Traveling Salesman Problem

- The salesman must visit every city in his territory exactly once and then return home covering the shortest distance.
- Variables: \( x_1, \ldots, x_n \) are \( n \) city names
- Representation: permutation of \( n \) cities.
- 5 cities example: \((a, d, g, b, e)\)

TSP Search Space

- Given \( n \) unique objects, \( n! \) permutations of the objects exist. Searching the shortest path is an NP-hard problem.
- In TSP, there are multiple equivalent solutions.
  - If starting point is not important, and the distance from city \( i \) to \( j \) is the same as that from city \( j \) to \( i \), each tour \( a-b-c-d-e \) can be represented in \( 2n \) ways and give the same distance.
- Search space: \( n!/(2n) = (n-1)!/2 \)

PSO for TSP

- The solution of a particle is a permutation of all cities.
  - Example: \((a, d, g, b, e)\)
- The velocity of a particle is a sequence of swap operators.
- Velocity examples:
  - swap operator (SO) = SO(1,2) //swap first visited city with the second visited city
  - swap sequence (SS) = (SO(1,2), SO(5,4), SO(5,1))

Particle Solution Update

\[
x(t) = x(t-1) + v(t)
\]

- Applying SO to a permutation:
  - \((a, d, g, b, e) + \text{SO}(1,2) = (d, a, g, b, e)\)
- Apply a sequence of SO (SS) to a permutation:
  - \(\text{SS} = (\text{SO}(1,2), \text{SO}(5,4), \text{SO}(5,1))\)
  - \((a, d, g, b, e) + \text{SS} \rightarrow (d,a,g,b,e) \rightarrow (d,a,g,e,b) \rightarrow (b,a,g,e,d)\)
Particle Velocity Update

- $v(t) = v(t-1) \alpha (p_{best} - x(t-1)) \beta (g_{best} - x(t-1))$

- Merging two Swap Sequences
  - $SS_1 = (SO(1,2), SO(5,4), SO(5,1))$
  - $SS_2 = (SO(1,2), SO(5,4), SO(5,1), SO(1,3), SO(5,1), SO(2,1))$

- Subtract Two Permutations
  - $A = (a, c, d, e, b)$, $B = (c, a, b, e, d)$
  - There is a SS that transforms $A$ to $B$.
    - $a$ is in position 1 in $A$ and 2 in $B$: \(SO(1,2)\)
    - $b$ is in position 5 in $A'$ and 3 in $B$: \(SO(5,3)\)
  - $SS = (SO(1,2), SO(5,3))$
  - $A - B = SS$

Particle Velocity Update

- Each velocity is a swap sequence.
- $v(t) = v(t-1) \alpha (p_{best} - x(t-1)) \beta (g_{best} - x(t-1))$
- $\alpha, \beta$ are random number between 0 and 1.
- The probability that all swap operators in swap sequence ($p_{best} - x(t-1)$) are included in the updated velocity is $\alpha$.
- The probability that all swap operators in swap sequence ($g_{best} - x(t-1)$) are included in the updated velocity is $\beta$.
- There is improvement on this in another paper.

TSP-PSO algorithm

- Random initialization of permutation and swap sequences.
- For each time step
  - Update $g_{best}$ if needed,
  - Update $p_{best}$ if needed.
  - For each particle in the swarm
    - $v(t) = v(t-1) \alpha (p_{best} - x(t-1)) \beta (g_{best} - x(t-1));$
    - $x(t) = x(t-1) + v(t);$
  - End
- End

Exercise

<table>
<thead>
<tr>
<th>particle</th>
<th>permutation</th>
<th>velocity</th>
<th>fitness</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>c a e f b d</td>
<td>(2,1),(3,2)</td>
<td>31.57401</td>
</tr>
<tr>
<td>B</td>
<td>b d e a f c</td>
<td>(2,3),(5,1)</td>
<td>28.95417</td>
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<tr>
<td>C</td>
<td>f d a b c e</td>
<td>(4,3),(1,5)</td>
<td>31.04489</td>
</tr>
<tr>
<td>D</td>
<td>d b f e a c</td>
<td>(3,6),(5,6)</td>
<td>31.57401</td>
</tr>
<tr>
<td>E</td>
<td>c d e b a f</td>
<td>(2,4),(1,3)</td>
<td>27.6283</td>
</tr>
</tbody>
</table>

\[
v(t) = v(t-1) \alpha (p_{best} - x(t-1)) \beta (g_{best} - x(t-1)); \\
x(t) = x(t-1) + v(t);
\]
Particle A:

- $E \cdot A = (2,1),(2,3),(2,5),(4,6),(2,6),(4,5),(5,6)$
- $v(t) = v(t-1) \alpha (p_{best} - x(t-1)) \beta (g_{best} - x(t-1));$
- $x(t) = x(t-1) + v(t);$

Exercise

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</tr>
</thead>
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<tr>
<td>A</td>
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<td>B</td>
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<td>C</td>
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