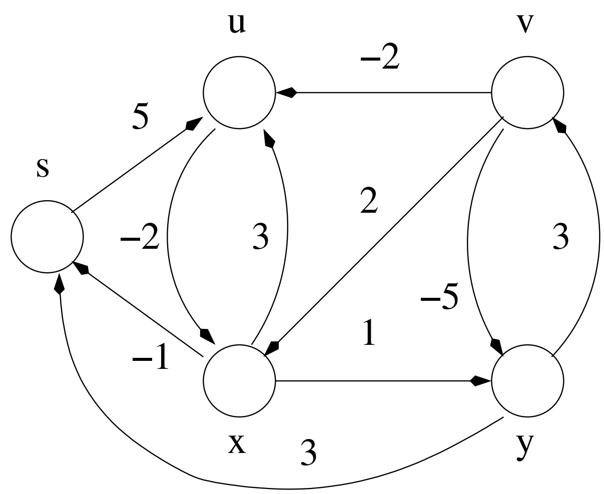
Computer Science 6901 (Fall 2026):
Problem-set #3

1. Consider the following edge-weighted directed graph:



a) Run Dijkstra's algorithm (p, 595) on the directed graph above using vertex x as the source vertex. In the style of Figure 24.6 in the textbook, show the d and π values and the vertices in set S after each iteration of the **while** loop.

b) Run the Bellman-Ford algorithm (p, 588) on the directed graph above using vertex x as the source vertex. Relax edges in lexicographic order in each pass, and in the style of Figure 24.4 on the textbook, show the d and π values after each pass. Finally, give the boolean value returned by the algorithm.

2. Consider the following decision problems:

DUMBBELL SUBGRAPH (DS)
Input: An undirected graph $G = (V, E)$ and two positive integers $k, l \geq 1$.
Question: Are there two cliques C_1 and C_2 and a simple path P in G such that C_1 and C_2 have $\geq k$ vertices apiece, P has $\geq l$ edges, P connects C_1 and C_2, the cliques and path do not have any edges in common, and the only vertices that P shares with C_1 (C_2) is its connection-vertex?

BOUNDED-WEIGHT SUBSET COVER (BWSSC)
Input: A set $I = \{i_1, \ldots, i_a\}$ of items, a set $R = \{r_1, \ldots, r_b\}$ of subsets of I, an integer-valued subset-weight function $w()$ such that for each $r_x \in R$, $w(r_x) > 0$, a subset $N \subseteq I$, and integers $0 < k_1 \leq k_2$.
Question: Is there is a subset $R' \subseteq R$ such that $\cup_{r \in R'} r = N$ and $k_1 \leq \sum_{r \in R'} w(r) \leq k_2$?

a) Prove that problem DS is NP-complete by (1) showing that this problem is in NP and (2) giving a polynomial-time many-one reduction (algorithm + proof of correctness) to this problem from an NP-hard problem.

b) Prove that problem BWSSC is NP-complete by (1) showing that this problem is in NP and (2) giving a polynomial-time many-one reduction (algorithm + proof of correctness) to this problem from an NP-hard problem.