MEMORIAL UNIVERSITY

COMP 4752
Computational Intelligence

Lecture 9

Game Theory

Game 1: Grades Game

- You choose α they choose α: B-
- You choose α they choose β: A
- You choose β they choose α : C
- You choose β they choose β: B+

Game 2: Choosing Numbers

- Everyone in class secretly choose a number between 1 and 100
- We will calculate the average number chosen for everyone in class
- The winner will be the person closest to 2/3 the average number chosen
- Winner gets \$5 – diff cents

Game 1: Grades Game

Partner

Me

	α	β
α	B-	A
β	C	B+

My Grades

Game 1: Grades Game

My Grades

Me \ Partner	α	β
α	B-	A
β	C	B+

Partner Grades

Me \ Partner	α	β
α	B-	C
β	A	B+

Grades Game

Partner

Me		α	B
	α	B-, B-	A, C
	β	C, A	B+, B+

Grade Results

Grades Game

- We have actions, strategies outcomes
- Not yet a game, what are we missing?
- Payoffs
  - What is our objective or goal in the game?
  - Game theory can’t help decide your objectives / payoffs
  - If you have payoffs, GT can help you achieve them
- Examples:
  - We care about our own grade
  - We care about others grades

Grades Game Payoffs

Partner

Me		ɑ	B
	ɑ	B-, B-	A, C
	β	C, A	B+, B+

Grade Results

Partner

Me		ɑ	β
	ɑ	0, 0	3, -1
	β	-1, 3	1, 1

Payoff Matrix

Grades Game Payoffs

Partner

Me	α	β
α	0, 0	3, -1
β	-1, 3	1, 1

Payoff Matrix

- Numbers = Utility
- Player should attempt to maximize utility
- This example
  - (B-,B-) -> (0)
  - (A, C) -> (3)
  - Player cares about self

Grades Game Payoffs

Partner

Me		α	β
	α	0, 0	3, -1
	β	-1, 3	1, 1

Payoff Matrix

- What should you do?
- Consider both choices
- Partner choose α
  - You α = 0, β = -1
- Partner choose β
  - You α = 3, β = 1
- α is best choice

Strategy Domination

- We say that a strategy (α) strictly dominates a strategy (β) if my payoff from alpha is strictly greater than that of beta regardless of what others do
- Do not play a strictly dominated strategy
  - The strategy that dominates it is better in every possible case

Prisoner’s Dilemma

- Two prisoners in separate cells
- Guard asks each to rat out the other
- If neither rat each other out
  - Both go to jail for a year
- If both rat each other out
  - Both go to jail for two years
- If one rats and the other doesn’t
  - The ratter goes free, other goes to jail for 3 years

Prisoner's Dilemma Payoffs

Prisoner 2

Prisoner 1

	Rat	!Rat
Rat	-1, -1	0, -5
!Rat	-5, 0	-2, -2

Payoff Matrix

- Prisoner 2 Rat
  - 1 Rat = -1, 1 !Rat = -5
- Prisoner 2 !Rat
  - 1 Rat = 0, 1 !Rat = -2

- Ratting out the other prisoner strictly dominates not ratting them out

Split or Steal Game Show

- Two contestants choose secretly and simultaneously reveal split or steal
- Both split: each gets half
- Both steal: both get 0
- Split/Steal:
  - Stealer gets it all

Split or Steal Payoffs

Player 2

Player 1

	Split	Steal
Split	½, ½	0, 1
Steal	1, 0	0, 0

Payoff Matrix

- Stealing strictly dominates Splitting
- Game creates drama by allowing players to talk to each other and try and make a deal

Grades Game – Diff Payoffs

Partner

	α	β
Me: α	0, 0	-1, -3
Me: β	-3, -1	1, 1

Payoff Matrix

- What if players cared about each other?
  - -1 = 3-4 (guilt)
  - -3 = -1-2 (anger)
- No dominated strategy
- Best case: β gets 1
- Worst case: α gets -1

Grade Game – Mixed Players

- What if we are greedy, and we know that the other player is not?
- Will choice be different if we know the strategy / payoffs of the other player are different?

Grades Game

Greedy Players

Me \ Partner	α	β
α	0, 0	3, -1
β	-1, 3	1, 1

Caring Players

Me \ Partner	α	β
α	0, 0	-1, -3
β	-3, -1	1, 1

Grades Game – Greedy Player

Partner (columns), Me (rows)

	α	β
α	0, 0	3, -1
β	-1, 3	1, 1

Greedy Players

Partner (columns), Me (rows)

	α	β
α	0, 0	-1, -3
β	-3, -1	1, 1

Caring Players

Grades Game – Greedy Player

Caring

Greedy		α	β
	α	0, 0	3, -3
	β	-1, -1	1, 1

Payoff Matrix

- As greedy player
- α dominates β

Grades Game – Caring Player

Partner

Me	α	β
α	0, 0	-1, -3
β	-3, -1	1, 1

Caring Players

Partner

Me	α	β
α	0, 0	3, -1
β	-1, 3	1, 1

Greedy Players

Grades Game – Caring Player

Greedy

Caring

	α	β
α	0, 0	-1, -1
β	-3, 3	1, 1

Payoff Matrix

- As caring player, which strategy to choose?
- Nothing dominates other
- But greedy α dominates β
- We know greedy will choose α
- Our best response to greedy selection of α is α

- Putting yourself in other's shoes to figure out what they will do

What makes a Game?

• Players	i, j	you	
• Strategy	s_i	particular strategy of i	13
	S_i	set of all strategies of i	[1..100]
	s	strategy profile	sheets
	s-i	choices for all but i	
• Payoff	$u_i(s)$	utility / payoff	$5 / 0

- Assume that these are all known values

Example Game

Player 2

Player 1

	L	C	R
T	5, -1	11, 3	0, 0
B	6, 4	0, 2	2, 0

Payoff Matrix

- Players = 1, 2
- Strategy Sets
  - S1 = [T, B]
  - S2 = [L, C, R]
- Payoffs
  - U1(T, C) = 11
  - U2(B, L) = 4

Example Game

Player 2

Player 1

	L	C	R
T	5, -1	11, 3	0, 0
B	6, 4	0, 2	2, 0

Payoff Matrix

- Does player 1 have a dominated strategy?
- No, it doesn't
- Does player 2 have a dominated strategy?
- Yes, C dominates R

Example Game

Player 2

Player 1

	L	C	R
T	5, -1	11, 3	0, 0
B	6, 4	0, 2	2, 0

Payoff Matrix

- Does player 1 have a dominated strategy?
- No, it doesn't
- Does player 2 have a dominated strategy?
- Yes, C dominates R

Strictly Dominated Strategy

- Player i's strategy s_i' is strictly dominated by player i's strategy s_i if:

- $u_i(s_i, s\text{-}i) > u_i(s_i', s\text{-}i)$ for all s-i

Game: Defense Choice

- You are a general defending a city
- There are two paths leading into the city
  - “Easy” path, “Hard” Path
- You can only defend one of these paths
- If attacker chooses hard path, loses 1 army
- If your armies meet, attacker loses 1 army

Defense Game

		Attacker	
		E	H
Defense	E	1, 1	1, 1
	H	0, 2	2, 0

Payoff Matrix

- Attacker Payoff
  - Armies that reach city
- Defense Payoff
  - Attacking armies killed
- Does defense have a dominated strategy?

Defense Game

Attacker

Defense

	E	H
E	1, 1	1, 1
H	0, 2	2, 0

Payoff Matrix

- Does attacker have a dominated strategy?
- Easy weakly dominates hard, in that it will do at least as well or better than hard

Weakly Dominated Strategy

- Player i's strategy s_i' is weakly dominated by player i's strategy s_i if:

- $u_i(s_i, s\text{-}i) >= u_i(s_i', s\text{-}i)$ for all s-i
- $u_i(s_i, s\text{-}i) > u_i(s_i', s\text{-}i)$ for at least one s-i