Fuzzy Inference Systems

A fuzzy inference system consists of four main components:

- **Fuzzifier**: an interface that transforms crisp inputs into the degrees of membership to the corresponding fuzzy sets.
- **Fuzzy rule base**: one or more if-then fuzzy rules.
- **Inference engine**: infer fuzzified inputs to produce fuzzy outputs.
- **Defuzzifier**: an interface that aggregates the inferred fuzzy outputs and produce a crisp output.

Fuzzy Operators Review

- **AND**: intersection
  - \( \min \): \( \mu_{A \land B}(x) = \min\{\mu_A(x), \mu_B(x)\}, \forall x \in X \)
  - \( \text{product} \): \( \mu_{A \land B}(x) = \mu_A(x) \times \mu_B(x), \forall x \in X \)
- **OR**: union
  - \( \max \): \( \mu_{A \lor B}(x) = \max\{\mu_A(x), \mu_B(x)\}, \forall x \in X \)
  - \( \text{sum} \): \( \mu_{A \lor B}(x) = \mu_A(x) + \mu_B(x), \forall x \in X \)
  - \( \text{probor} \): probabilistic OR, algebraic sum
  - \( \mu_{A \lor B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \times \mu_B(x), \forall x \in X \)

Fuzzy Operators Examples

- **price is expensive**: \( A = \mu_{\text{high}}(\text{price} = 45) = 0.4 \)
- **temperature is pleasant**: \( B = \mu_{\text{high}}(\text{temperature} = 20) = 0.9 \)
- **time is FridayNight**: \( C = \mu_{\text{high}}(\text{time} = 16:40) = 0.5 \)
- **A and B and C**
  - \( \min \): \( \min\{0.4, 0.9, 0.5\} = 0.4 \)
  - \( \text{product} \): \( 0.4 \times 0.9 \times 0.5 = 0.18 \)
- **A and B or C**
  - \( \min \): ?
  - \( \text{product} \): ?
  - \( \text{probor} \): ?

Fuzzy Inference Systems

- Fuzzy Inference Systems
- Sugeno (TSK) fuzzy inference system
- ANFIS
- fuzzy inference system for PSO maximum velocity control.

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Fuzzification

- Design membership functions for fuzzy variables and map crisp values to the degrees of membership to the fuzzy sets.

Sugeno (TSK) fuzzy rules

- The antecedent is a combination of fuzzy variables with fuzzy values while the consequent is a singleton (a linear equation):
- A singleton, or more precisely a fuzzy singleton, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.
- Examples:
  If \( x \) is \( A_1 \) and \( y \) is \( B_1 \) then \( k_1 = p_1x + q_1y + r_1 \)
  If \( x \) is \( A_2 \) and \( y \) is \( B_2 \) then \( k_2 = p_2x + q_2y + r_2 \)

Sugeno-style Rule Inference

Sugeno-style Defuzzification

- Defuzzification is the process of transforming the aggregate output fuzzy set to a single number.

Weighted Average (WA)

\[
\mu_{z} = \frac{\mu(A_1) \cdot k_1 + \mu(A_2) \cdot k_2 + \mu(A_3) \cdot k_3}{\mu(A_1) + \mu(A_2) + \mu(A_3)}
\]

Sugeno-style defuzzification

- A fuzzy inference system with an adaptive network structure.
- ANFIS represents a TSK fuzzy inference system.
- A clustering algorithm to design membership function.
- A gradient decent algorithm to optimize fuzzy rules.

ANFIS: Adaptive Neuro-Fuzzy Inference Systems
TSK fuzzy inference system

If $x$ is $A_1$ and $y$ is $B_1$ then $k_1 = p_1x + q_1y + r_1$
If $x$ is $A_2$ and $y$ is $B_2$ then $k_2 = p_2x + q_2y + r_2$

$w_1 = \mu_{A_1}(x) \times \mu_{B_1}(y)$

Layer 1

• Compute the degree of membership for all inputs to the fuzzy sets:
• $A_1$ is a Gaussian membership function with (mean, std).

Layer 2

• Compute the weight of the antecedent of each rule.

Layer 3 & 4

• Layer 3:
  – Normalize weight for all $n$ rules
    \[ \pi_i = \frac{\pi_i}{\sum_{j=1}^{n} \pi_j} \]
  – Layer 4:
    – Compute the output of each rule:
      \[ \text{out}_i = \bar{w}_i \times (p_i x + q_i y + r_i) \]

Layer 5

• Compute the output of the fuzzy inference system:
  – Weighted average:
    \[ \text{output}_{out} = \sum_{i} \pi_i f_i = \frac{\sum_{i} \pi_i f_i}{\sum_{i} \pi_i} \]
Matlab fuzzy tool

- One copy is available in the cs lab.
- export LM_LICENSE_FILE=27001@irma
- cd /usr/local/matlab7/bin/
- ./matlab
- Inside matlab
  - cd toolbox/fuzzy/fuzzy
  - anfisedit