COMP 4752
Computational Intelligence

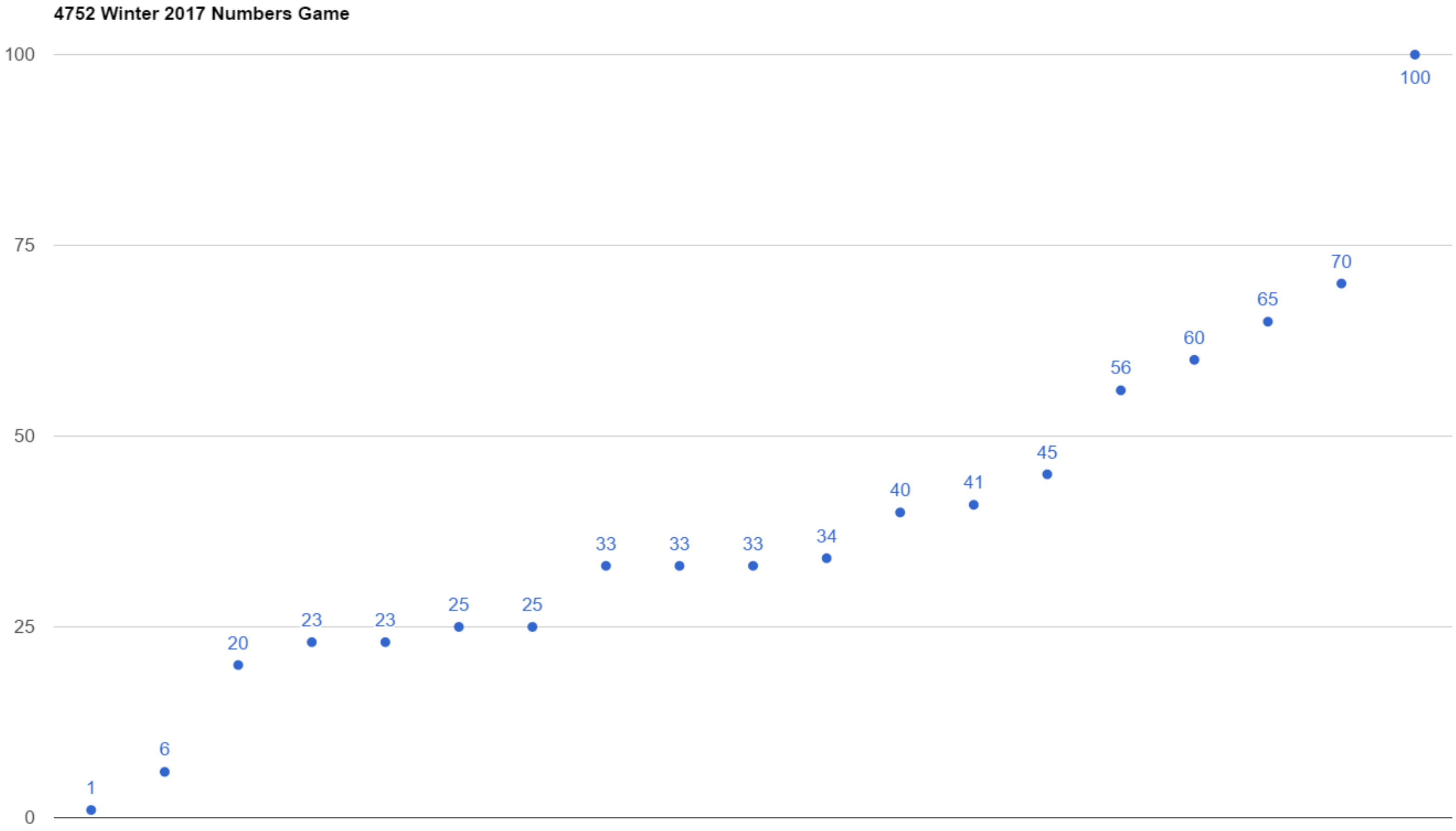
Lecture 11

Game Theory 2

Nash Equilibrium

Last Time: Choosing Numbers

- Everyone in class secretly choose a number between 1 and 100
- We will calculate the average number chosen for everyone in class
- The winner will be the person closest to 2/3 the average number chosen
- Winner gets $5 – diff cents


4752 Winter 2017 Numbers Game
100
75
50
25
0
1
6
20
23
23
25
25
33
33
33
34
40
41
45
56
60
65
70
100

Numbers Game

- Class Results
  - 19 Numbers Chosen
  - Average = 38.578
  - 2/3 Average = 25.719
- No Troll Results
  - Average = 35.167
  - 2/3 Average = 23.44

1
6
20
23
23
25
25
33
33
33
34
40
41
45
56
60
65
70
100

Numbers Game

- Class Results
  - 19 Numbers Chosen
  - Average = 38.578
  - 2/3 Average = 25.719
- No Troll Results
  - Average = 35.167
  - 2/3 Average = 23.44

	1
	6
	20
Kris Hart	23
Josh Davis	23
Rick Kelly	25
Matthew Randell	25
	33
	33
	33
	34
	40
	41
	45
	56
	60
	65
	70
	100

Numbers Game

- Are there dominated strategies?

- If everyone chose max value of 100
  - Average = 100
  - 2/3 Average = 67

- [68... 100] dominated by 67

Numbers Game

- Can we keep going?
- If nobody plays a dominated strategy
- Strategy set is now [1... 67]
- Everyone picks max of 67
  - Average is 67
  - 2/3 of average is 45
- So once we delete [68... 100], [46... 67] become dominated strategies (dominated by 45)
- Keep repeating... down to 1

Rationality Chain

- [68.... 100] Rational
- [46... 67] Rational + Know Others Are
- [31... 45] Rational + KR + KKR
- ...
- Common Knowledge
  - I know that you know that I know... (to inf)
  - Different from Mutual Knowledge

Best Response

- "The best you can do, given your belief about what other people are doing"

Grades Game – Greedy Player

Partner

Me

	α	β
α	0, 0	3, -1
β	-1, 3	1, 1

Greedy Players

Partner

Me

	α	β
α	0, 0	-1, -3
β	-3, -1	1, 1

Caring Players

Grades Game – Greedy Player

Caring

Greedy		α	β
	α	0, 0	3, -3
	β	-1, -1	1, 1

Payoff Matrix

- As greedy player
- α dominates β

Grades Game – Caring Player

Partner

Me	α	β
α	0, 0	-1, -3
β	-3, -1	1, 1

Caring Players

Partner

Me	α	β
α	0, 0	3, -1
β	-1, 3	1, 1

Greedy Players

Grades Game – Caring Player

Greedy

Caring		α	β
	α	0, 0	-1, -1
	β	-3, 3	1, 1

Payoff Matrix

- As caring player, which strategy to choose?
- Nothing dominates other
- But greedy α dominates β
- We know greedy will choose α

- Our best response to greedy selection of α is α

Nash Equilibrium (NE)

- A strategy profile (s_1*, s_2*, ..., s_n*) is a NE if for each player i, their choice is a best response to the other players' choices

Motivations for NE

1. No Regrets
    - If we hold the strategies of everyone else fixed, no individual player i has any strict incentive to deviate (change their strategy)
2. Self Fulfilling
    - If everyone believes everyone else is going to play the NE, then they will

Nash Equilibrium Example

Player 2

Player 1

	L	C	R
U	0, 4	4, 0	5, 3
M	4, 0	0, 4	5, 3
D	3, 5	3, 5	6, 6

Payoff Matrix

Nash Equilibrium Example

Player 2

Player 1

	L	C	R
U	0, 4	4, 0	5, 3
M	4, 0	0, 4	5, 3
D	3, 5	3, 5	6, 6

Payoff Matrix

- What is $BR_1(L)$?

Nash Equilibrium Example

Player 2

Player 1		L	C	R
	U	0, 4	4, 0	5, 3
	M	4, 0	0, 4	5, 3
	D	3, 5	3, 5	6, 6

Payoff Matrix

- What is $BR_1(L)$? M

Nash Equilibrium Example

Player 2

Player 1	L	C	R
U	0, 4	4, 0	5, 3
M	4, 0	0, 4	5, 3
D	3, 5	3, 5	6, 6

Payoff Matrix

- What is $BR_1(L)$? M
- What is $BR_1(C)$?

Nash Equilibrium Example

Player 2

Player 1		L	C	R
	U	0, 4	4, 0	5, 3
	M	4, 0	0, 4	5, 3
	D	3, 5	3, 5	6, 6

Payoff Matrix

- What is $BR_1(L)$? M
- What is $BR_1(C)$? U

Nash Equilibrium Example

Player 2

Player 1	L	C	R
U	0, 4	4, 0	5, 3
M	4, 0	0, 4	5, 3
D	3, 5	3, 5	6, 6

Payoff Matrix

- What is $BR_1(L)$? M
- What is $BR_1(C)$? U
- What is $BR_1(R)$?

Nash Equilibrium Example

Player 2

Player 1		L	C	R
	U	0, 4	4, 0	5, 3
	M	4, 0	0, 4	5, 3
	D	3, 5	3, 5	6, 6

Payoff Matrix

- What is $BR_1(L)$? M
- What is $BR_1(C)$? U
- What is $BR_1(R)$? D

Nash Equilibrium Example

Player 2

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	L	C	R
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M	4, 0	0, 4	5, 3
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Payoff Matrix

- What is $BR_1(L)$? M
- What is $BR_1(C)$? U
- What is $BR_1(R)$? D

- What is $BR_2(U)$?

Nash Equilibrium Example

Player 2

Player 1

	L	C	R
U	0, 4	4, 0	5, 3
M	4, 0	0, 4	5, 3
D	3, 5	3, 5	6, 6

Payoff Matrix

- What is $BR_1(L)$? M
- What is $BR_1(C)$? U
- What is $BR_1(R)$? D

- What is $BR_2(U)$? L

Nash Equilibrium Example

Player 2

Player 1	L	C	R
U	0, 4	4, 0	5, 3
M	4, 0	0, 4	5, 3
D	3, 5	3, 5	6, 6

Payoff Matrix

- What is $BR_1(L)$? M
- What is $BR_1(C)$? U
- What is $BR_1(R)$? D

- What is $BR_2(U)$? L
- What is $BR_2(M)$?

Nash Equilibrium Example

Player 2

Player 1

	L	C	R
U	0, 4	4, 0	5, 3
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Payoff Matrix

- What is $BR_1(L)$? M
- What is $BR_1(C)$? U
- What is $BR_1(R)$? D

- What is $BR_2(U)$? L
- What is $BR_2(M)$? C

Nash Equilibrium Example

Player 2

Player 1	L	C	R
U	0, 4	4, 0	5, 3
M	4, 0	0, 4	5, 3
D	3, 5	3, 5	6, 6

Payoff Matrix

- What is $BR_1(L)$? M
- What is $BR_1(C)$? U
- What is $BR_1(R)$? D

- What is $BR_2(U)$? L
- What is $BR_2(M)$? C
- What is $BR_2(M)$?

Nash Equilibrium Example

Player 2

Player 1

	L	C	R
U	0, 4	4, 0	5, 3
M	4, 0	0, 4	5, 3
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Payoff Matrix

- What is $BR_1(L)$? M
- What is $BR_1(C)$? U
- What is $BR_1(R)$? D

- What is $BR_2(U)$? L
- What is $BR_2(M)$? C
- What is $BR_2(D)$? R

Nash Equilibrium Example

Player 2

Player 1	L	C	R
U	0, 4	4, 0	5, 3
M	4, 0	0, 4	5, 3
D	3, 5	3, 5	6, 6

Payoff Matrix

- What is $BR_1(L)$? M
- What is $BR_1(C)$? U
- What is $BR_1(R)$? D

- What is $BR_2(U)$? L
- What is $BR_2(M)$? C
- What is $BR_2(D)$? R

- What is the NE?

Nash Equilibrium Example

Player 2

Player 1

	L	C	R
U	0, 4	4, 0	5, 3
M	4, 0	0, 4	5, 3
D	3, 5	3, 5	6, 6

Payoff Matrix

- What is $BR_1(L)$? M
- What is $BR_1(C)$? U
- What is $BR_1(R)$? D

- What is $BR_2(U)$? L
- What is $BR_2(M)$? C
- What is $BR_2(D)$? R

- What is the NE? (D,R)

Nash Equilibrium Example 2

Player 2

Player 1

	L	C	R
U	0, 2	2, 3	4, 3
M	11, 1	3, 2	0, 0
D	0, 3	1, 0	8, 0

Payoff Matrix

Nash Equilibrium Example 2

Player 2

Player 1

	L	C	R
U	0, 2	2, 3	4, 3
M	11, 1	3, 2	0, 0
D	0, 3	1, 0	8, 0

Payoff Matrix

Nash Equilibrium Example 2

Player 2

Player 1

	L	C	R
U	0, 2	2, 3	4, 3
M	11, 1	3, 2	0, 0
D	0, 3	1, 0	8, 0

Payoff Matrix

Nash Equilibrium Example 2

Player 2

Player 1	L	C	R
U	0, 2	2, 3	4, 3
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D	0, 3	1, 0	8, 0

Payoff Matrix

Nash Equilibrium Example 2

Player 2

Player 1

	L	C	R
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Payoff Matrix

Nash Equilibrium Example 2

Player 2

Player 1

	L	C	R
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Payoff Matrix

Nash Equilibrium Example 2

Player 2

Player 1

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Payoff Matrix

Nash Equilibrium Example 2

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	D	0, 3	1, 0	8, 0

Payoff Matrix

Relate NE to Dominance

Partner

Me

	α	β
α	0, 0	3, -1
β	-1, 3	1, 1

Payoff Matrix

- α strictly dominates β
- Let's find the NE

Relate NE to Dominance

Partner

		α	β
Me	α	0, 0	3, -1
	β	-1, 3	1, 1

Payoff Matrix

- α strictly dominates β
- Let’s find the NE

Relate NE to Dominance

Partner

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Payoff Matrix

- α strictly dominates β
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Relate NE to Dominance

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Me

Payoff Matrix

- α strictly dominates β
- Let’s find the NE

Relate NE to Dominance

Partner

		α	β
Me	α	0, 0	3, -1
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Payoff Matrix

- α strictly dominates β
- Let's find the NE

Relate NE to Dominance

Partner

Me

	α	β
α	0, 0	3, -1
β	-1, 3	1, 1

Payoff Matrix

- α strictly dominates β
- Let's find the NE
- No strictly dominated strategy can ever be played in an NE

Relate NE to Dominance

Partner

Me

	α	β
α	1, 1	0, 0
β	0, 0	0, 0

Payoff Matrix

Relate NE to Dominance

Partner

Me		α	β
	α	1, 1	0, 0
	β	0, 0	0, 0

Payoff Matrix

- More than one NE can exist in a game
- Weakly dominated strategies can be chosen
- In a NE, no player can do STRICTLY better by changing their strategy

Investment Game

- Players: Everyone
- Strategy Set: [Invest $10, Don't Invest]
- Payoffs:
  - If you don't invest, you gain/lost nothing
  - If you invest
    - If >= 90% of people invested, you gain $5
    - If < 90% of people invested, you lost $10

Investment Game NE

- What is the NE for Investment Game?

- Two Nash Equilibria:
  - Nobody Invests
  - Everyone Invests