Mixed-Integer-Discrete-Continuous Optimization


Approach

• The search space is continuous in that all optimizing variables, regardless of their types, are stored as floating-point numbers.
• During fitness evaluation:
  – Real-value variables:
    • use the stored values as the real values
  – Integer-value variables:
    • use the truncated values as the integer values
  – Discrete-value variables:
    • use the truncated values as the indices to get the discrete values.

Angle Modulated Binary DE

Approach

- DE evolves the real-value coefficients \((a, b, c, d)\) of a trigonometric function that is used to generate binary bits.
- If the output of the trigonometric function (with an evolved \(a, b, c, d\) vector) on an \(x\) input is positive, a bit value 1 is given. Otherwise, a bit value 0 is given.
- DE evolves a 4-dimensional \((a, b, c, d)\) vector to generate binary vectors of any dimension.

The Trigonometric Function

\[
g(x) = \sin(2\pi(x - a) \times b \times \cos(A)) + d
\]
\[
A = 2\pi \times c(x - a)
\]

- \(x\) is a value from a set of evenly separated intervals determined by the required number of bits that need to be generated for the binary vector.
- For example, for a vector of 10 bits, the values between 1 and 10 are: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

The Coefficients

- Coefficient \(a\):
  - Gives the horizontal shift of the generating function.
- Coefficient \(b\):
  - Gives the maximum frequency of the composed \(\sin\) function.
- Coefficient \(c\):
  - Gives the frequency of the composed \(\cos\) function.
- Coefficient \(d\):
  - Gives the vertical shift of the generating function.

Examples

-2 <= \(x\) <= 2

0110 and 01-10 give the same outputs: \(\cos(y) = \cos(-y)\)
Transformations: 8 Bits

- $x = \{-1.9, -1.4, -0.9, -0.4, 0.1, 0.6, 1.1, 1.6\}$
- DE vector: $a=0$, $b=1$, $c=1$, $d=0$.
  - Binary vector: 1, 1, 1, 1, 0, 0, 0
- DE vector: $a=1$, $b=1$, $c=1$, $d=0$.
  - Binary vector: 0, 0, 0, 0, 0, 0, 0
- DE vector: $a=0$, $b=-1$, $c=1$, $d=1$.
  - Binary vector: 0, 0, 0, 0, 0, 1, 0

Advantages

- Only 4 real-values ($a$, $b$, $c$, $d$) are evolved, regardless of the dimension of the binary string.
- The search space is continuous where the original DE would work without any modification.

Angle Modulated DE Algorithm

**Algorithm 1** AMDE Algorithm

Initialise a population and set control parameter values $(a, b, c, d)$

repeat
  Select required individuals for reproduction scheme
  Produce an offspring individual
  Evaluate the fitness of the offspring individual by generating bit string and passing to fitness function
  if $F_{AMDE}(offspring) \leq F_{AMDE}(parent)$ then
    Replace parent individual with offspring
  else
    Retain parent individual
  end if
until Stopping condition is met

Experiments

<table>
<thead>
<tr>
<th>Function</th>
<th>Dimensions</th>
<th>Common name</th>
<th>Function domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>3</td>
<td>Spherical</td>
<td>(-5.12, 5.12)</td>
</tr>
<tr>
<td>$f_2$</td>
<td>2</td>
<td>2D Rosenbrock</td>
<td>(-2.048, 2.048)</td>
</tr>
<tr>
<td>$f_3$</td>
<td>5</td>
<td>Step Function</td>
<td>(-5.12, 5.12)</td>
</tr>
<tr>
<td>$f_4$</td>
<td>30</td>
<td>Quadric</td>
<td>(-1.28, 1.28)</td>
</tr>
<tr>
<td>$f_5$</td>
<td>2</td>
<td>Foxholes</td>
<td>(-65536.0, 65536.0)</td>
</tr>
<tr>
<td>$f_6$</td>
<td>2</td>
<td>Schaffer’s F6</td>
<td>(-100.0, 100.0)</td>
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<tr>
<td>$f_7$</td>
<td>30</td>
<td>Griewank</td>
<td>(-300.0, 300.0)</td>
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<tr>
<td>$f_8$</td>
<td>30</td>
<td>Ackley</td>
<td>(-30.0, 30.0)</td>
</tr>
<tr>
<td>$f_9$</td>
<td>30</td>
<td>Rosenbrock</td>
<td>(-2.048, 2.048)</td>
</tr>
<tr>
<td>$f_{10}$</td>
<td>30</td>
<td>Rastrigin</td>
<td>(-5.12, 5.12)</td>
</tr>
</tbody>
</table>
Implementation

- Each real-value is represented with $m$ bits, hence an $n$-dimension problem, the number of bits to be generated from the trigonometric function is $n \times m$.
- The initial population is randomly generated with 40 4-dimensional $(a,b,c,d)$ vectors with value between 1 & -1.
- Weight factor $\beta$: 1.0; the differential vector is fully used as step size to mutate a base vector.
- Crossover point Pr: 0.25; at least 1 of the 4 coefficients is modified during crossover.

Results - Continued

<table>
<thead>
<tr>
<th>Function</th>
<th>AMDE Iterations</th>
<th>AMPSO Iterations</th>
<th>BinPSO Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>3.33</td>
<td>1.00</td>
<td>801.85</td>
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<tr>
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<td>279.25</td>
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<tr>
<td>$f_3$</td>
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<td>1.00</td>
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<td>17.67</td>
<td>1017.88</td>
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<td>$f_5$</td>
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<td>$f_{14}$</td>
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<td>$f_{15}$</td>
<td>8920.67</td>
<td>451.18</td>
<td>4302.67</td>
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</tbody>
</table>

Take longer