The Critical Transmitting Range for Connectivity in Mobile Ad Hoc Networks

Paolo Santi

Abstract—In this paper, we study the critical transmitting range (CTR) for connectivity in mobile ad hoc networks. We prove that $r_{\mathcal{M}} = c\sqrt{\frac{\ln n}{\pi n}}$ for some constant $c \geq 1$, where $r_{\mathcal{M}}$ is the CTR in the presence of $\mathcal{M}$-like node mobility and n is the number of network nodes. Our result holds for an arbitrary mobility model $\mathcal{M}$ such that: 1) $\mathcal{M}$ is obstacle free and 2) nodes are allowed to move only within a certain bounded area. We also investigate in detail the case of random waypoint mobility, which is the most common mobility model used in the simulation of ad hoc networks. Denoting with r_p^w the CTR with random waypoint mobility when the pause time is set to p and node velocity is set to v, we prove that $r_p^w = \frac{p+\frac{0.521405}{v}}{p}\sqrt{\frac{\ln n}{\pi n}}$ if $p > 0$ and that $r_0^w \gg \sqrt{\frac{\ln n}{n}}$. The results of our simulations also suggest that if n is large enough ($n \geq 50$), r_0^w is well approximated by $\frac{r}{4}\ln n$, where r is the critical range in case of uniformly distributed nodes. The results presented in this paper provide a better understanding of the behavior of a fundamental network parameter in the presence of mobility and can be used to improve the accuracy of mobile ad hoc network simulations.

Index Terms—Critical transmitting range, connectivity, random waypoint model, mobility modeling, ad hoc networks.

1 Introduction

One of the prominent features of ad hoc networks is mobility. As actual implementations of medium to large scale ad hoc networks are scarce to date, the performance of ad hoc networking protocols is evaluated primarily through simulation. Due to the lack of real world movement patterns, mobility is usually simulated by implementing synthetic mobility models. Several such models have been proposed in the literature, modeling both individual and group movement [6], [13], [14]. For a survey on mobility models for ad hoc networks, the reader is referred to [2], [9].

Several recent papers have investigated the accuracy of ad hoc network simulation in the presence of node mobility. In particular, two properties of mobile networks that could affect simulation accuracy have been identified: the *border effect* and, more recently, the *speed decay* phenomenon.

The border effect, which was first noticed in [4] and investigated in detail in [3], [5], [8], [21], [22], may occur when node mobility is constrained within a limited area. Depending on the mobility model and/or the mobility parameters, the presence of the border may bias the long-term node spatial distribution in the presence of mobility, which is no longer uniform. If this occurs, the accuracy of the mobile network's simulation could be impaired: In fact, the initial node distribution, which is usually uniform, is different from the long-term distribution, which is not uniform.

The speed decay phenomenon, which was first noticed in [25] and further investigated in [26], occurs when nodes' velocities are chosen at random in a certain interval $[v_{min}, v_{max}]$. If $v_{min} < v_{max}$, the asymptotic average node velocity is different from the average node velocity at the beginning of the simulation. This phenomenon becomes critical when $v_{min} = 0$ (as is often assumed when simulating ad hoc networks) since, in this case, the mobile system will eventually converge to a stationary one.

Due to the combination of the border effect and the speed decay phenomenon, the results obtained when simulating mobile networks could be quite inaccurate unless the simulation methodology is carefully chosen. This is especially true in the case of random-waypoint mobility (RWP) [14], which is the most common mobility model used in the literature. In this model, nodes are initially distributed in a (usually bidimensional) region R. Then, every node chooses uniformly at random a destination in R and moves toward it along a straight line with a velocity chosen uniformly at random in the interval $[v_{min}, v_{max}]$. When the node reaches the destination, it remains stationary for a predefined pause time p and, then, it starts moving again according to the same rule. It is known that the RWP model suffers from both the border effect [2], [4], [8] and the speed decay phenomenon [25], [26]. Thus, the performance of RWP mobile networks should be evaluated only after a certain "warm-up" period, which must be sufficient for the system to reach the "steady state" in terms of both node spatial distribution and average node velocity. If the system performance is evaluated during the "warm-up" period, the obtained results might be quite misleading. For instance, in [25], it is shown that the inaccuracy can be as high as 40 percent when evaluating certain ad hoc routing metrics.

In this paper, we make a step forward toward the definition of a methodology for accurate simulation of mobile networks. We consider the *critical transmitting range* (CTR) for connectivity, and we study how this network metric changes in the presence of node mobility. The CTR

- *The author is with the Istituto di Informatica e Telematica del CNR, Via G. Moruzzi 1, 56124, Pisa, Italy. E-mail: paolo.santi@iit.cnr.it.*

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corresponds to the minimum *common* value of the nodes' transmitting range that produces a connected communication graph.[1] It is known that setting the nodes' transmitting range to the critical value minimizes energy consumption while maximizing network capacity [12], [15].

Due to the possible occurrence of the border effect, the CTR in the presence of mobility is, in general, different from the critical transmitting range in the stationary case with uniformly distributed nodes. This observation discloses another potential source of inaccuracy in the simulation of mobile networks. Suppose we want to evaluate the performance of a routing protocol for mobile networks. The main effects of mobility on a routing protocol are: 1) frequent route reconfigurations and 2) occasional network disconnections. In order to fully understand the behavior of the protocol, the relative effects of items 1 and 2 on the routing performance should be carefully evaluated. It is clear that the frequency of network disconnections depends on the choice of the nodes' transmitting range: The larger the range, the less likely it is that the network becomes disconnected. On the other hand, for the reasons described above (energy consumption and network capacity), the nodes' transmitting range cannot be excessively large. Thus, setting the transmitting range to the critical value for connectivity is a reasonable choice. However, a wrong setting of the CTR might lead to an incorrect interpretation of the simulation results. For instance, if the CTR in the presence of mobility is larger than in the case of stationary networks[2] and the CTR is wrongly set as if the network were stationary, then there is a relatively high likelihood of generating a disconnected topology as the nodes move. In turn, this causes a relatively low packet delivery rate, which could be erroneously interpreted as a scarce protocol's ability to perform route maintenance (which might not be the case).

The example above outlines the importance of a correct estimation of the CTR when simulating mobile ad hoc networks. In this paper, we investigate the relation between the CTR in case of stationary networks with uniformly distributed nodes and the CTR in the presence of $\mathcal{M}$-like node mobility, where $\mathcal{M}$ is an arbitrary bounded and obstacle free mobility model (see Section 4 for the definition). Denoting with r and $r_{\mathcal{M}}$ the critical range in case of stationary and mobile networks, and denoting with n the number of network nodes, we prove that $\lim_{n\to\infty} \frac{r_{\mathcal{M}}}{r} = c$ for some constant $c \geq 1$.

We also investigate in detail the case of RWP mobility, which is the most common mobility model used in the simulation of ad hoc networks. Denoting with r_p^w the CTR with random waypoint mobility when the pause time is set to p and node velocity is $v_{min} = v_{max} = v$, we prove that

$$r_p^w = \frac{p + \frac{0.521405}{v}}{p} \sqrt{\frac{\ln n}{\pi n}}$$

if $p > 0$ and that $r_0^w \gg \sqrt{\frac{\ln n}{n}}$, otherwise. Besides studying the asymptotic properties of the CTR, we also give a formula that can be used to derive a good estimate of the CTR in the presence of mobility given the critical range for stationary networks. Based on the results of extensive simulations, we argue that, if n is large enough ($n \geq 50$), r_0^w is well approximated by $\frac{r}{4} \ln n$. The error obtained using this formula is below 6.4 percent. Since r_p^w when $p > 0$ is smaller than r_0^w, our formula provides an upper bound to the CTR is presence of arbitrary RWP mobility.

The rest of this paper is organized as follows: In Section 2, we overview some related work on the characterization of the CTR. In Section 3, we introduce a result taken from the theory of geometric random graphs, which we use in the remainder of the paper. In Section 4, we provide a partial characterization the CTR of mobile networks, and a more accurate characterization for the case of RWP mobility. This latter characterization is qualitatively confirmed by the simulation results presented in Section 5. Section 6 concludes.

2 Related Work

The critical transmitting range for connectivity has been investigated in several papers, mostly under the assumption of uniform node distribution.

In [11], R is the disk of unit area. The authors show that if the units' transmitting range is set to $r = \sqrt{\frac{\ln n + c(n)}{\pi n}}$, then the resulting network is connected with high probability (i.e., with probability 1 as n grows to infinity) if and only if $c(n) \to +\infty$. The minimum value of r which ensures connectivity with high probability is called the *critical transmitting range*. In [16], Panchapakesan and Manjunath obtain a similar expression for the critical transmitting range when nodes are distributed in the unit square $[0,1]^2$, which is shown to be $r = c\sqrt{\frac{\ln n}{n}}$, for some constant $c > 0$. The CTR for k-connectivity has been investigated in [1].

The results above assume that the deployment region R is fixed and investigate the asymptotic behavior of r as n grows to infinity, i.e., for increasing node density. This means that the results of [1], [11], [16] in principle can be used only for very dense networks. On the other hand, it is known that real ad hoc networks cannot be too dense due to the problem of spatial reuse: When a node is receiving a message, all the nodes within its interference range must be silent in order not to corrupt the transmission. If the node density is very high, many nodes must remain silent when a node is receiving, and the overall network capacity is compromised [12]. In order to circumvent this problem, we can let the area a of the deployment region R be a further parameter and characterize the critical transmitting range as $a \to +\infty$. Necessary and sufficient conditions for asymptotic connectivity in this model have been obtained in [24].

Although interesting, the results above are valid only for *stationary* wireless networks. To the best of our knowledge, the only analytical investigation of the CTR in the presence of mobility is the one reported in [7], where a lower bound to the CTR is computed through numerical integration. However, no explicit formula of the CTR in the presence of

1. In this paper, we assume that nodes use omnidirectional antennas and that any two nodes u and v can communicate through a bidirectional wireless link if and only if u is within v's range, and v is within u's range.

2. As proven in this paper, this is actually the case, at least for bounded, obstacle free mobility.

RWP mobility is given. Other papers have investigated the CTR in the presence of mobility by means of simulation [23], [24]: For instance, in [24], it is shown that a moderate increase in the transmitting range (on the order of 25 percent) with respect to the stationary case is sufficient to provide good connectivity in the presence of mobility.

3 GEOMETRIC RANDOM GRAPHS

The theory of geometric random graphs (GRG) has been often used in the derivation of analytical characterizations of the CTR.

In the theory of GRG, a set of points is distributed according to some probability density function (pdf) in a d-dimensional region, and some property of the resulting node placement is investigated. For example, the longest nearest neighbor link, the longest edge of the Euclidean Minimum Spanning Tree (MST), and the total cost of the MST have been investigated. For a survey of GRG, the reader is referred to [10].

Some of these results can be applied to the study of connectivity in ad hoc networks. For instance, consider a set S of points distributed in the deployment region R. It is known that the minimum common value of the transmitting range such that the resulting communication graph is connected equals the length of the longest edge of the Euclidean Minimum Spanning Tree built on S [19]. Hence, results concerning the asymptotic distribution of the longest MST edge [18], [19] can be used to characterize the critical transmitting range, as has been done by the authors of [16]. Another notable result of the theory of GRG is that, under the assumption of uniformly distributed points, the longest nearest neighbor link and the longest MST edge have the same value (asymptotically). In terms of the resulting communication graph, this means that connectivity occurs (asymptotically) when the last isolated node disappears from the graph. This observation can be generalized to the case of k-connectivity: When the minimum node degree becomes k, the graph becomes k-connected [20]. This result has been used in [1] to characterize the k-connectivity of ad hoc networks.

In the next section, we will use the following result due to Penrose [18], which characterizes the distribution of the longest MST edge for points distributed according to an arbitrary pdf with connected and compact support:[3]

Theorem 1 (Penrose 1999). *Let $X_1, X_2, X_3 \ldots$ be independent random points in $\mathbb{R}^2$, and assume that the points are distributed according to a common pdf f, having connected and compact support Ω with smooth boundary $\partial\Omega$. Further, assume that f is continuous on $\partial\Omega$. Let M_n denote the length of the longest MST edge built on the first n points of this random process. Then,*

$$\lim_{n\to\infty} \frac{n\pi(M_n)^2}{\ln n} = \frac{1}{\min_\Omega f},$$

almost surely.

We recall that the boundary $\partial\Omega$ is smooth if and only if it is twice differentiable (technically, if it is in C^2).

3. We recall that the support of a pdf is the set of points in which it has nonzero value.

Theorem 1 holds in the hypothesis that $\min_\Omega f > 0$. However, Penrose states that, given the similarities with the result on the largest nearest neighbor link of [17], the theorem also holds when $\min_\Omega f = 0$ (up to details which are not written in the paper). In this case, the value of the limit must be intended as $+\infty$.

In words, Theorem 1 states that the asymptotic behavior of the longest MST edge (and, consequently, of the critical transmitting range) depends only on the *minimum value* of the pdf used to distribute the nodes in Ω. In the next section, we use this result to characterize the asymptotic behavior of the CTR in the presence of mobility.

4 THE CRITICAL TRANSMITTING RANGE OF MOBILE NETWORKS

Our first result is the characterization of the CTR in the presence of *bounded* and *obstacle free* mobility, which we now define.

Definition 1. *Let R be a bounded region, and let ∂R be its boundary. Let $\mathcal{M}$ be an arbitrary mobility model, and let $f_\mathcal{M}$ be the pdf that resembles the long-term node spatial distribution generated by $\mathcal{M}$-like mobility. $\mathcal{M}$ is bounded within R if the support of $f_\mathcal{M}$ is contained in R. $\mathcal{M}$ is obstacle free if the support of $f_\mathcal{M}$ contains $R - \partial R$.*

In words, a mobility model is bounded within R if the nodes are allowed to move only within that region, while it is obstacle free if the probability of finding a mobile node in any subregion of R (excluding the border) is greater than 0.

Note that most of the mobility models used in the simulation of ad hoc networks are bounded (provided a suitable border rule is implemented [4]) and obstacle free: For instance, the random waypoint model, the random direction model, and the Brownian-like model are bounded and obstacle free.

For simplicity, in the rest of this paper, we assume that $R = [0,1]^2$, i.e., it is the unit square.

Theorem 2. *Let $\mathcal{M}$ be an arbitrary mobility model which is bounded within $R = [0,1]^2$ and obstacle free. Furthermore, assume that $f_\mathcal{M}$ is continuous on ∂R and $\min_R f_\mathcal{M} > 0$. The critical transmitting range for connectivity of an ad hoc network with $\mathcal{M}$-like mobility is $r_\mathcal{M} = c\sqrt{\frac{\ln n}{\pi n}}$ with high probability, for some constant $c \geq 1$.*

Proof. We observe that, if the hypotheses of Penrose's theorem hold, our result follows immediately since $\min_R f_\mathcal{M} > 0$ by hypothesis. Thus, we only have to show that the hypotheses of Penrose's theorem are satisfied.

First, we observe that, since $\mathcal{M}$ is bounded within R and obstacle free, the support of $f_\mathcal{M}$ is contained in R and contains $R - \partial R$. Since R is connected and compact, it follows that the support of $f_\mathcal{M}$ is connected and compact also. Furthermore, $f_\mathcal{M}$ is continuous on ∂R by hypothesis. The only hypothesis left to prove is that ∂R is smooth. Unfortunately, this is not true due to the presence of the corners. This problem can be circumvented by considering the region R_ε, obtained by "rounding" the corners of ∂R with a portion of the circle of radius ε (see Fig. 1). The boundary of R_ε is smooth for any value of $0 < \varepsilon < 1/2$ and $\lim_{\varepsilon\to 0} R_\varepsilon = R$.

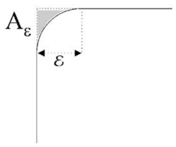

Fig. 1. The corner of ∂R is rounded using a portion of the circle of radius ε. For each corner, the density residing in the shaded area A_ε is uniformly redistributed in R_ε.

In order to preserve the probability measure when considering the restricted region R_ε, the fraction of the original density $f_{\mathcal{M}}$ residing in the areas such as area A_ε in Fig. 1 is uniformly redistributed in R_ε. Denoting with $f^\varepsilon_{\mathcal{M}}$ this modified pdf, it is immediate to see that $\min_R f_{\mathcal{M}} \leq \min_{R_\varepsilon} f^\varepsilon_{\mathcal{M}}$, $\lim_{\varepsilon\to 0}(\min_{R_\varepsilon} f^\varepsilon_{\mathcal{M}}) = \min_R f_{\mathcal{M}}$, and $\lim_{\varepsilon\to 0} f^\varepsilon_{\mathcal{M}} = f_{\mathcal{M}}$. By Theorem 1, we can write:

$$\lim_{n\to\infty,\varepsilon\to 0} \frac{n\pi\left(M_n^{(\varepsilon)}\right)^2}{\ln n} = \lim_{n\to\infty} \frac{n\pi(M_n)^2}{\ln n} = \frac{1}{\min_R f_{\mathcal{M}}} = c \geq 1,$$

almost surely, where $M_n^{(\varepsilon)}$ denotes the longest MST edge (hence, the critical transmitting range) when nodes are distributed according to $f^\varepsilon_{\mathcal{M}}$ on R_ε. The fact that $c \geq 1$ follows by observing that, since the area of R is 1 and $f_{\mathcal{M}}$ is a pdf with support that contains $R - \partial R$ and is contained in R (i.e., $\int_R f_{\mathcal{M}} = 1$), the minimum value of $f_{\mathcal{M}}$ on R is at most equal to 1. This concludes the proof of the theorem. □

Corollary 1. *Let r denote the CTR for connectivity in stationary (uniformly distributed) networks, and let $r_{\mathcal{M}}$ denote the same range in the presence of $\mathcal{M}$-like mobility, where $\mathcal{M}$ satisfies the hypotheses of Theorem 2. Then, $\lim_{n\to\infty} \frac{r_{\mathcal{M}}}{r} = c$, for some constant $c \geq 1$.*

Proof. Denoting by f_u the pdf that distributes uniformly at random the nodes in R, we have $\min_R f_u = 1$. As observed in the proof of Theorem 2, the minimum value on R of any other pdf with support contained in R is at most 1, and the corollary is proven. □

In words, Corollary 1 states that *every bounded and obstacle free type of mobility is detrimental for network connectivity* since the CTR can only increase. Note that this result is asymptotic, so it is true only for very large networks. If the network is small, the situation might even be reversed (see the simulation results reported in the next section).

The characterization of the CTR for mobile networks stated in Theorem 2 is only partial since it leaves the value of the constant c unspecified. A more accurate characterization can be given only if the actual formula of $f_{\mathcal{M}}$ is known. To the best of our knowledge, the only mobility model for which an accurate approximation of the pdf that models the long-term node spatial distribution has been derived is the RWP model. This approximation is given in [3].[4] The pdf is the following:

4. Indeed, the authors consider a generalized version of the random waypoint model in which nodes are allowed to remain stationary during the entire network operational time with a given probability p_{stat}, and the pause time is chosen independently at random at each waypoint. Up to straightforward modifications, our characterization can also be extended to this generalized RWP model.

Theorem 3. (BettstetterRestaSanti 2003). *The asymptotic spatial density function of a node moving in $R = [0,1]^2$ according to the random waypoint model with pause time p and $v_{min} = v_{max} = v$ is accurately approximated by*

$$f_{RW}(x,y) = \begin{cases} P_p + (1-P_p) f_m(x,y) & \text{if}(x,y) \in [0,1]^2 \\ 0 & \text{otherwise,} \end{cases}$$

where $P_p = \frac{p}{p + \frac{0.521405}{v}}$ and

$$f_m(x,y) = \begin{cases} 0 & \text{if } (x=0) \text{ or } (y=0) \\ f_{\mathcal{R}}(x,y) & \text{otherwise.} \end{cases}$$

The function $f_{\mathcal{R}}(x,y)$ is defined as follows:

$$\begin{aligned} &f_{\mathcal{R}}(x,y) = \\ &6y + \frac{3}{4}(1-2x+2x^2)\left[\frac{y}{y-1} + \frac{y^2}{(x-1)x}\right] + \\ &\frac{3}{2}\left[(2x-1)y(1+y)\log\left(\frac{1-x}{x}\right) + \right. \\ &\qquad \left. y(1-2x+2x^2+y)\log\left(\frac{1-y}{y}\right)\right]. \end{aligned}$$

We remark that the expression of $f_m(x,y)$ above is valid only for $(x,y) \in \mathcal{R} = \{(x,y) \in [0,1]^2 \mid (x \geq y) \wedge (x \leq 1/2)\}$. The expression of $f_m(x,y)$ on the remainder of $[0,1]^2$ can be easily obtained observing that, by symmetry, we have $f_m(x,y) = f_m(y,x) = f_m(1-x,y) = f_m(x,1-y)$. The 3D and contour plot of $f_m(x,y)$ are reported in Fig. 2, which is taken from [3].

The pdf f_{RW} is composed of the sum of two distinct components: The first component accounts for the time a node is resting at the waypoint, and it is uniform because, in the RWP model, the waypoints are chosen uniformly at random; the second component is not uniform and is responsible for the border effect.

Note that the nonuniform component of f_{RW} has a minimum value of 0 on the boundary ∂R of R. Since the other component of f_{RW} is uniform, it follows that the minimum value of f_{RW} is achieved on ∂R. It is immediate to see that the actual value of this minimum equals $P_p = \frac{p}{p+\frac{0.521405}{v}}$. We can then state the following corollary of Theorem 3:

Corollary 2. *Let f^p_{RW} denote the long-term node spatial density generated by RWP mobile networks with pause time equal to p and $v_{min} = v_{max} = v$. The minimum value of f^p_{RW} is achieved on ∂R, and it equals $P_p = \frac{p}{p+\frac{0.521405}{v}}$. When $p \to \infty$, f^p_{RW} becomes the uniform distribution on $[0,1]^2$ and $\min_R f^\infty_{RW} = 1$.*

We are now ready to prove the following theorem:

Theorem 4. *The critical transmitting range for connectivity of an ad hoc network whose nodes move in $R = [0,1]^2$ according to the RWP model when the pause time is p and $v_{min} = v_{max} = v$ is $r^w_p = \frac{p+\frac{0.521405}{v}}{p}\sqrt{\frac{\ln n}{\pi n}}$ with high probability if $p > 0$. When $p = 0$, we have $r^w_0 \gg \sqrt{\frac{\ln n}{n}}$ with high probability.*

Proof. If $p > 0$, the proof follows immediately by Theorems 1 and 2 and Corollary 2. When $p = 0$, the minimum of f^0_{RW} on R is 0 (Corollary 2), and the limit in the statement of Theorem 1 must be interpreted as $+\infty$. □

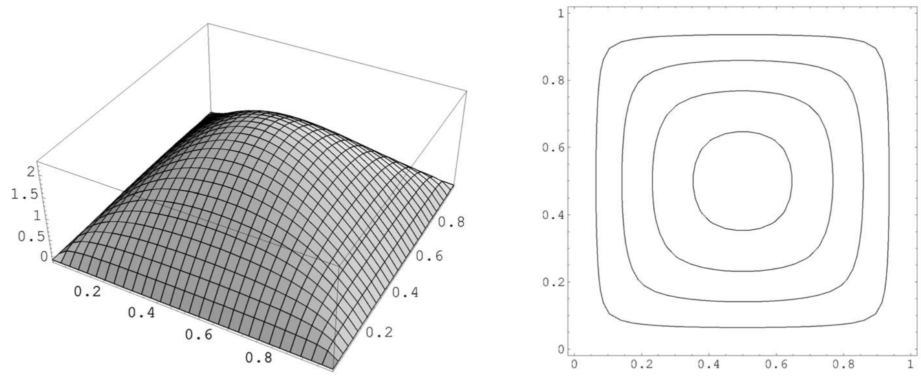

Fig. 2. Three-dimensional plot of $f_m(x, y)$ and contour lines corresponding to the values $f_m(x, y) = 0.5$, 1, 1.5, and 2.

In words, Theorem 4 states that, as long as the node spatial distribution has a nonnull uniform component, the critical transmitting range with RWP mobility differs from the stationary one at most by a constant factor. On the contrary, when the uniform component is null, there is an asymptotic gap between the mobile and stationary case.

As observed in [18], the critical range to achieve k-connectivity, for any constant k, should have the same asymptotic behavior as M_n also in the case of arbitrary node distribution. Hence, a result similar to that of Theorems 2 and 4 should hold for k-connectivity also.

5 Experimental Evaluation

In order to investigate whether the asymptotic result of Theorem 4 holds for reasonable values of n, we have performed some simulations.

The simulator takes as input the number n of nodes to distribute, the number $\sharp sim$ of simulations to run, and the number $\sharp steps$ of mobility steps to perform for each simulation. The simulator also takes as input the parameters of the random waypoint model, i.e., t_p (expressed as the number of steps that the node remains stationary between two movements) and the minimum (v_{min}) and maximum (v_{max}) velocity (expressed in units of length per step). Initially, nodes are distributed uniformly and independently at random in $[0, 1]^2$; then, they start moving according to the random waypoint mobility model. At the end of the $\sharp steps$ steps of mobility, the longest MST edge is calculated and recorded in the output file. The values contained in the output file are used to determine the critical transmitting range, which is defined as the 0.99 quantile of the experimental data. We recall that the q quantile of a series of data gives the location before which $100q$ percent of the data lie.

First, we have investigated the rate of convergence of the CTR to the asymptotic value stated in Theorem 4. We have set $v_{min} = v_{max} = 0.01$, considered two values of the pause time ($p = 100$ and $p = 200$), and computed the asymptotic value of the CTR, which is $1.52\sqrt{\frac{\ln n}{\pi n}}$ when $p = 100$ and $1.26\sqrt{\frac{\ln n}{\pi n}}$ when $p = 200$. Then, we have performed a large number of simulations to evaluate the CTR experimentally: We have distributed a number of nodes ranging from 10 to 2,500, and we have performed 10,000 mobility steps. As the experimental analysis of [3] has shown, this number of mobility steps ensures the convergence to the asymptotic node spatial distribution. The results of our simulations, averaged over 10,000 experiments, are shown in Figs. 3 and 4. The rate of convergence to the asymptotic value is quite slow for both settings of the pause time: Only for large networks ($n = 1,000$ and above) is the value obtained with the formula of Theorem 4 quite accurate.

We have also experimentally verified the *quality* of Theorem 4 which, we recall, states that the CTR in case of RWP mobility has a different asymptotic behavior with respect to the uniform case only when the pause time is 0. To this purpose, we have simulated four different scenarios. First, we have considered the stationary case, obtained by setting $\sharp steps = 1$. Then, we have set $v_{min} = v_{max} = 0.01$ and considered three settings of p: 0, 100, and 200. In the first case, f_{RW} is composed only of the nonuniform component, and the border effect is maximum. In the second case, the intensity of the uniform component of f_{RW} equals $P_{100} = 0.657$; with $p = 200$, the intensity of the uniform component is $P_{200} = 0.793$. In each scenario considered, we have distributed a number of nodes ranging from 10 to 2,500 and, in case of mobility, we have performed 10,000 mobility steps.

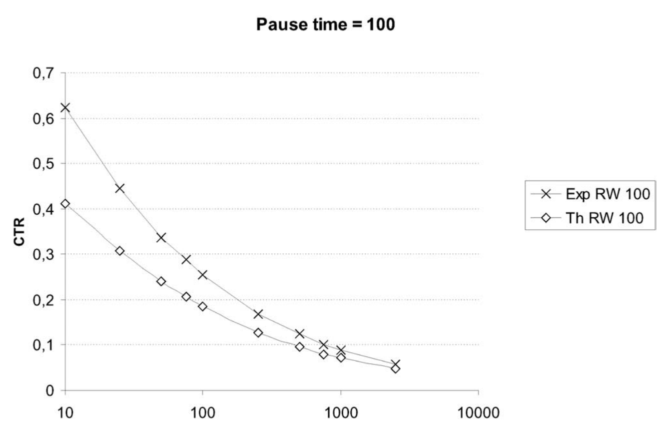

Fig. 3. Theoretical and experimental value of the critical transmitting range for values of n ranging from 10 to 2,500 when the pause time is set to 100 and $v_{min} = v_{max} = 0.01$. The x-axis is in logarithmic scale.

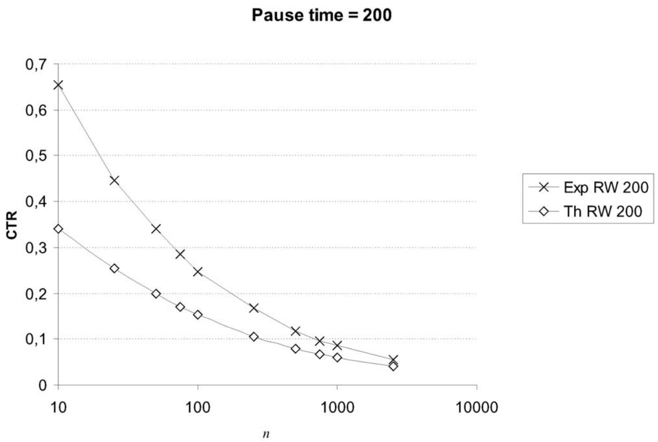

Fig. 4. Theoretical and experimental value of the critical transmitting range for values of n ranging from 10 to 2,500 when the pause time is set to 200 and $v_{min} = v_{max} = 0.01$. The x-axis is in logarithmic scale.

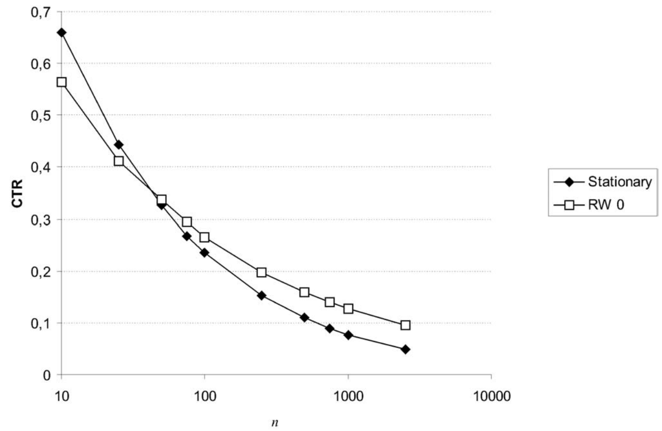

Fig. 5. Critical transmitting range in the stationary case and with extreme RWP mobility for values of n ranging from 10 to 2,500. The x-axis is in logarithmic scale.

The results of our simulations, averaged over 10,000 experiments, are reported in Table 1 and plotted in Figs. 5 and 6. As it is seen, the qualitative result of Theorem 4 is fully confirmed: The curves referring to the stationary scenario and to the case of RWP mobility with positive pause times are very close to each other (see Fig. 6), while those corresponding to the extreme case of mobility ($p = 0$) are far apart (see Fig. 5). As predicted by Theorem 4, the critical transmitting range in the case of "uniform" mobility is larger than in the stationary case, although only marginally. Note that setting $p = 100$ is already sufficient to render the CTR very close to the stationary one. Conversely, the critical transmitting range in the presence of extreme mobility is much larger than in the stationary case for large values of n. Quite interestingly, the situation is reversed for small values of n ($n \leq 50$). This is due to the fact that, when n is small, the probability of finding at least one node close to the border is very low, and the critical transmitting range is smaller than in the stationary case (nodes are concentrated in the center of the deployment region). However, when n is large enough, some of the nodes actually lie close to the border of R, causing the asymptotic gap with respect to the uniform case.

We have found that, given the value r of the CTR in the stationary case, the critical range in case of extreme mobility is well approximated by the formula $r_0^w = \frac{r}{4} \ln n$, for sufficiently large n ($n \geq 50$). Fig. 7 reports the plot obtained with our formula and that produced by the simulations. For $n \geq 100$, our approximation provides a quite accurate upper bound to the actual CTR. Table 2 reports the relative error of our formula with respect to the actual CTR, defined as $\frac{r_{app} - r_0^w}{r_0^w}$, where r_{app} is the approximate value of the CTR. We observe that the relative error is below 6.4 percent when $n \geq 50$.

We remark that the formula above can be used to improve the accuracy of mobile network simulation. As noticed in the Introduction and proven in this paper, setting the nodes' transmitting range to the critical value computed as if the nodes were uniformly distributed is likely to produce inaccurate results in many scenarios (for instance,

TABLE 1
Values of the CTR for Increasing Values of n

n	Uni	RWP 200	RWP 100	RWP 0
10	0.6587	0.65422	0.62409	0.56423
25	0.44249	0.44531	0.44454	0.41203
50	0.32763	0.34132	0.33717	0.33644
75	0.268	0.28439	0.28761	0.,29454
100	0.23494	0.2478	0.25395	0.26526
250	0.15183	0.16691	0.16933	0.19761
500	0.10926	0.11812	0.12442	0.15955
750	0.08838	0.09676	0.10216	0.13963
1000	0.07727	0.08543	0.08978	0.12708
2500	0.04977	0.05478	0.0586	0.09482

The columns refer to the case of stationary (uniformly distributed) nodes and to the case of RWP mobility with different settings of the pause time.

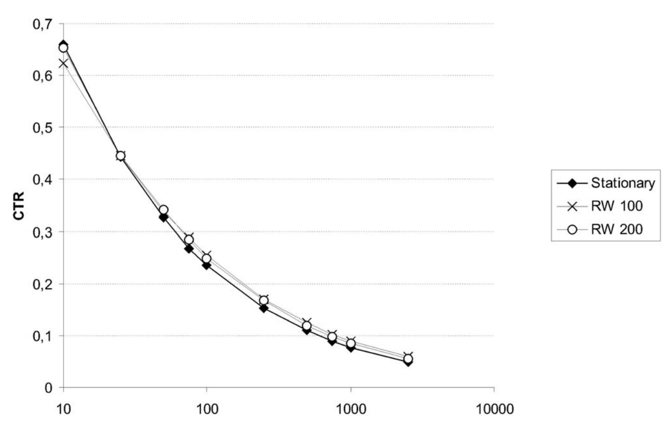

Fig. 6. Critical transmitting range in the stationary case and with intermediate RWP mobility ($p = 100$ and $p = 200$) for values of n ranging from 10 to 2,500. The x-axis is in logarithmic scale.

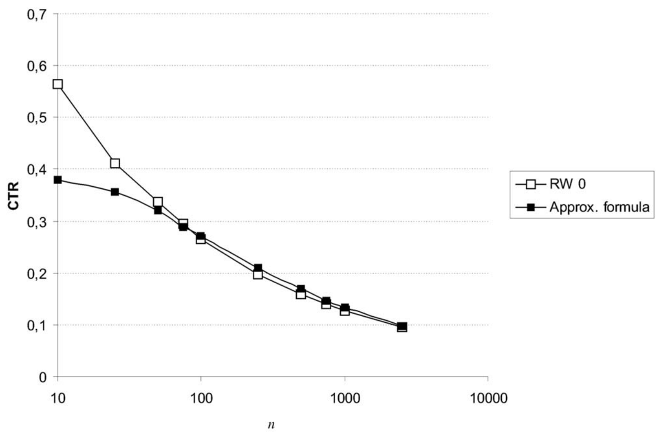

Fig. 7. Critical transmitting range with RWP mobility when $p = 0$ (RW 0) and values of the CTR as computed with our formula (Approx. formula). The values of n range from 10 to 2,500, and are reported in logarithmic scale.

when simulating an ad hoc routing protocol in the presence of RWP mobility). Given the critical range r with uniformly distributed nodes, our formula can be used to compute a tight upper bound on the CTR in the presence of RWP mobility, thus considerably improving the accuracy of the simulation results. The fact that our formula depends on r is not a problem since the value of the CTR with uniformly distributed nodes is a widely studied network parameter (see, for instance, the table reported in [24]).

6 Concluding Remarks

In this paper, we have investigated the critical transmitting range for connectivity in mobile ad hoc networks. We have proven that, in the presence of bounded and obstacle free mobility, the CTR in the mobile case is at least as large as the CTR in the case of uniformly distributed points (asymptotically). For the case of RWP mobility, we have proven a more accurate characterization of the CTR and shown that, if the pause time is 0, there is an asymptotic gap between the mobile and uniform scenario.

We have verified the quality of our results through simulation. We have also presented a formula that, given the value of the CTR in the uniform case, provides a good approximation of the CTR in the most extreme case of RWP mobility, i.e., when the pause time is set to 0.

We want to remark that the approach presented in this paper can be easily extended to other mobility models: If the expression of the pdf $f_{\mathcal{M}}$ that resembles the long-term node distribution is known and satisfies certain properties, it is sufficient to compute the minimum value of $f_{\mathcal{M}}$ on R to determine the value of the critical range for connectivity.

We believe that the results presented in this paper provide a better understanding of the behavior of a fundamental network parameter in the presence of mobility and, in particular, of RWP mobility. From a practical point of view, our results can be used to improve the accuracy of RWP mobile ad hoc networks simulation, which is commonly used to evaluate the performance of ad hoc networking protocols.

TABLE 2
Relative Error Obtained Using Our Formula

n	Relative error
10	-0.32797
25	-0.13579
50	-0.0476
75	-0.01789
100	0.019696
250	0.060578
500	0.063942
750	0.047558
1000	0.050052
2500	0.026689

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Paolo Santi received the MS and PhD degrees in computer science from the University of Pisa, Italy, in 1994 and 2000, respectively. During 2000, he was a research assistant at the Istituto di Elaborazione dell'Informazione, Pisa. From January to June 2001, he visited the School of Electrical and Computer Enginerring, Georgia Institute of Technology, Atlanta, GA. In September 2001, he joined the Italian National Research Council, Istituto di Informatica e Telematica, in Pisa. From May to December 2003, he visited the Department of Computer Science at Carnegie Mellon University, Pittsburgh, Pennsylvania. His research interests include fault diagnosis in multiprocessors and safety-critical systems (during the PhD studies), the study of structural properties of wireless ad hoc networks (such as connectivity, mobility modeling, topology control), and combinatorial auctions. Dr. Santi is a member of the ACM and SIGMOBILE.