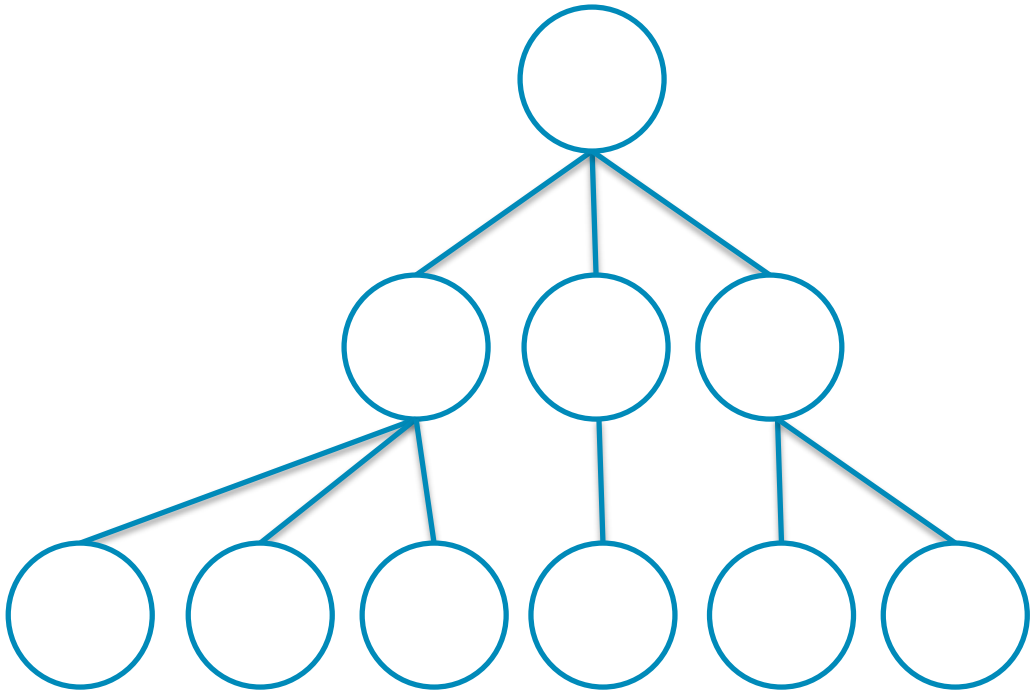
- Some number of actions at each state
  - b = “Branching Factor”
- Game ends after some number of moves
  - d = Search Depth
- Game tree = b^d
- Chess = 10^{120}

Game-Playing Computer

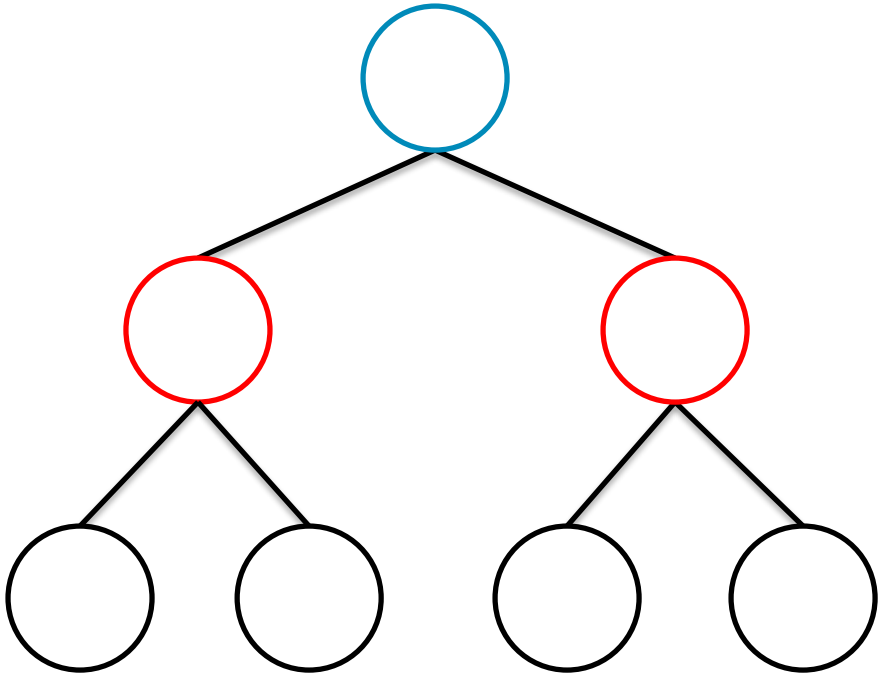
- How can we design an algorithm to play a two-player alternating move game?

1. Analysis / Strategy / Tactics
2. Thousands of If-Then Statements
3. Look-Ahead and Evaluate
4. Search the Entire Game Tree
5. Look-Ahead as Far as Possible

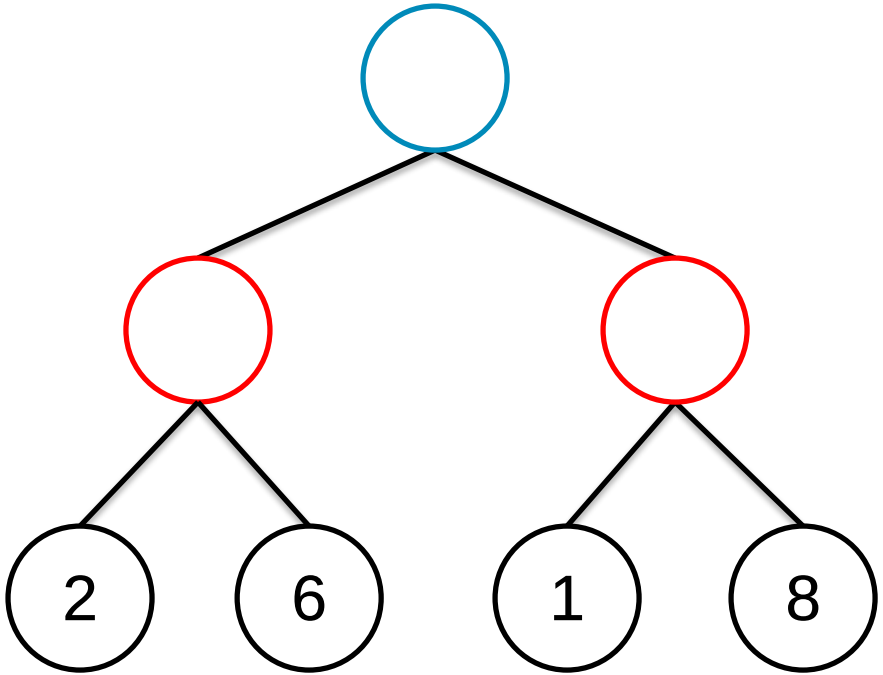
Look Ahead as Far as Possible



Competing Players



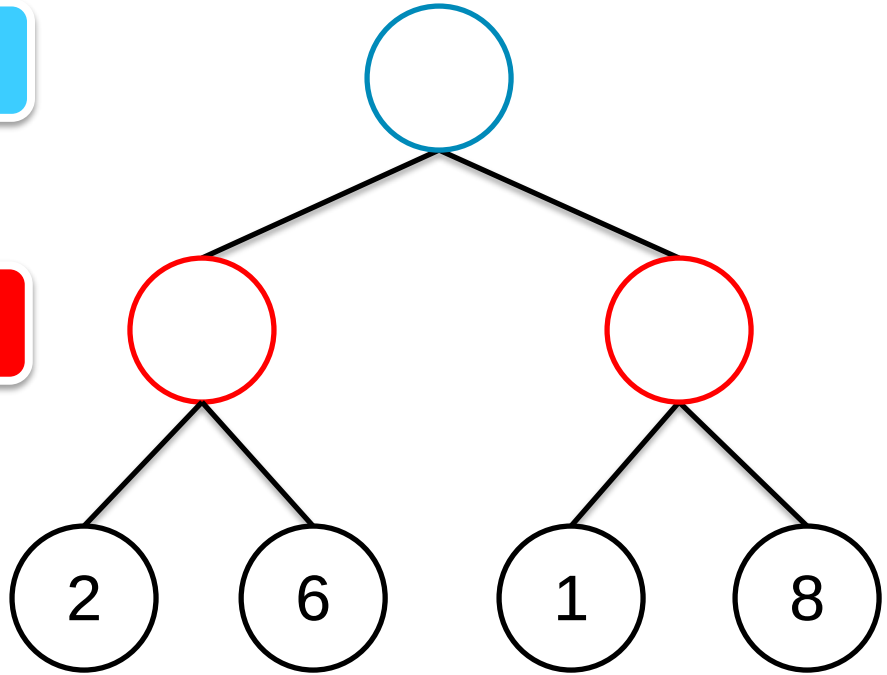
Competing Players



Competing Players

Max Player

Min Player



Competing Players

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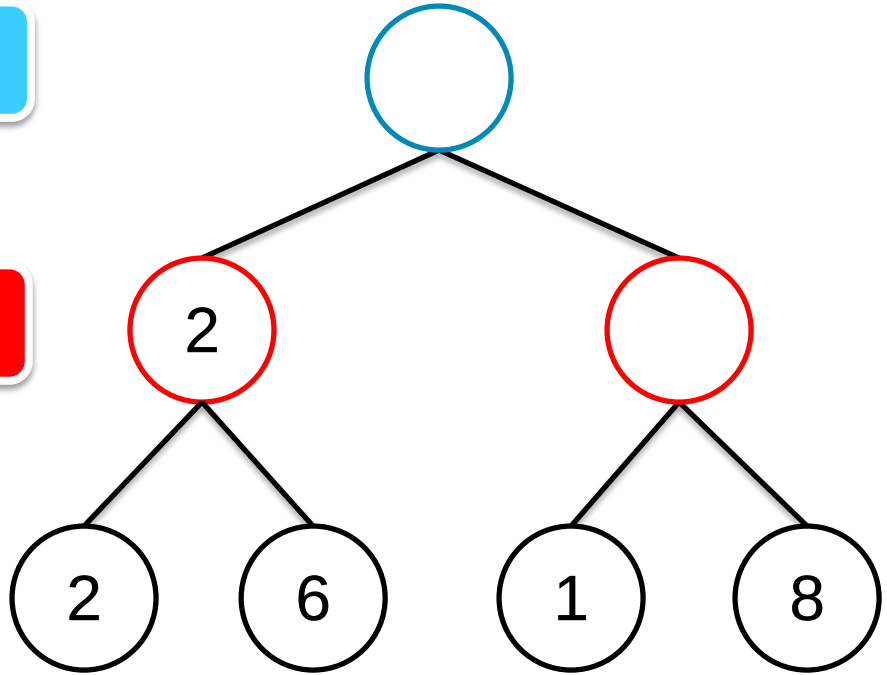

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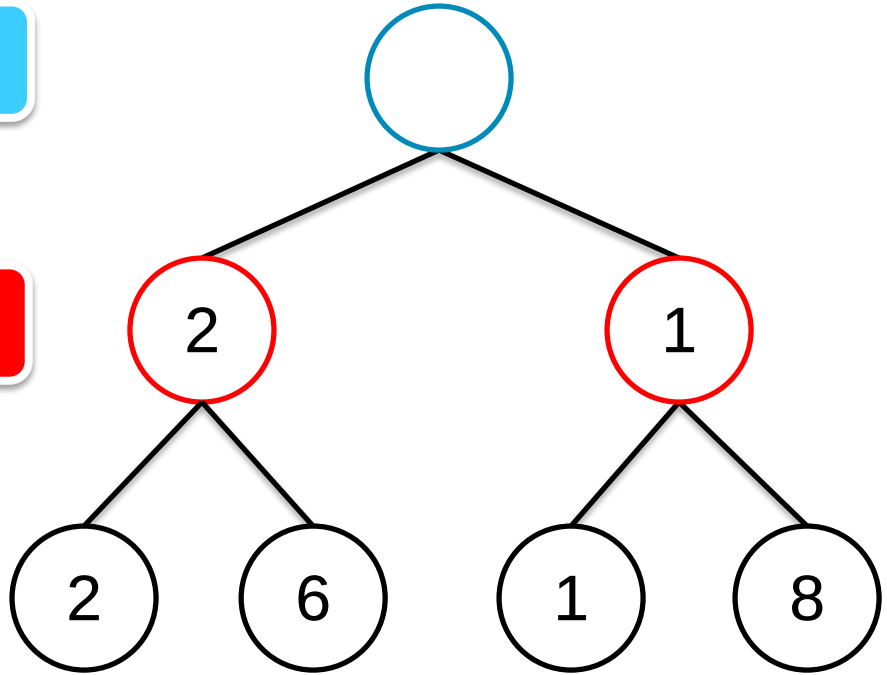

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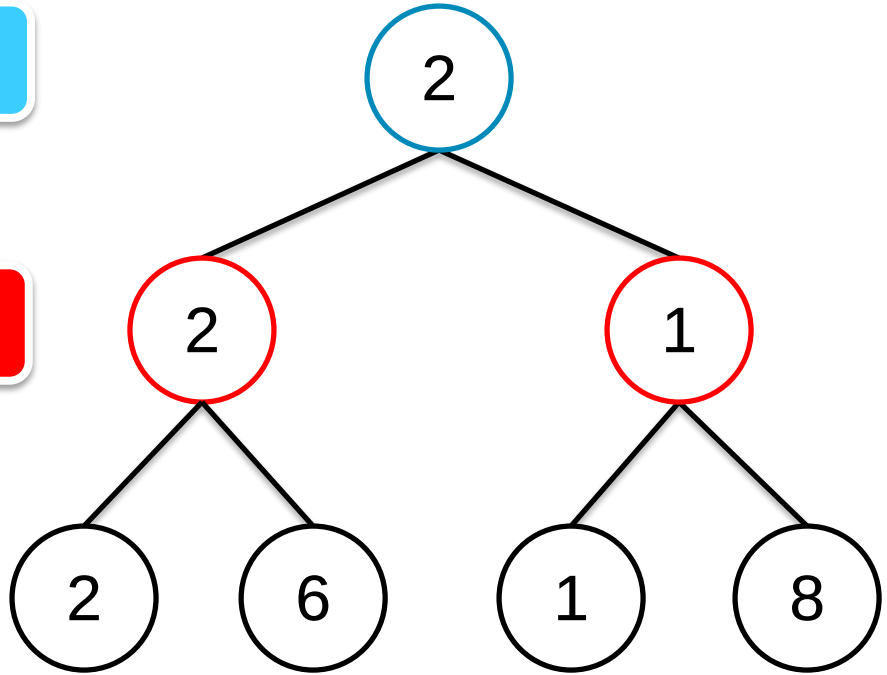

Minimax Algorithm

```
1.  Function MiniMax(node, depth, maxPlayer)
2.          if (depth > maxDepth or node.terminal)
3.              return eval(node)
4.          if (node.playerToMove == maxPlayer)
5.              bestValue = -infinity
6.              for each child of node
7.                  value = MiniMax(child, depth+1, maxPlayer)
8.                  bestValue = max(bestValue, value)
9.          else
10.             bestValue = infinity
11.             for each child of node
12.                 value = MiniMax(child, depth+1, maxPlayer)
13.                 bestValue = min(bestValue, value)
14. Initial Call: MiniMax(rootNode, 0, maxPlayer)
```

Negamax Algorithm

```
Function NegaMax(node, depth, player)
        if (depth > maxDepth or node.terminal)
            return eval(node)
        bestValue = -infinity
        for each child of node
            value = -NegaMax(child, depth+1, opponent(player))
            bestValue = max(bestValue, value)
```

Initial Call: NegaMax(root, 0, player)

Mini-Max Properties

- Complete and Optimal: Will find the optimal solution to a given max depth
- Each player plays a best response to the possible actions of the other player
- Mini-Max plays the Nash Equilibrium

Nash Equilibrium (NE)

- For two-player, finite, zero-sum games, a Nash Equilibrium exists
- Recall that in a NE:
  - Each player is best responding to the other
  - Neither player can gain by deviating
  - Neither player has any regrets
- Playing a NE is a very strong strategy

Improving MiniMax

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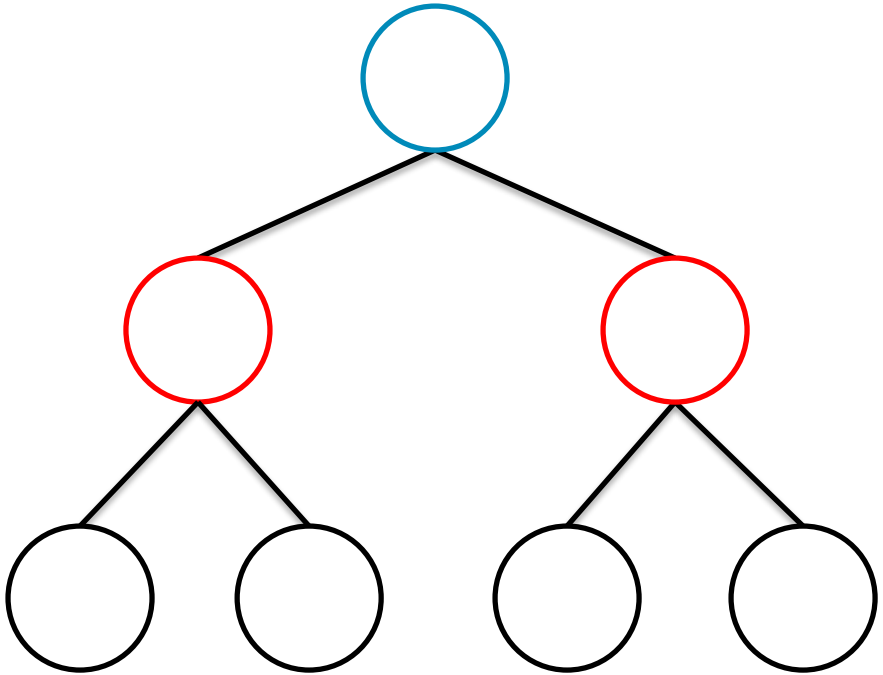

Improving MiniMax

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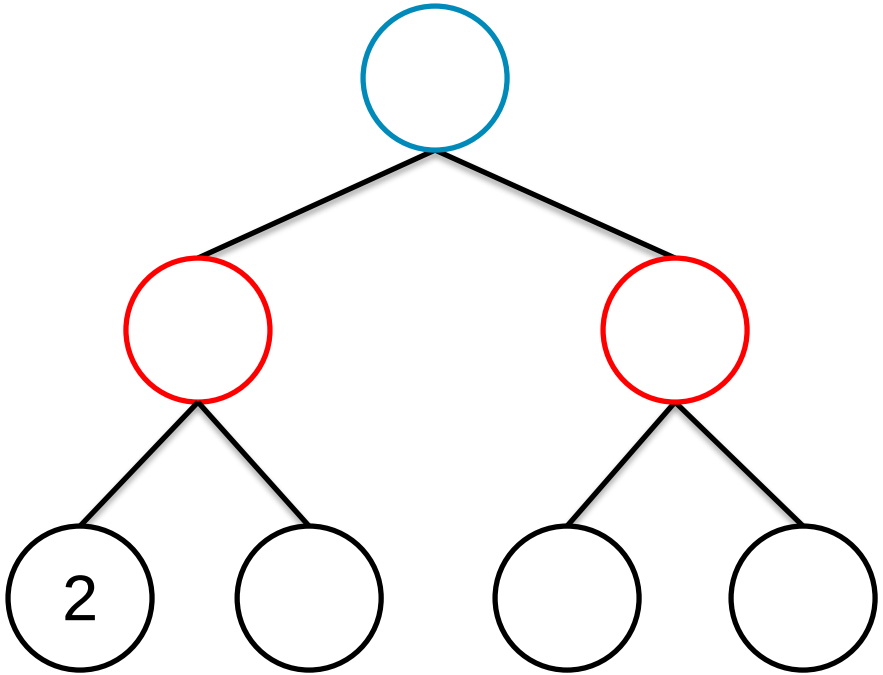

Improving MiniMax

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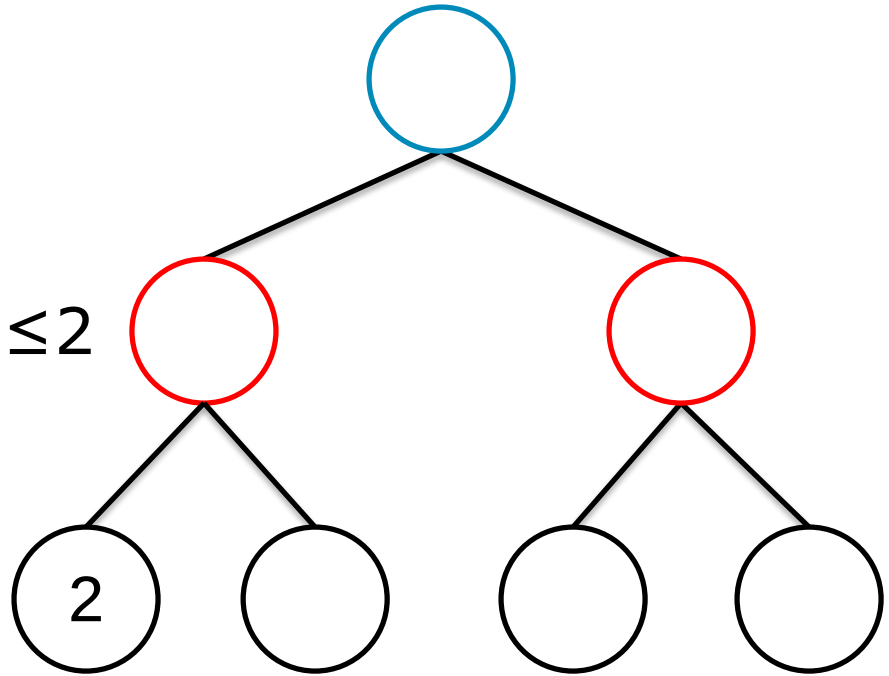

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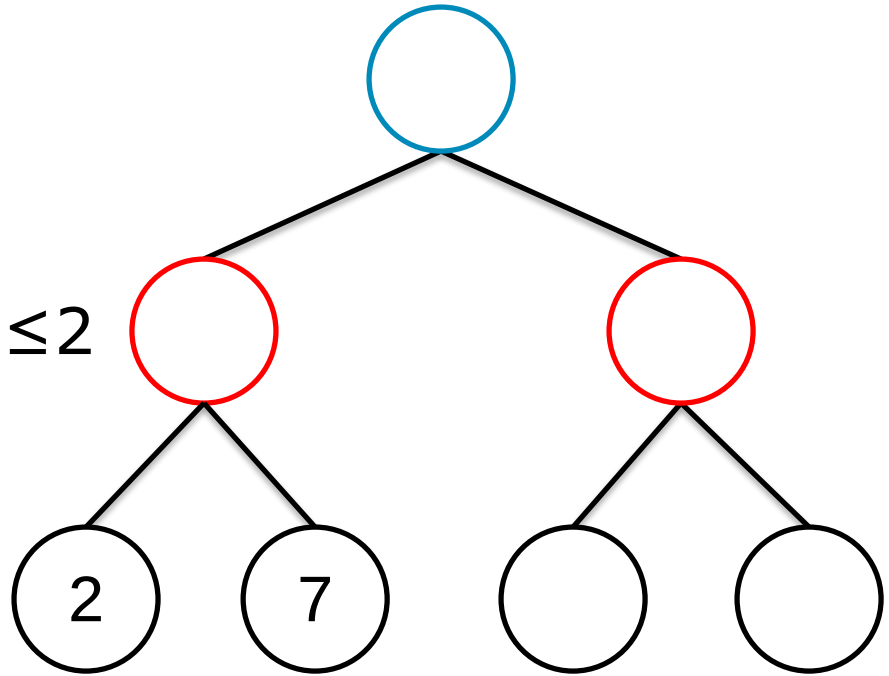

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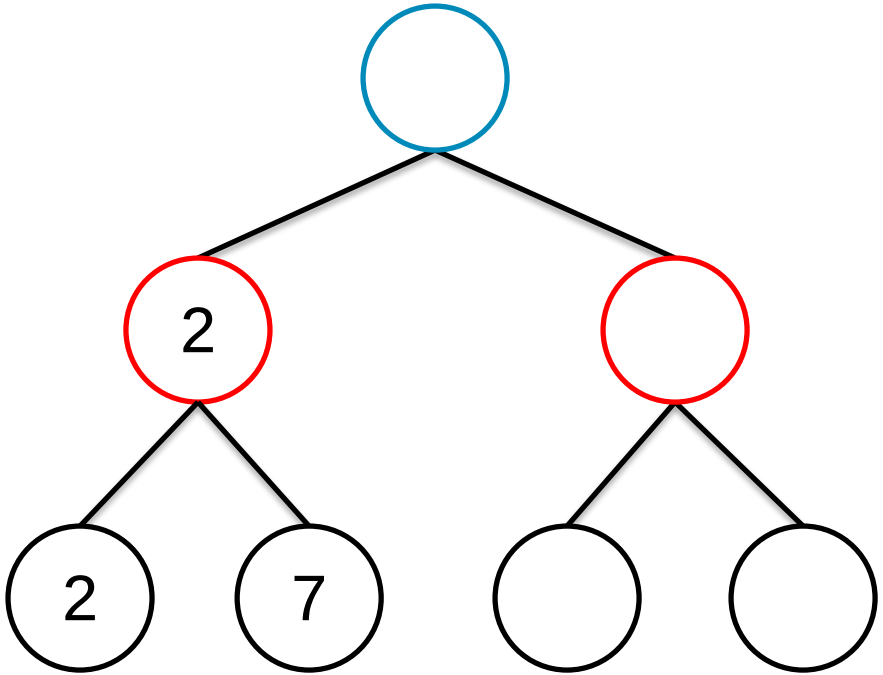

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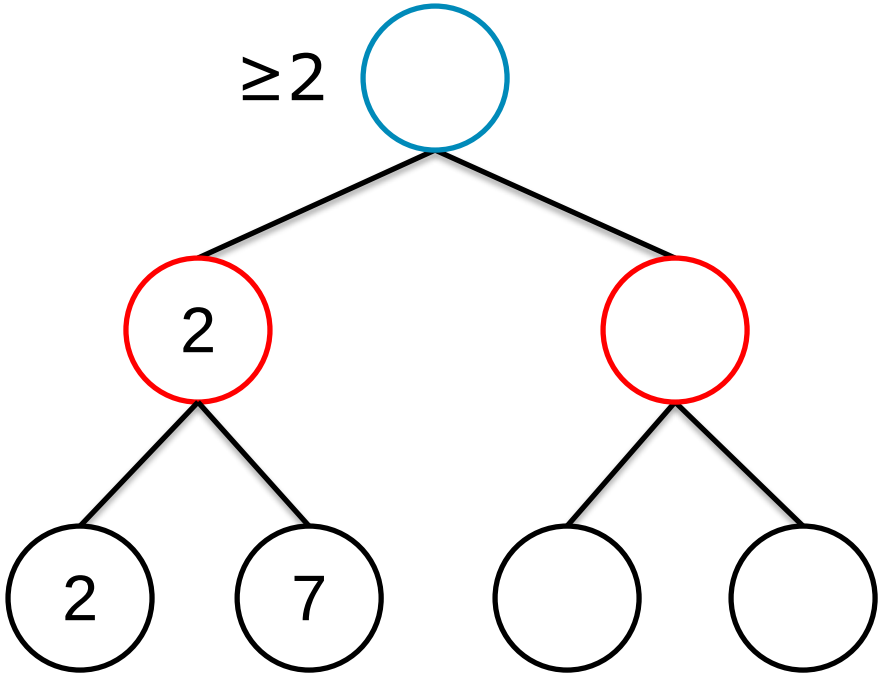

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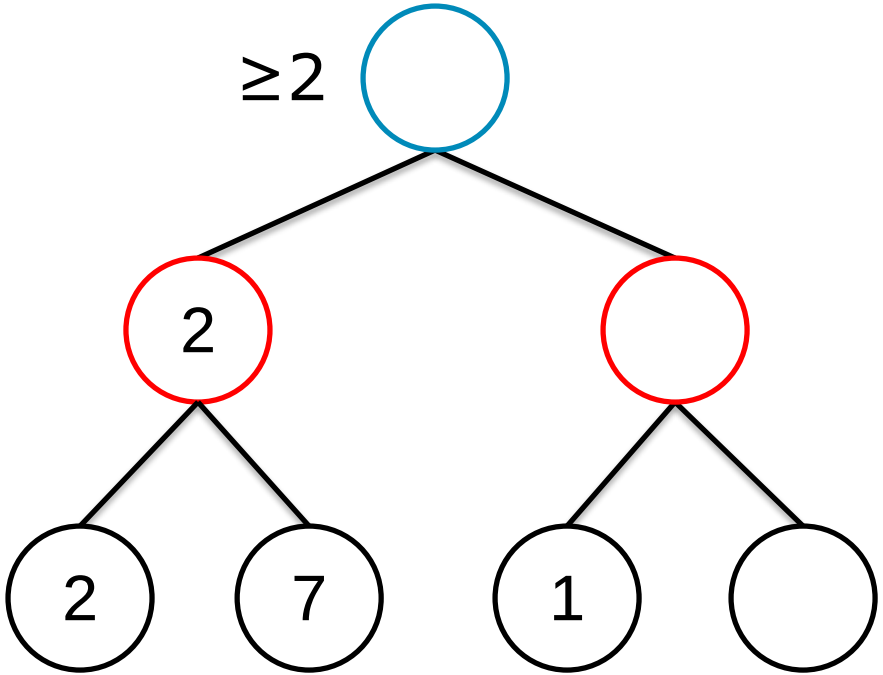

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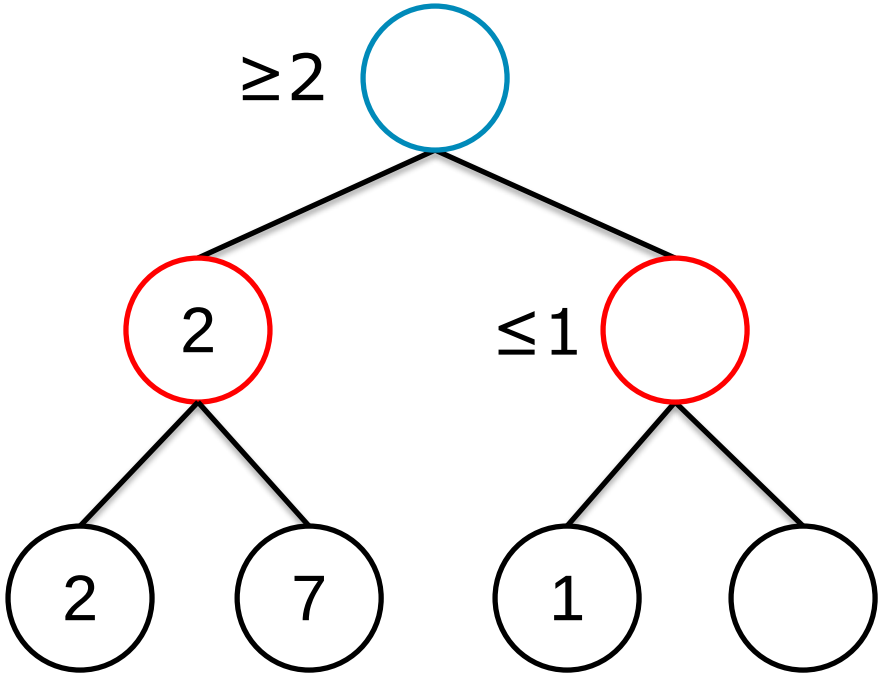

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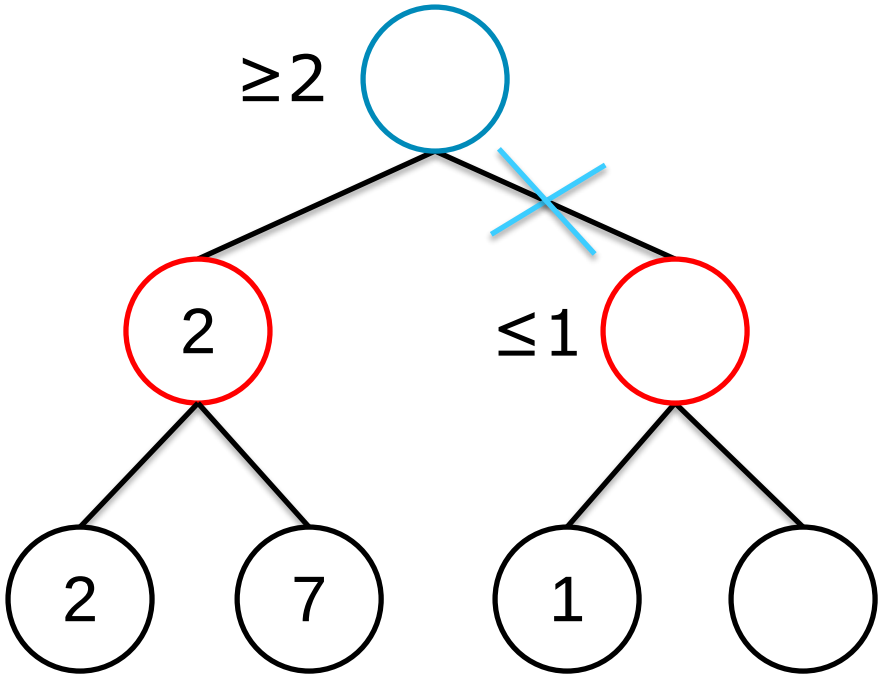

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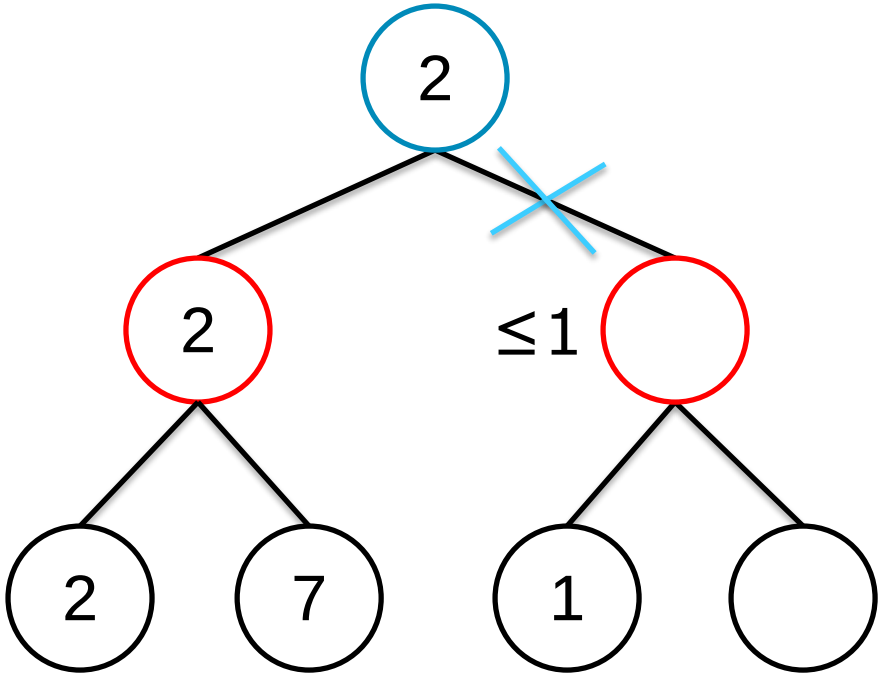

Improving MiniMax

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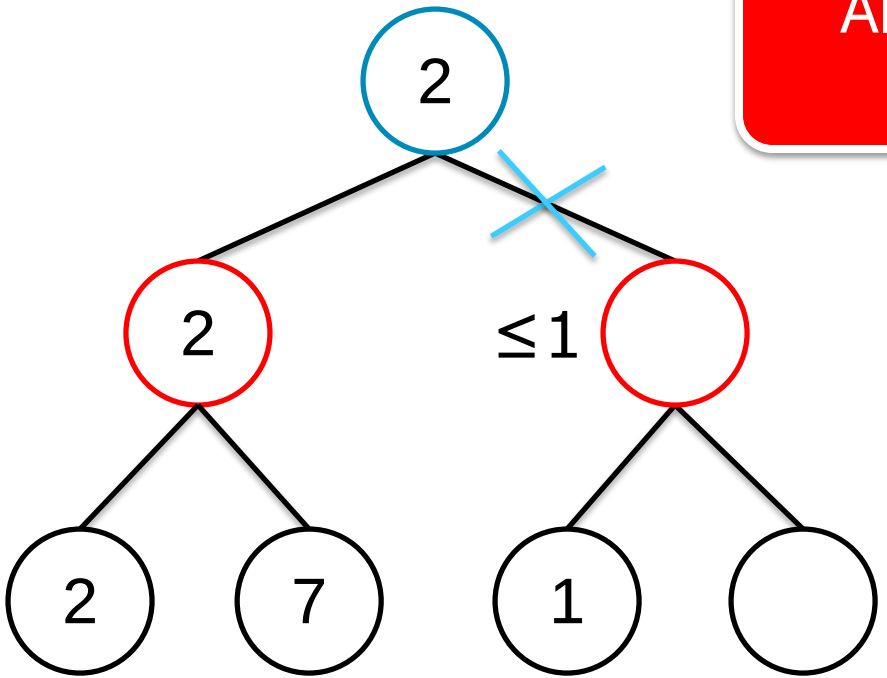
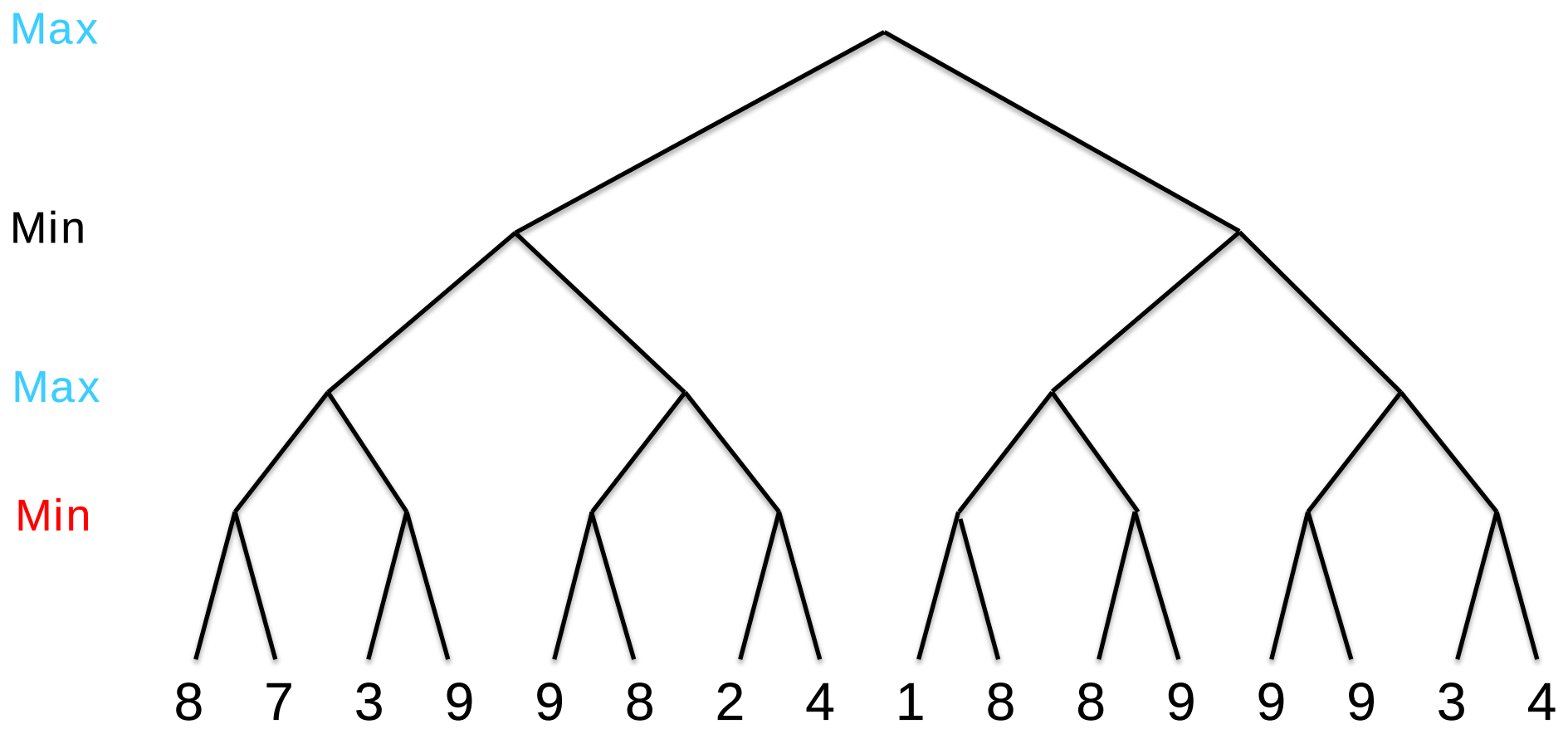
Alpha-Beta Pruning



Alpha-Beta Pruning

- Alpha-Beta is not a different algorithm than MiniMax, it is an optimization
- Maintains all of the properties of minimax but is strictly better
- Cutting off branches of the search tree yields exponential savings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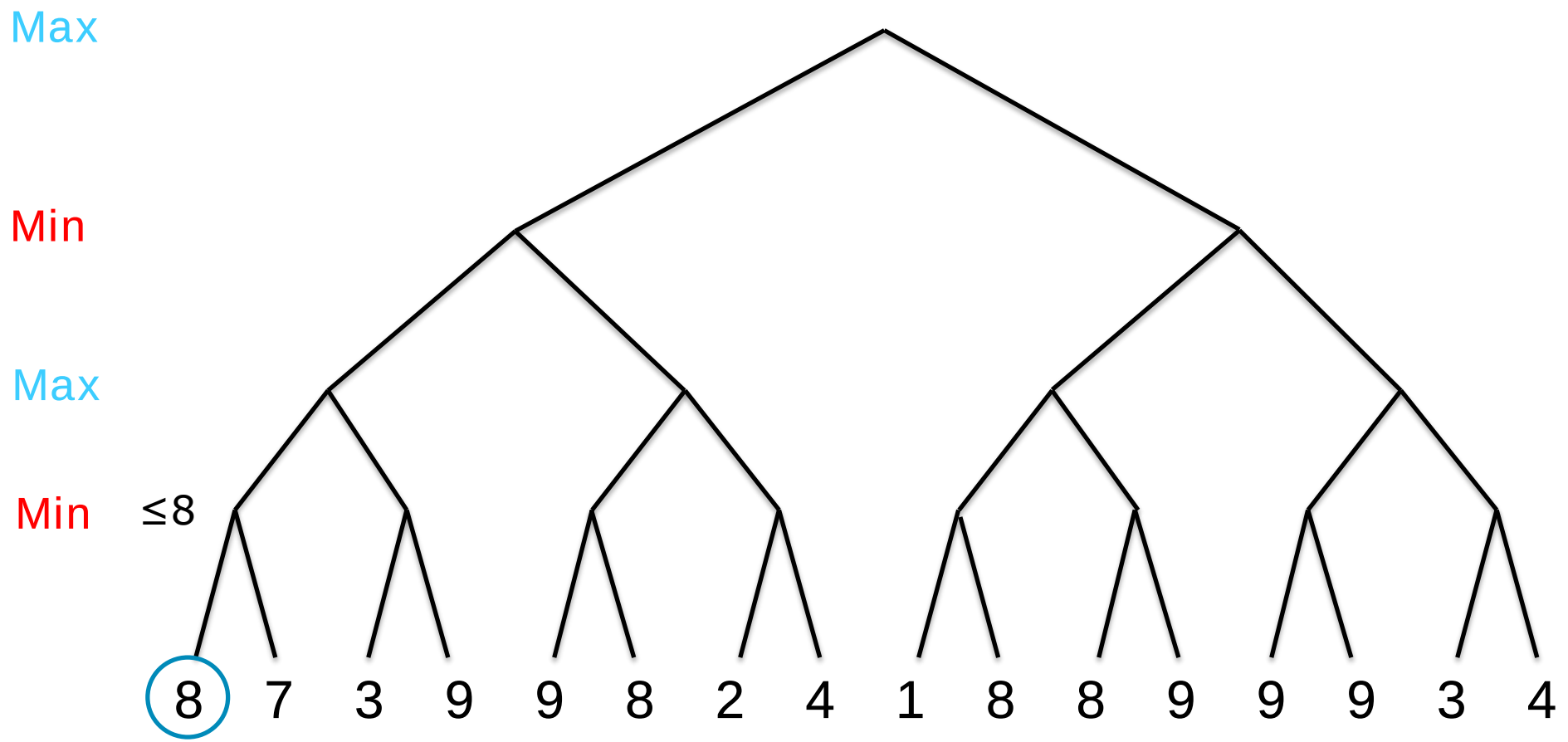
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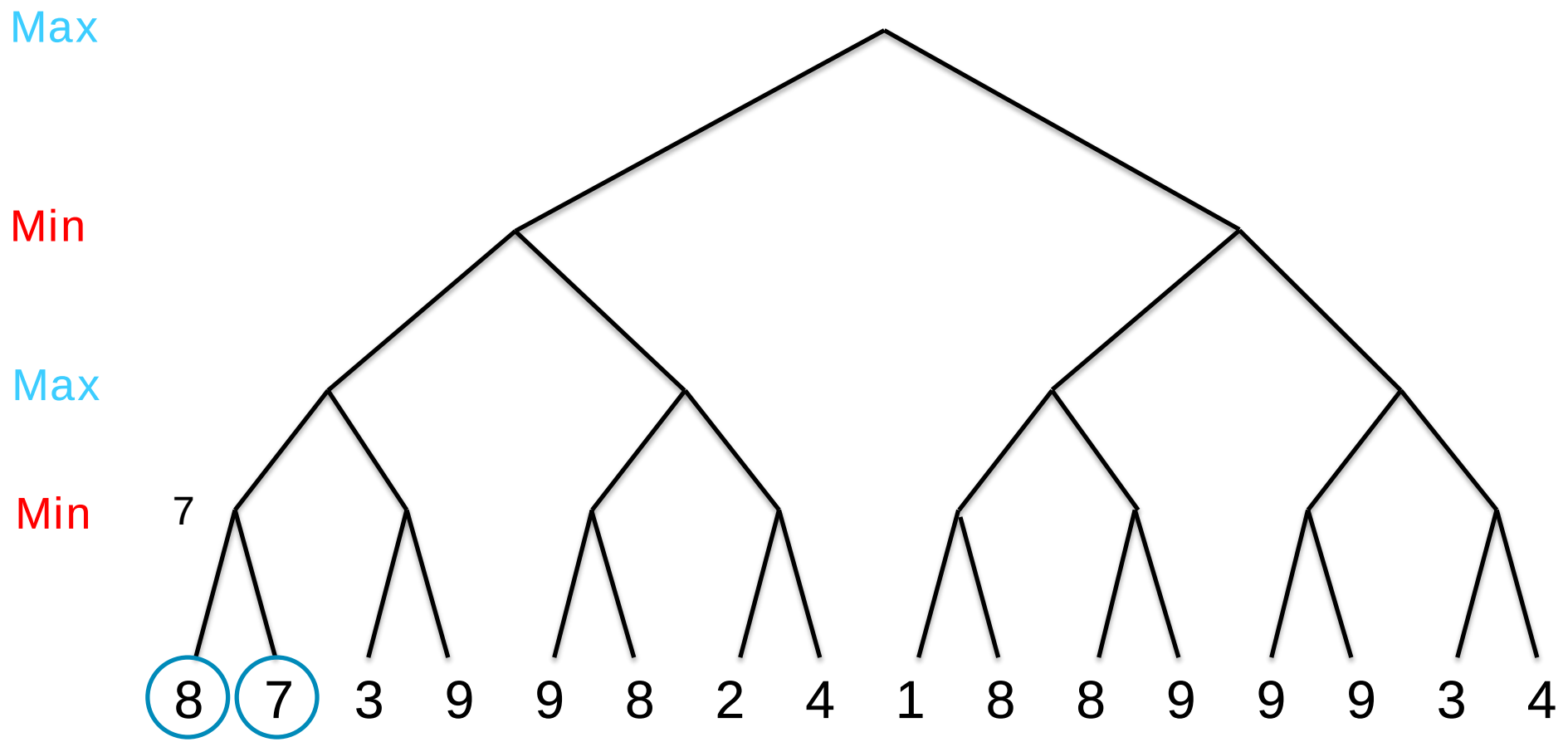
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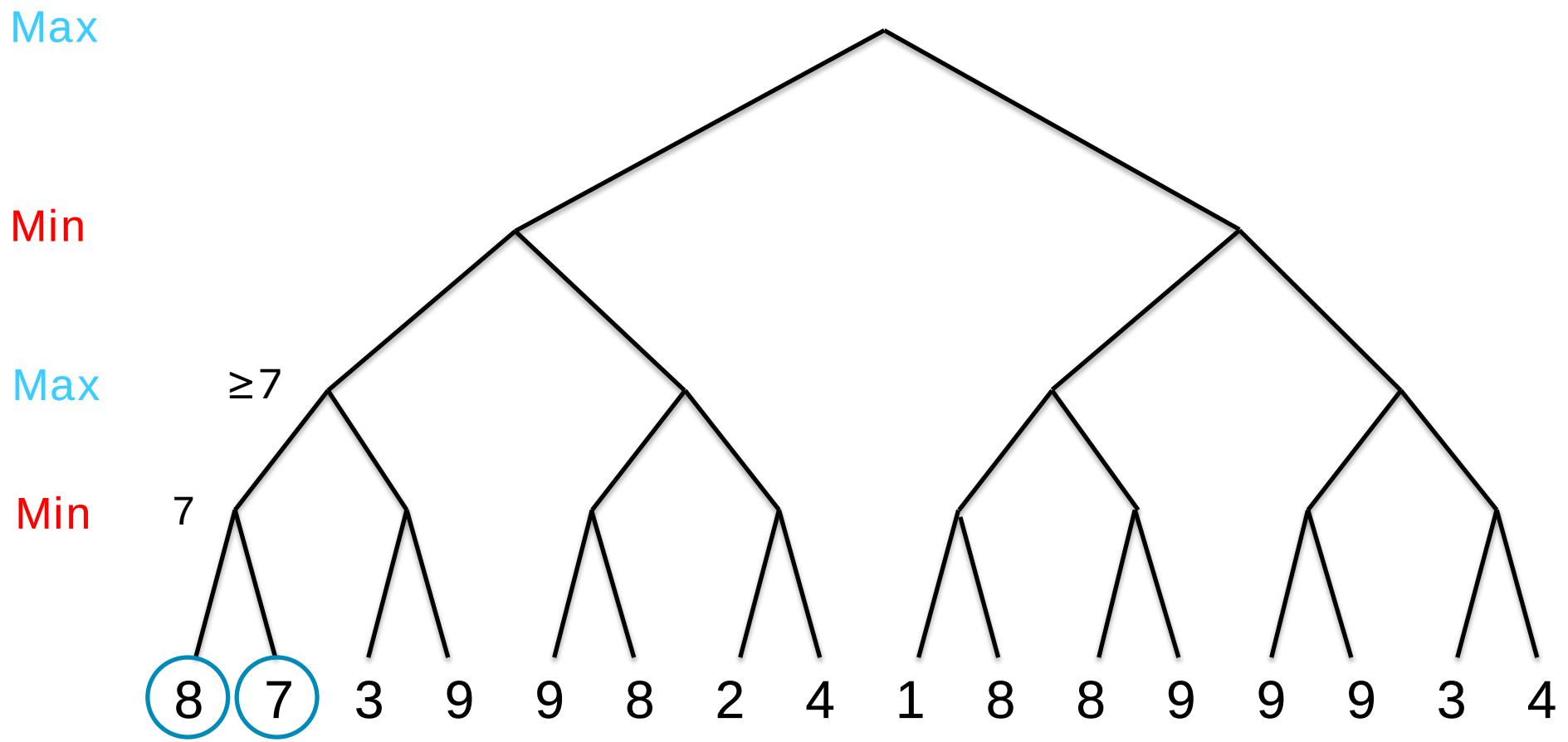
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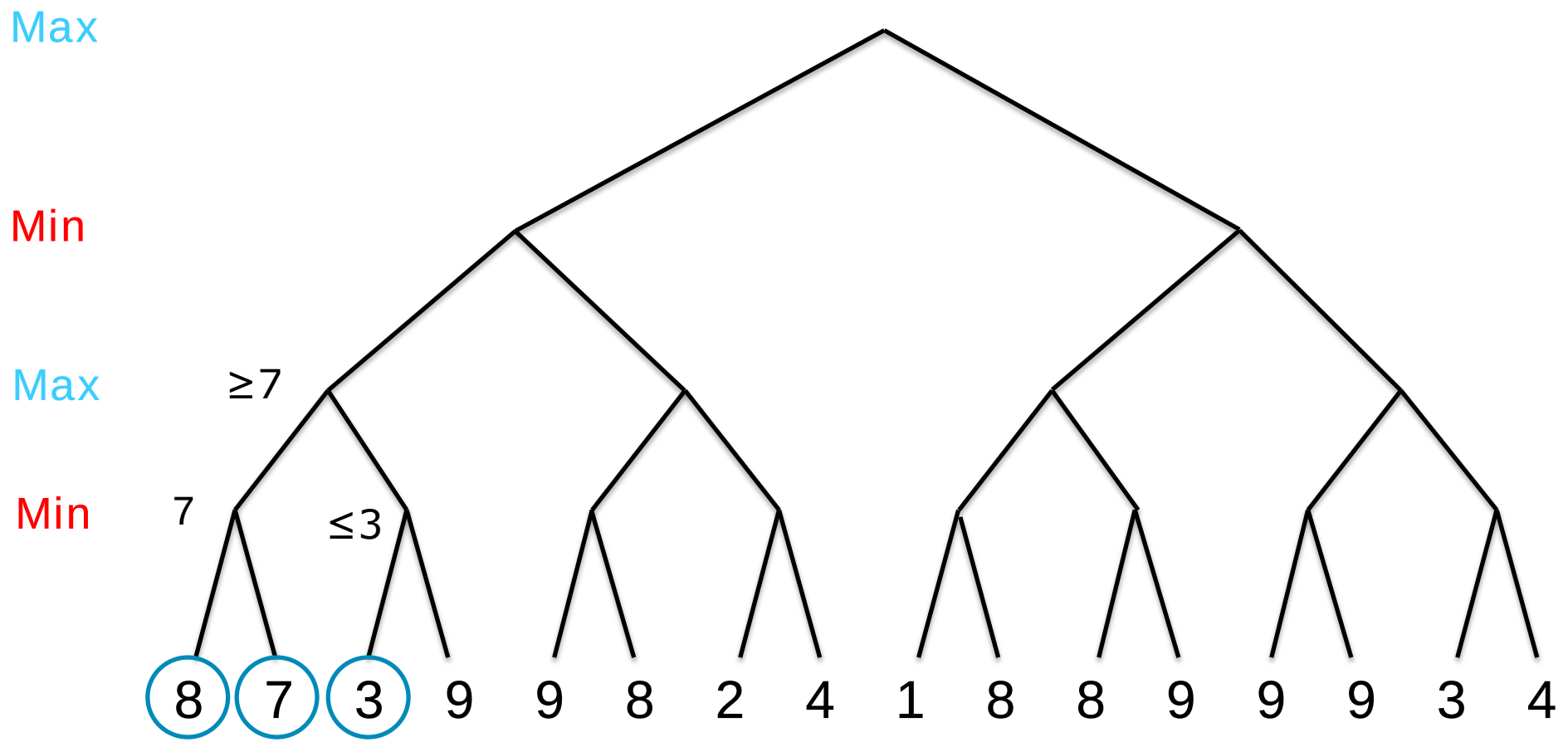
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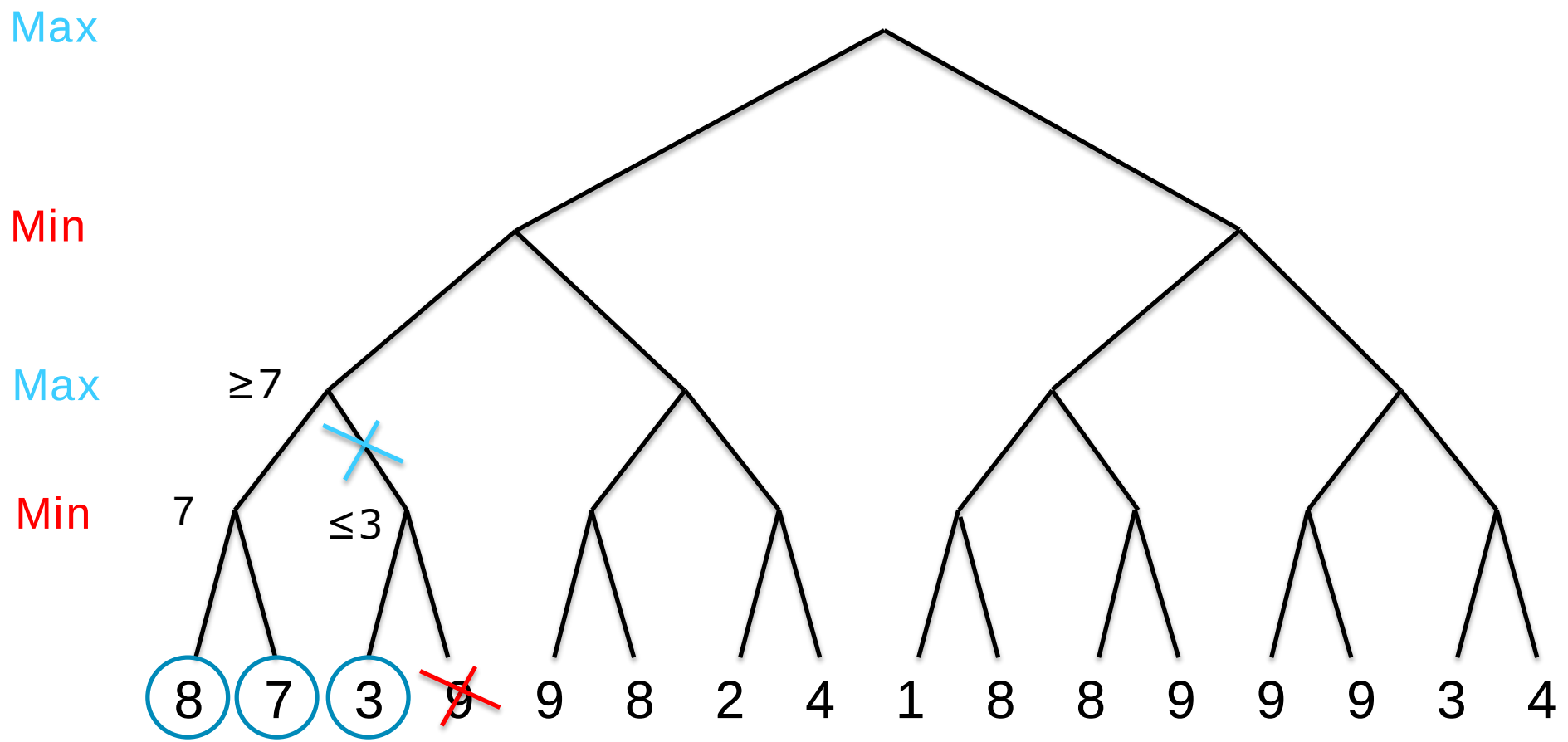
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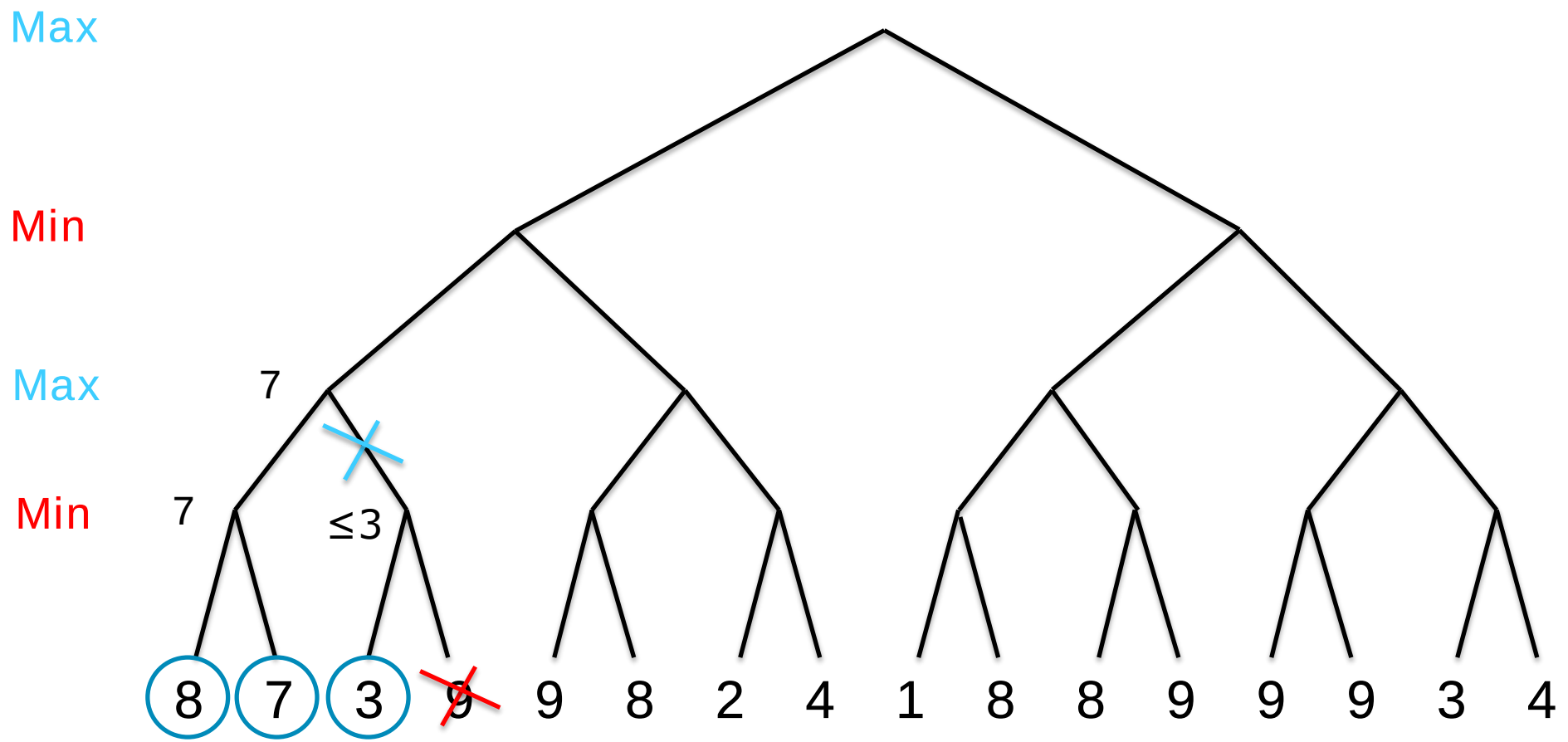
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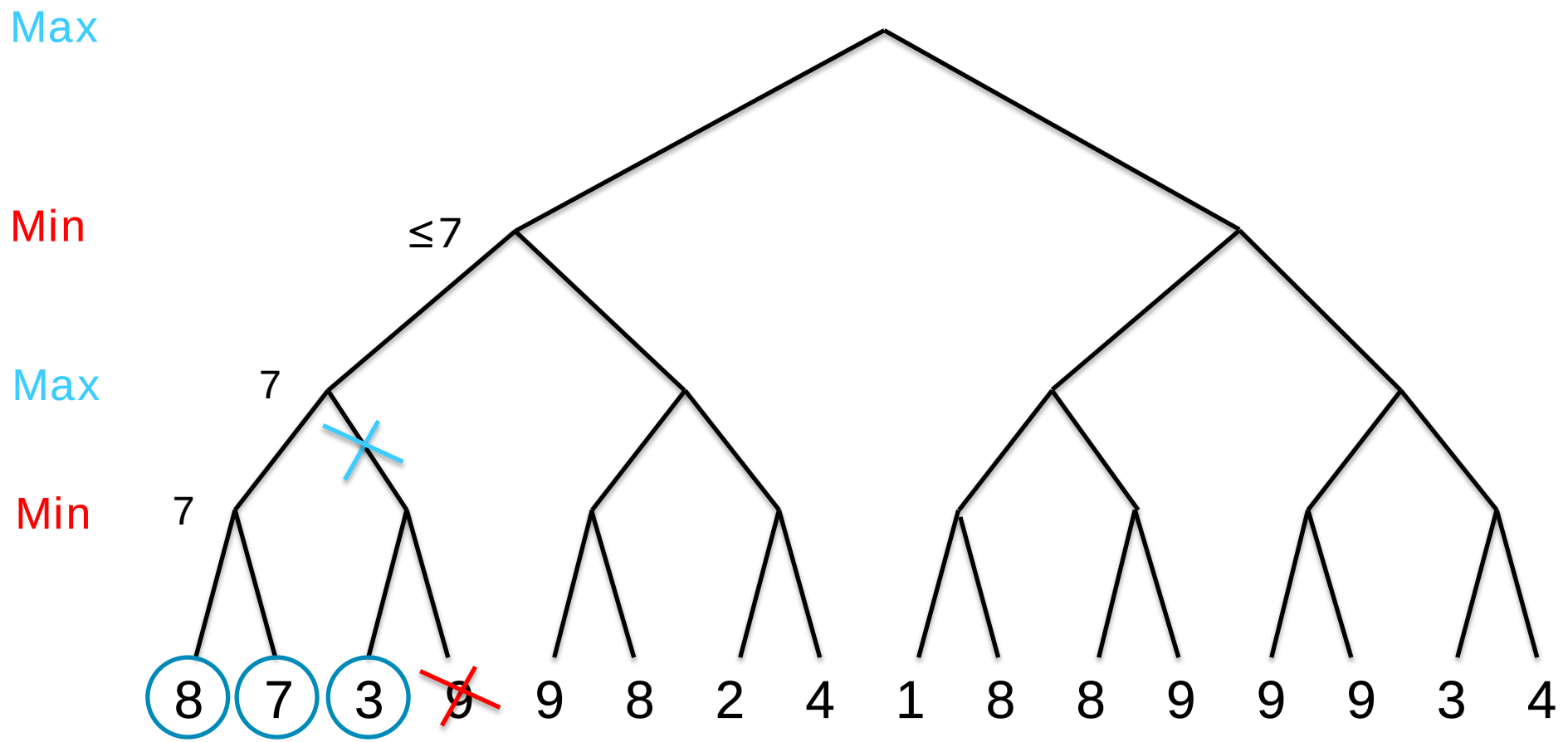
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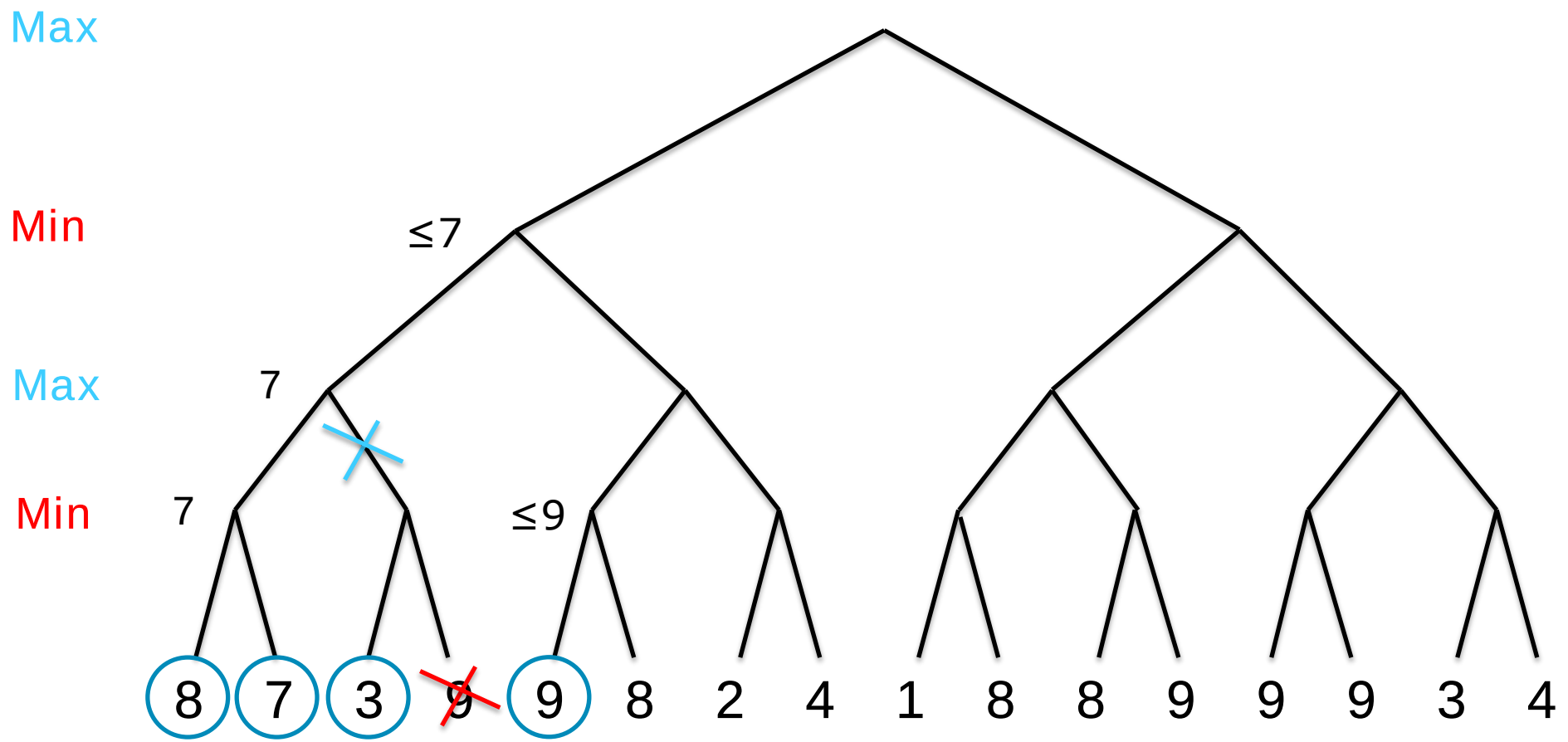
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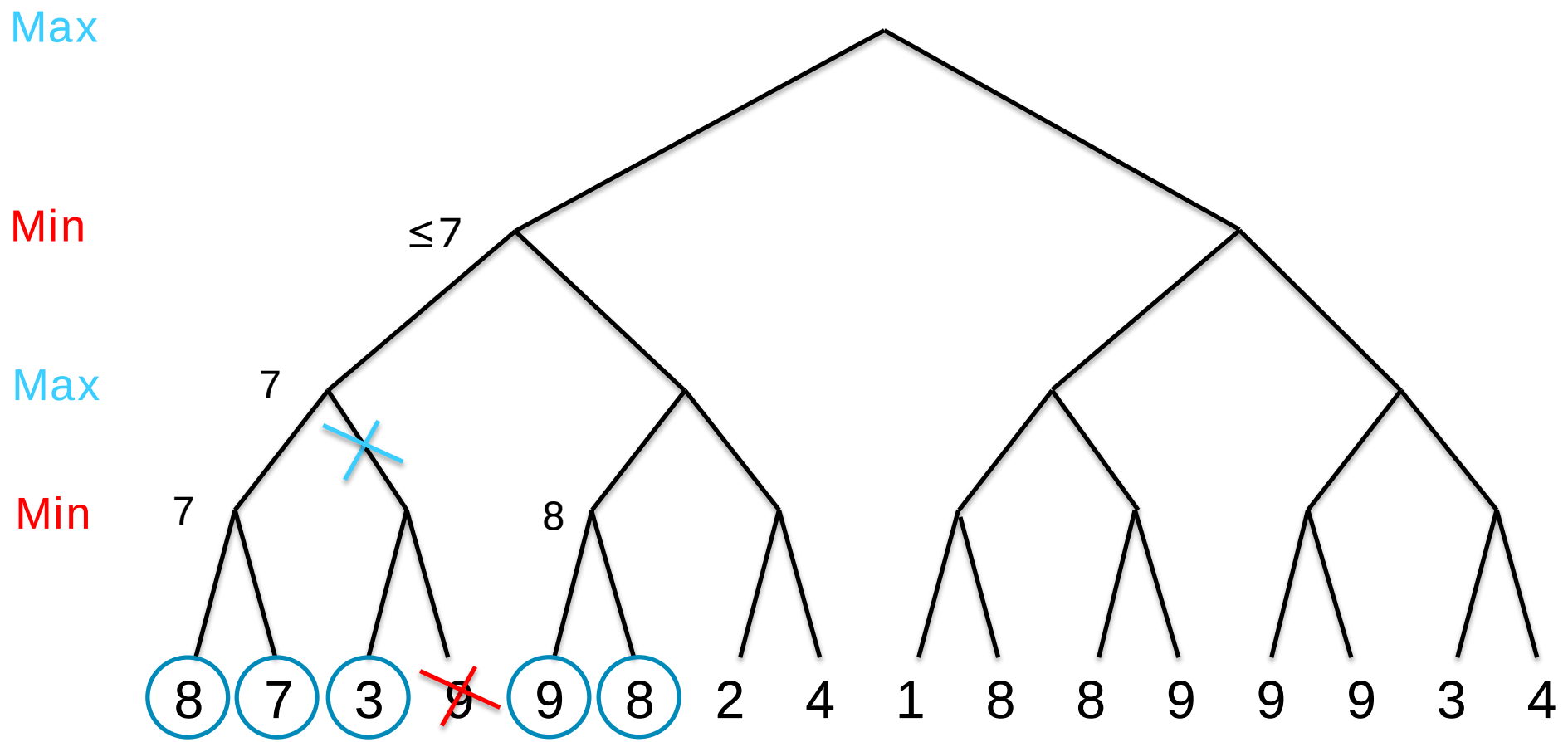
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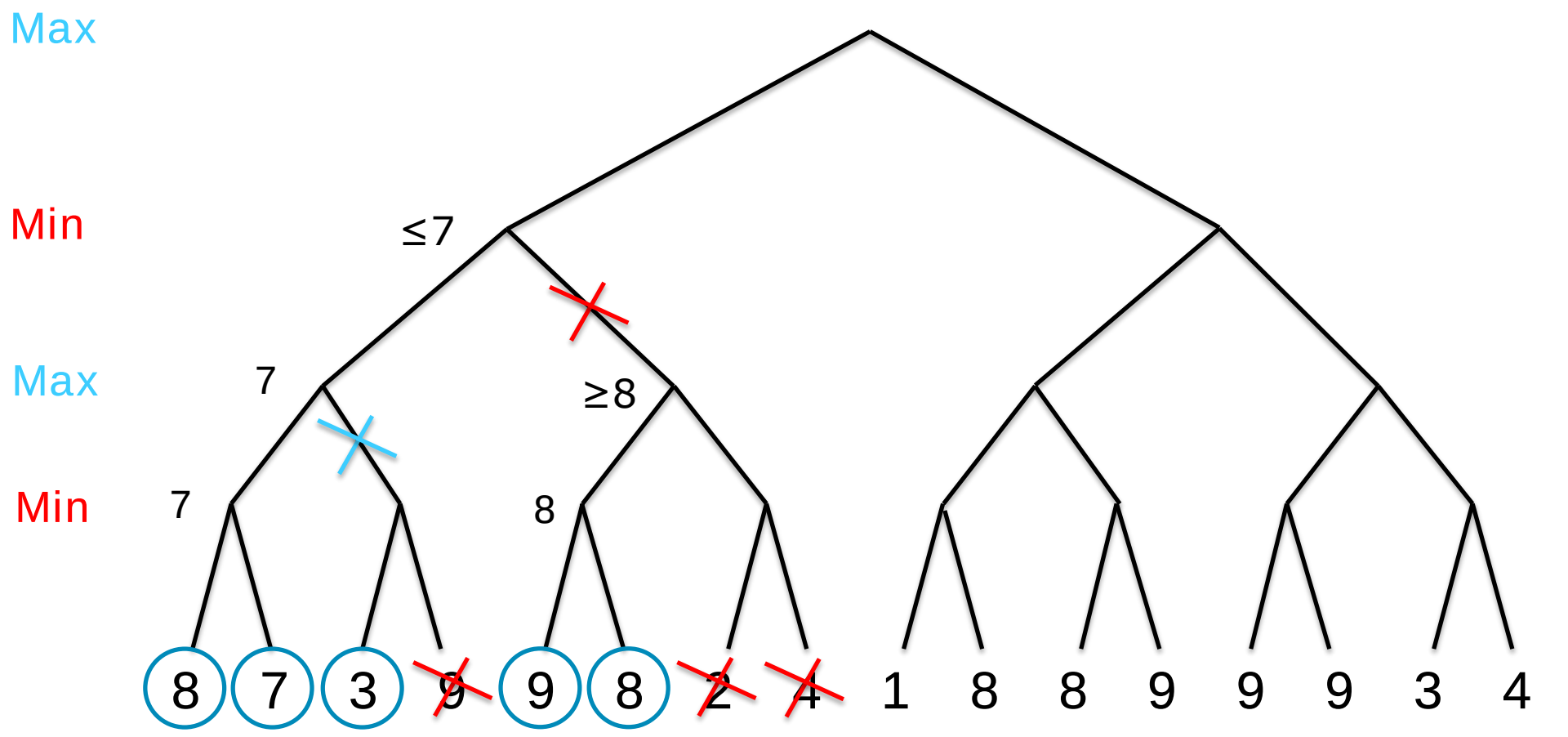
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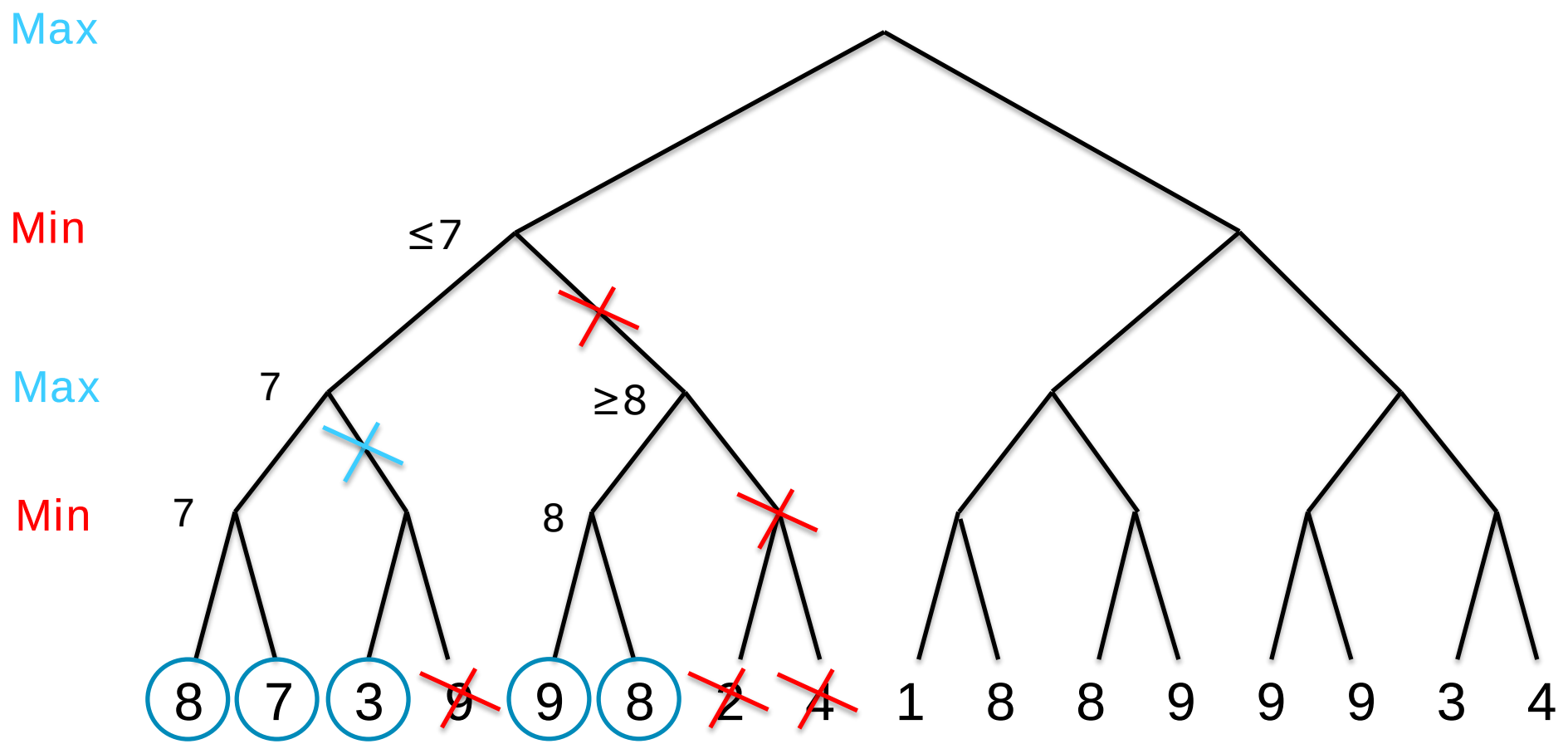
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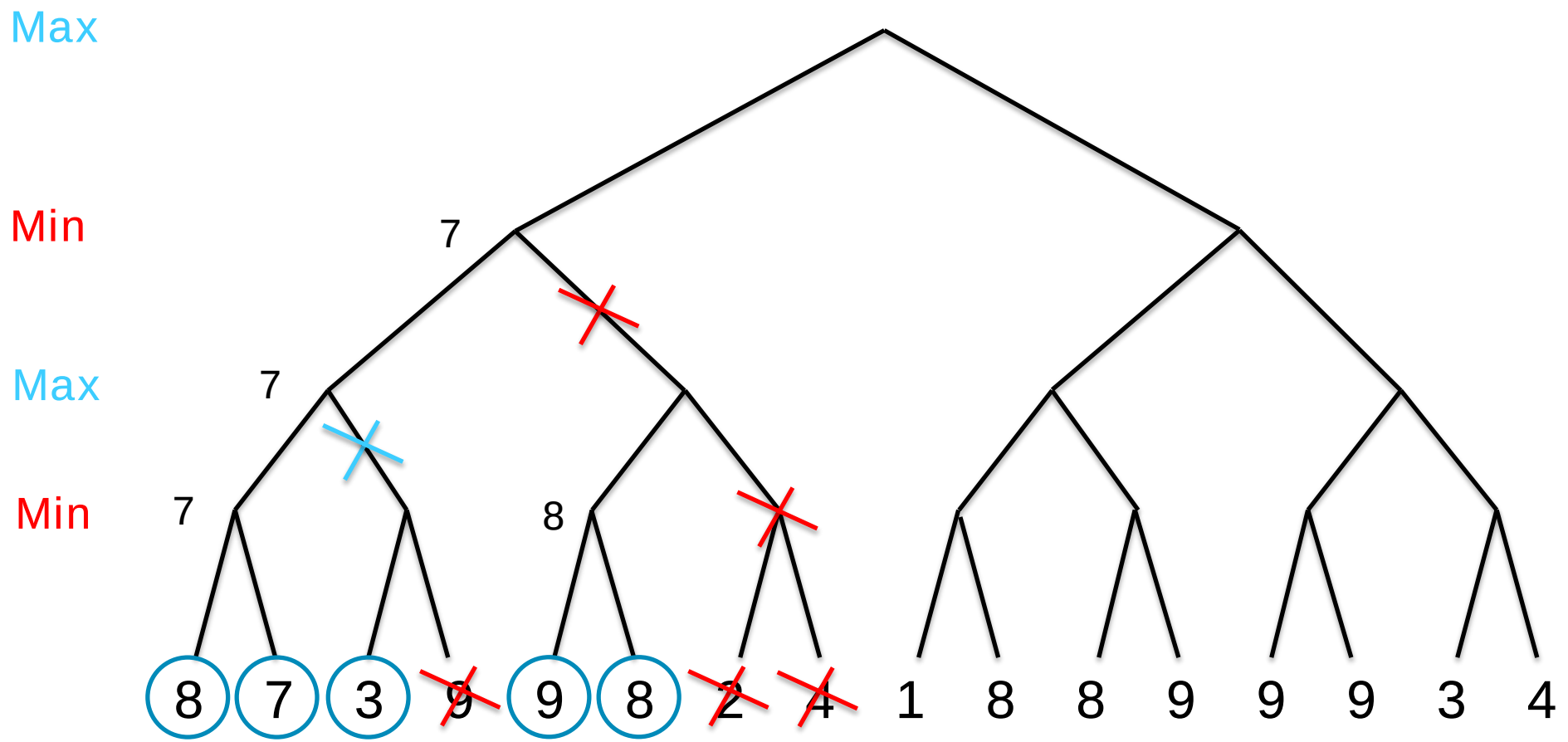
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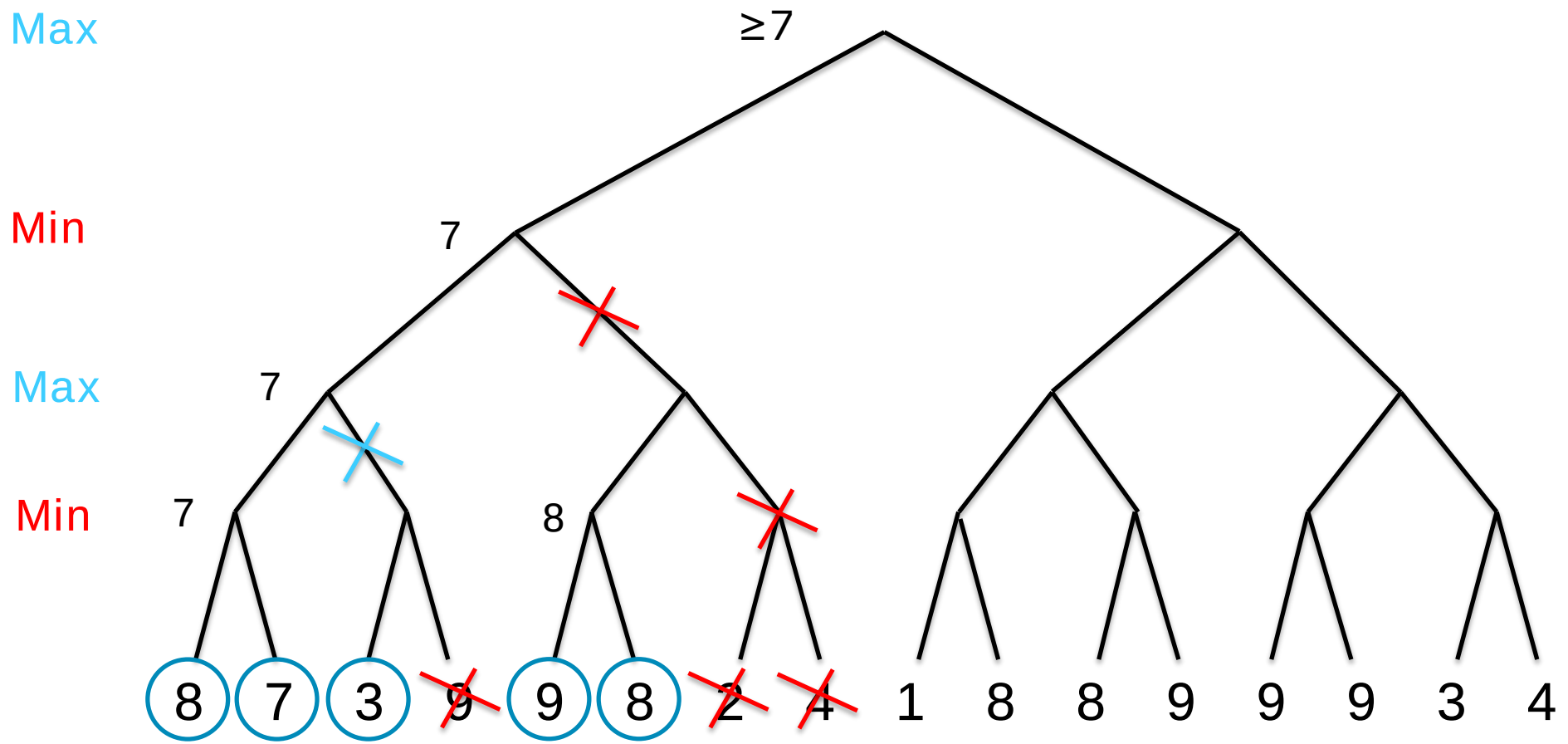
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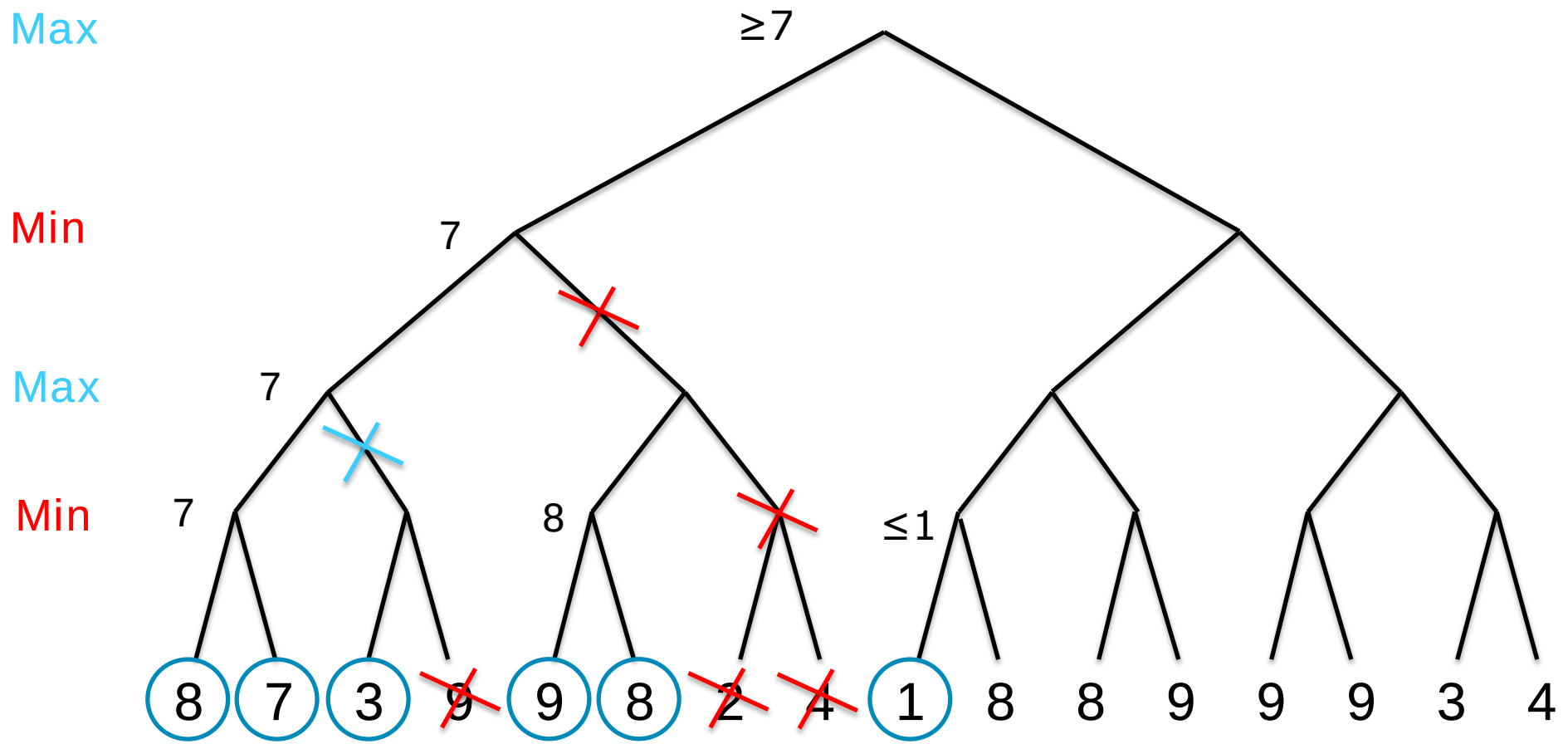
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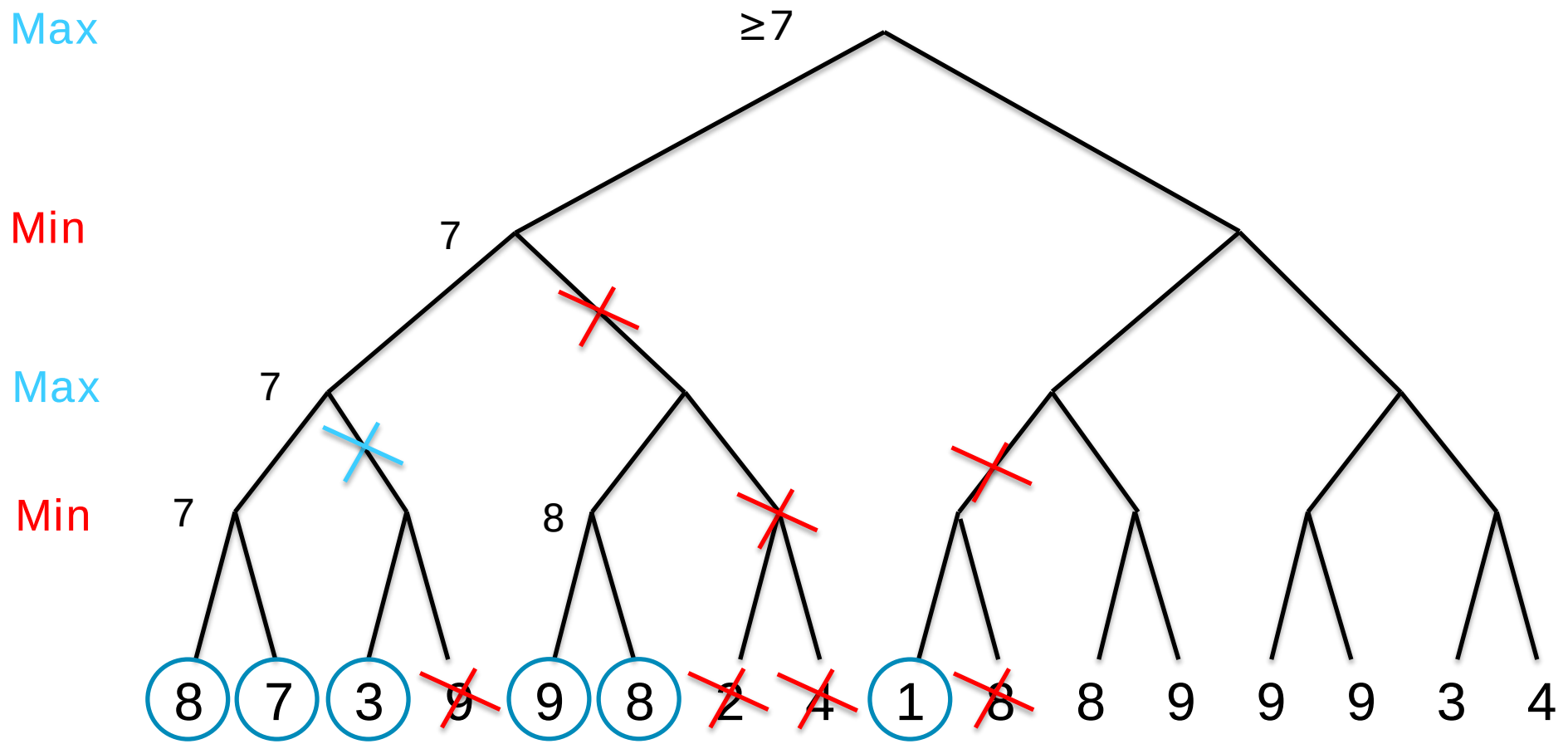
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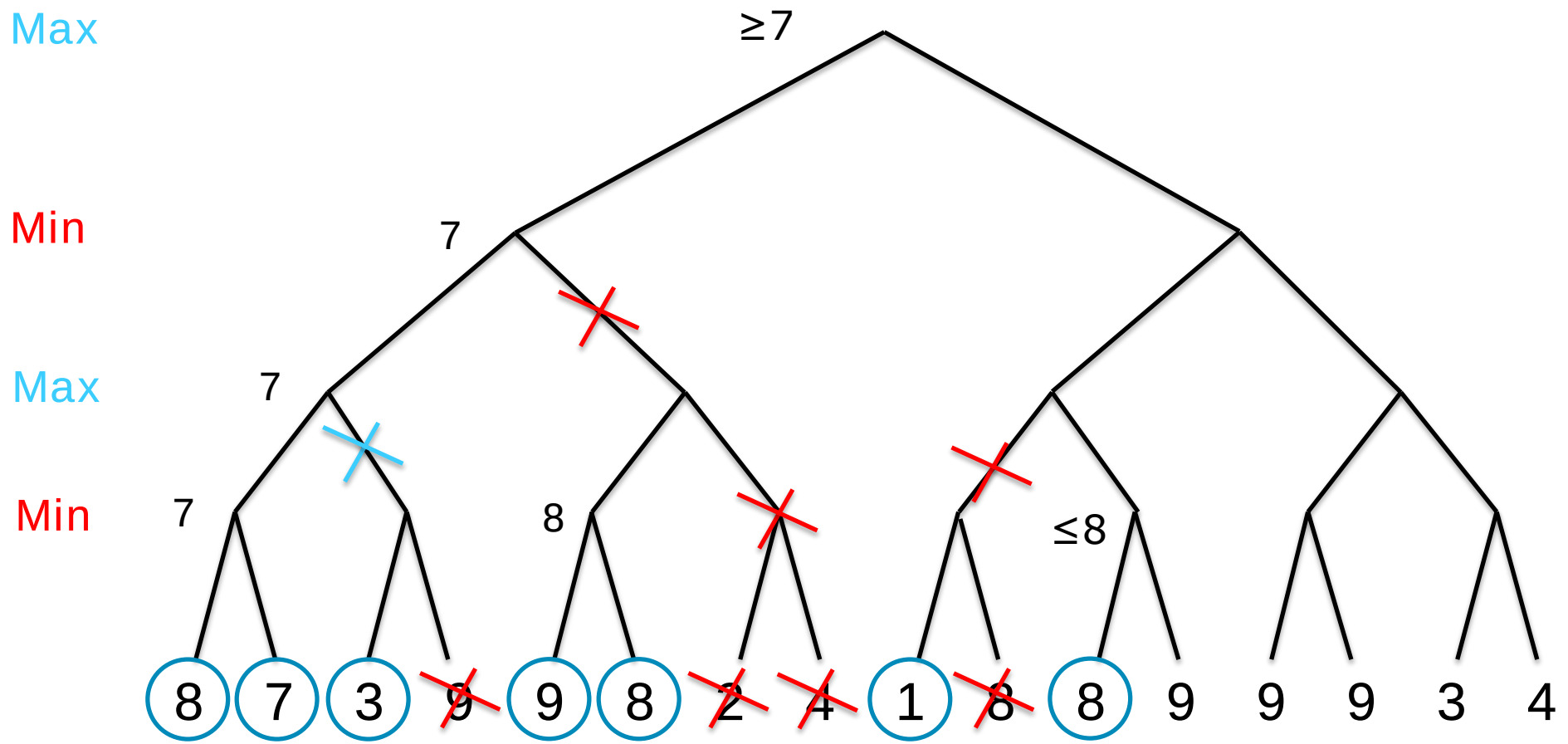
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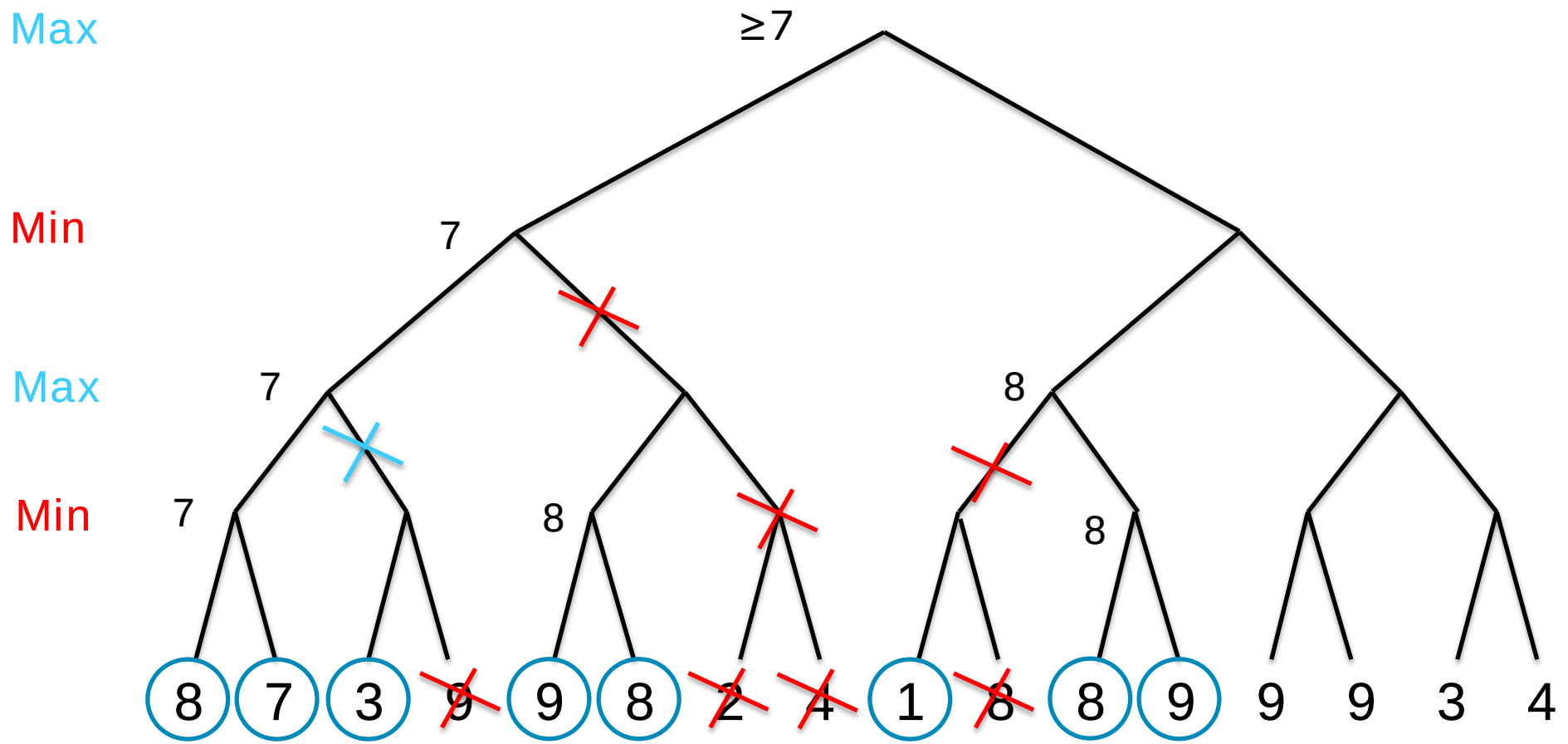
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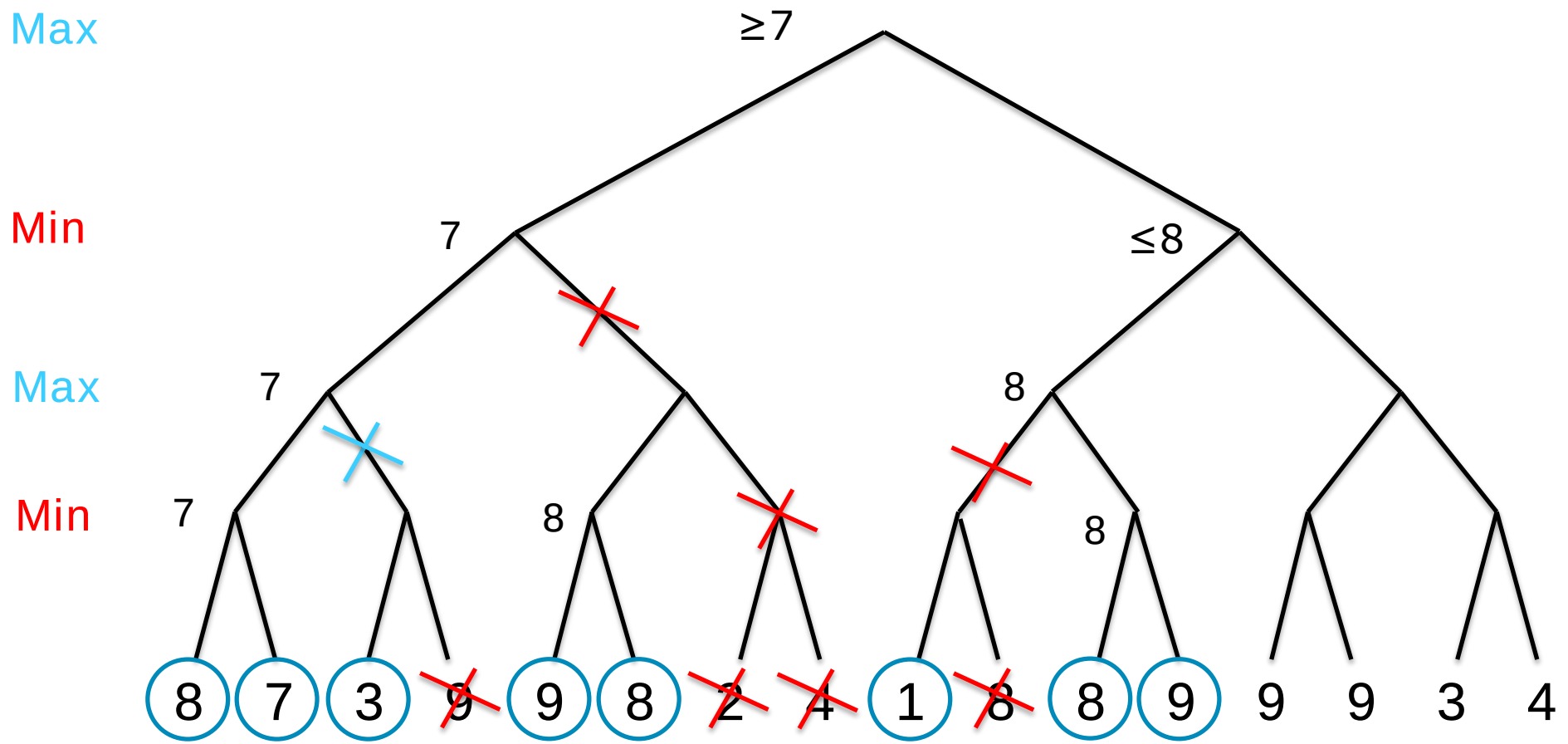
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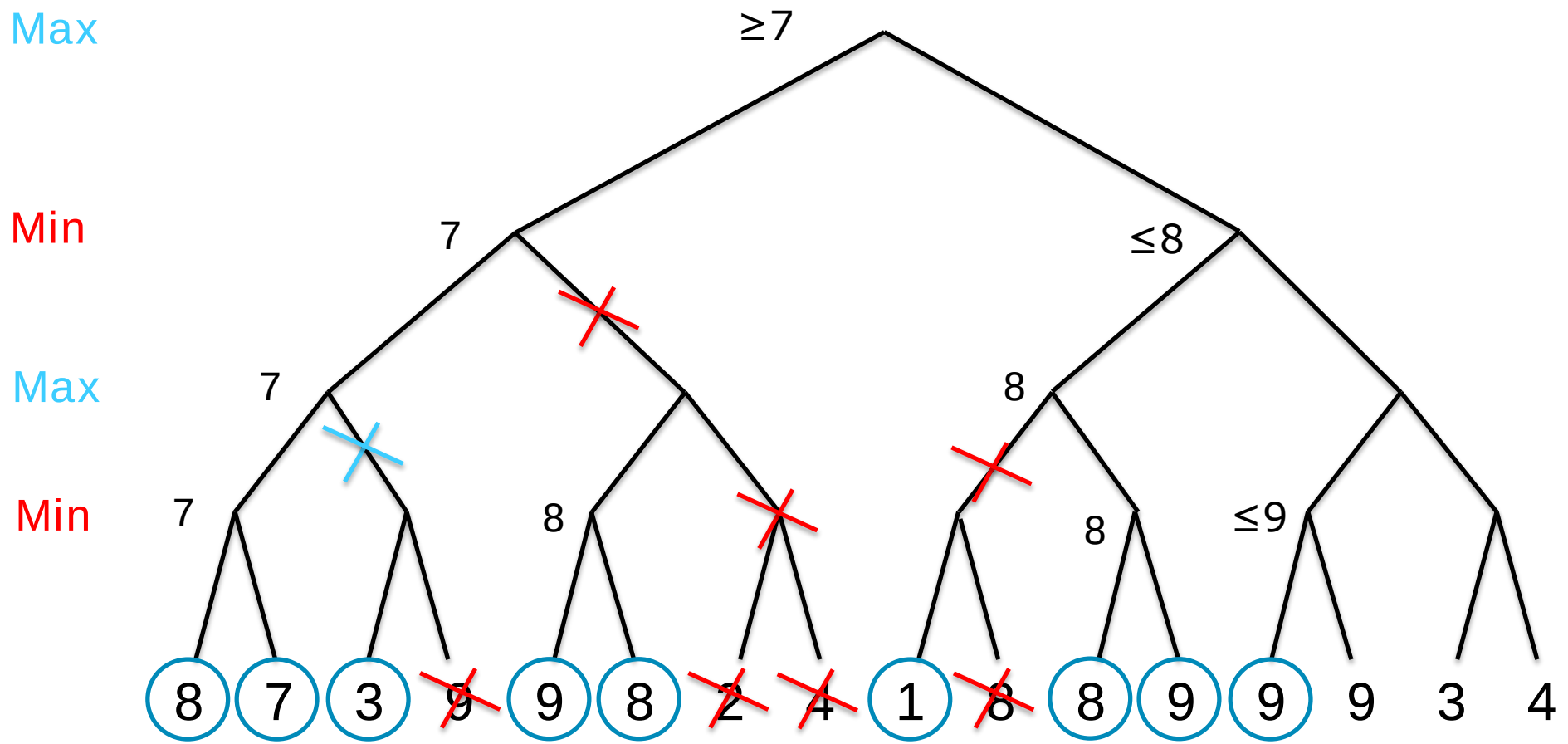
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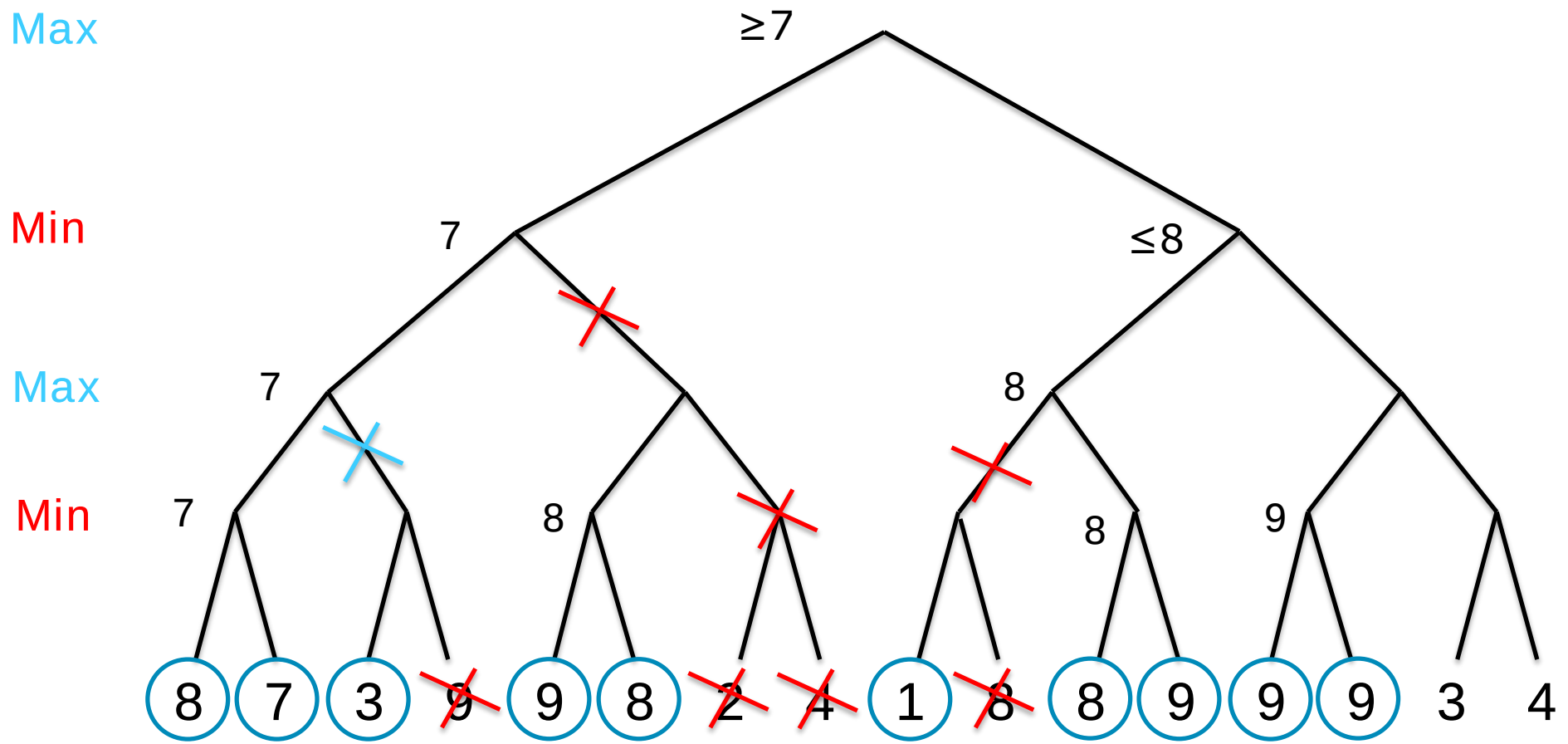
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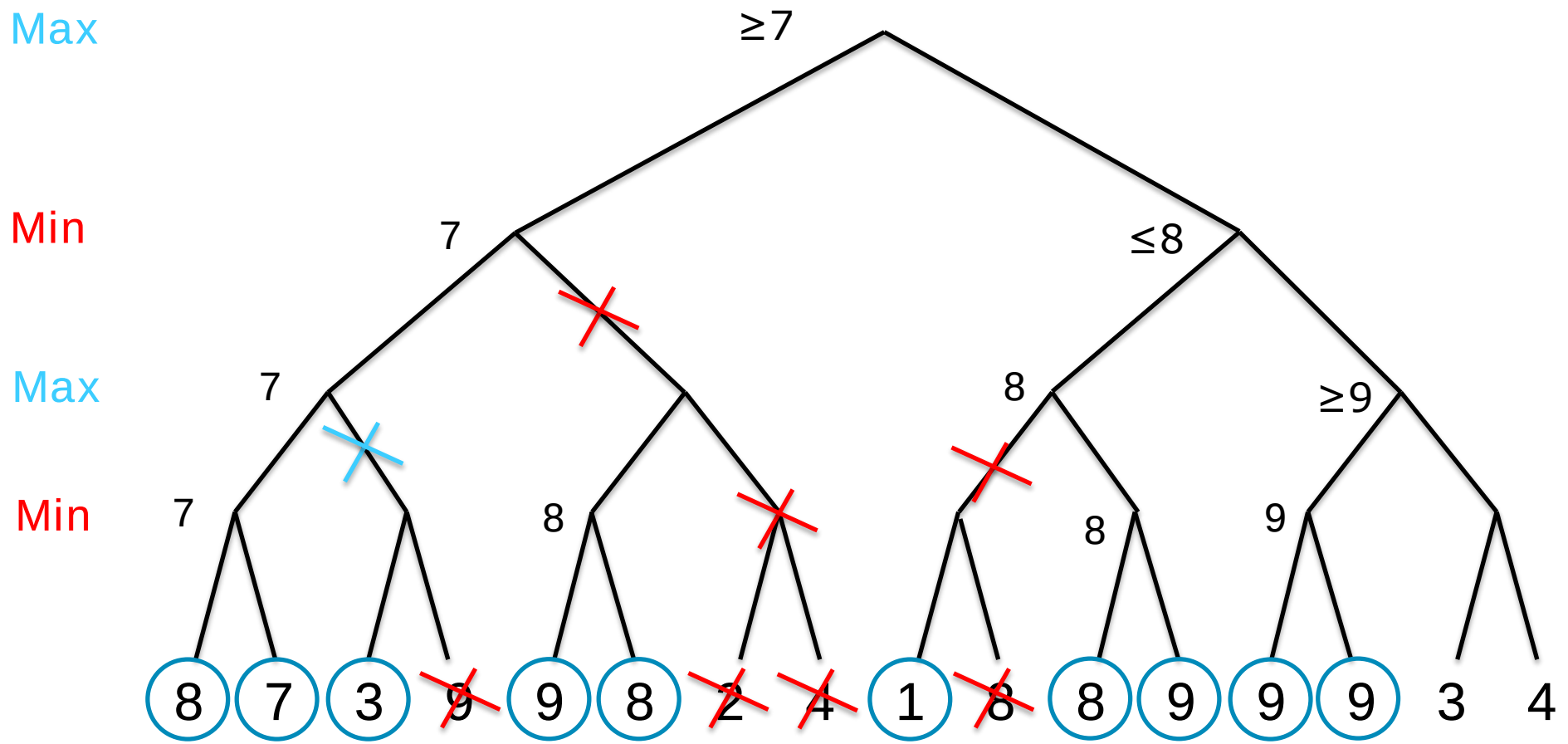
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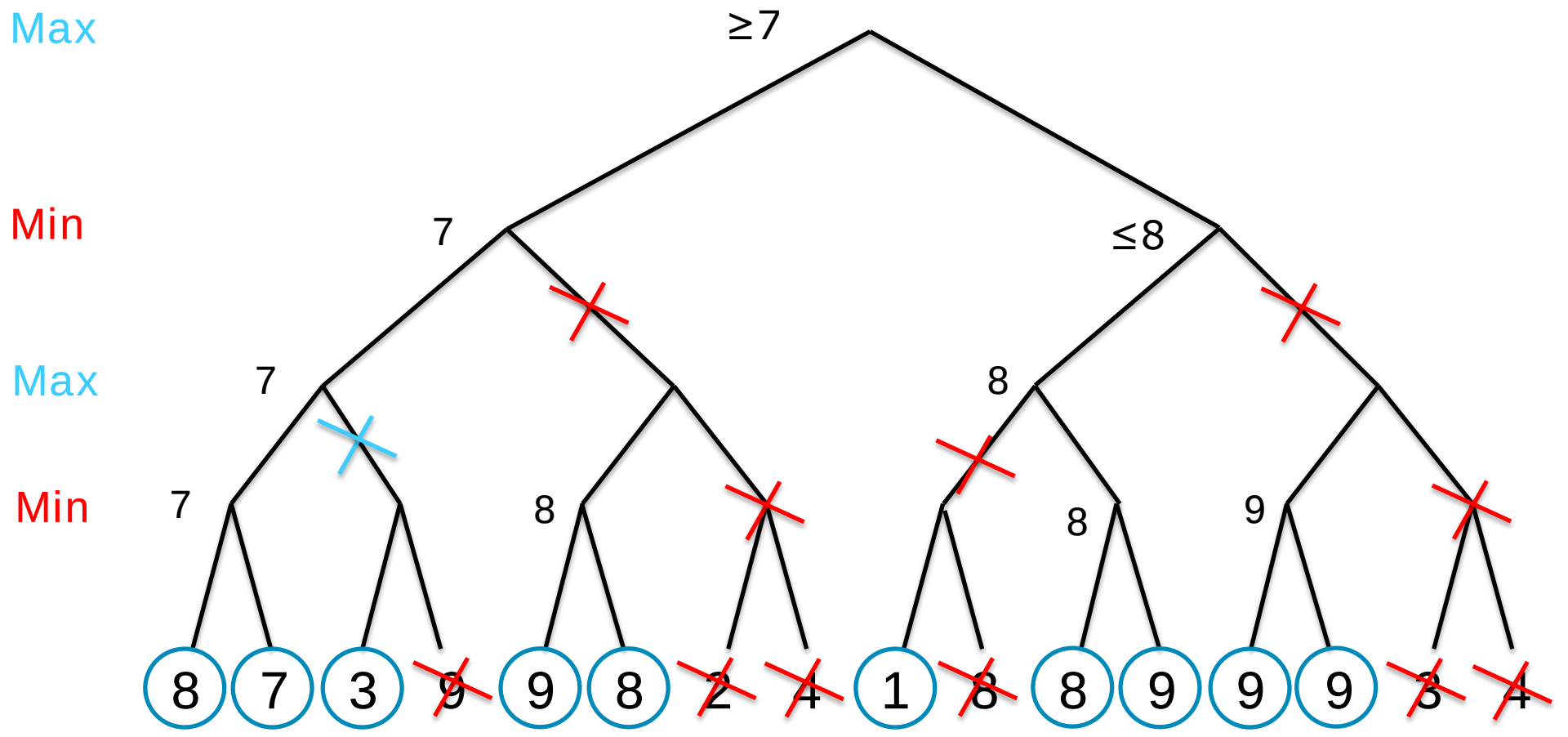
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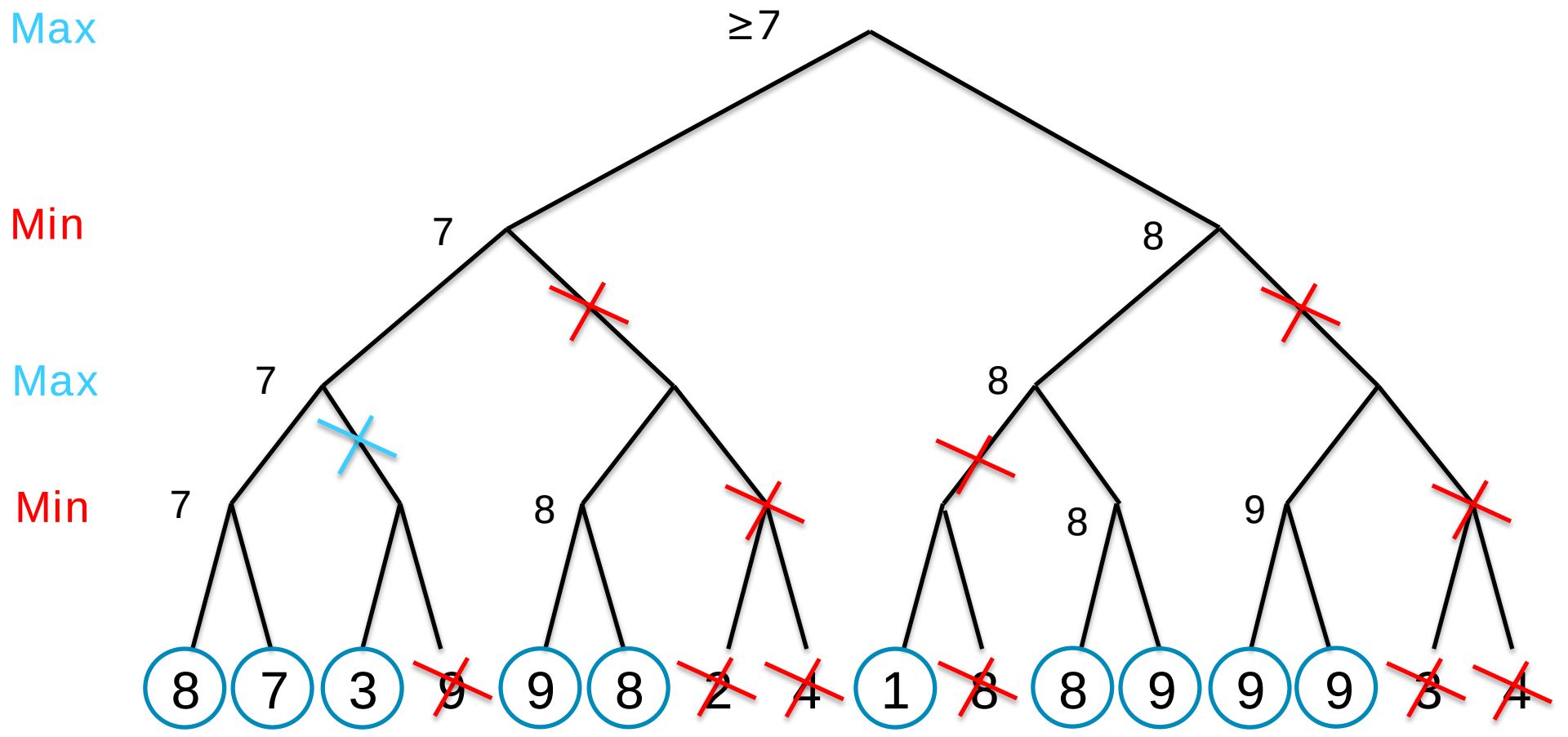
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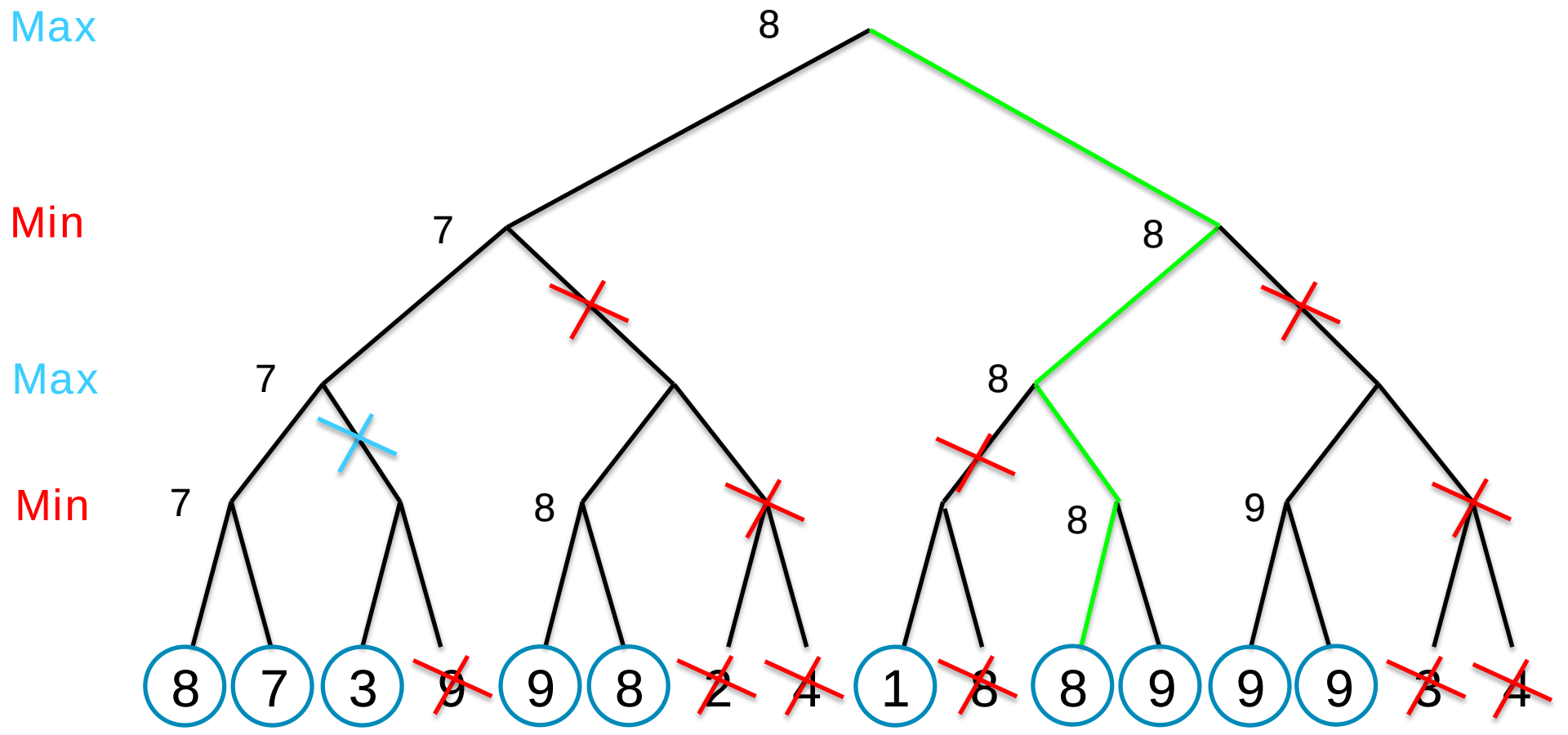
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Computational Savings

- MiniMax Tree Search
  - Nodes searched = b^d
- Alpha-Beta Search
  - Nodes searched ~ $2b^{d/2}$ (optimal savings)
- Depth 7 search becomes depth 14
  - Bad program beats world champion

Alpha-Beta Algorithm

```
Function AlphaBeta(node, depth, alpha, beta, maxPlayer)
        if (depth > maxDepth or node.terminal)
            return eval(node)
        if (maxPlayer)
            value = -infinity
            for each child of node
                value = max(value, AlphaBeta(child, depth+1, alpha, beta, false))
                if (depth == 0 and value > alpha) best_action = child.action
                alpha = max(alpha, value)
                if (alpha >= beta) break
            return value
        else
            value = infinity
            for each child of node
                value = min(value, AlphaBeta(child, depth+1, alpha, beta, true))
                beta = min(beta, value)
                if (beta <= alpha) break
            return value
Initial Call: AlphaBeta(root, 0, -infinity, +infinity, true)
```

Alpha-Beta Algorithm (shorter)

```
Function AlphaBeta(node, depth, alpha, beta, maxPlayer)
        if (depth > maxDepth or node.terminal)
            return eval(node)
        for each child of node
            value = AlphaBeta(child, depth+1, alpha, beta, !maxPlayer)
            if (maxPlayer and (value > alpha))
                if (depth == 0) best_action = child.action
                alpha = value
            if (!maxPlayer and (value < beta)) beta = value
            if (alpha >= beta) break
        return alpha if maxPlayer else beta
Initial Call: AlphaBeta(root, 0, -infinity, +infinity, true)
```