**Constraint Handling in EA**

- Goldberg, D. 1989, Genetic algorithms in search, optimization and machine learning, Addison-Wesley.

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**Constrained Optimization Problems**

\[
\text{maximize} \quad f(x) = \sum_{i=1}^{n} \cos(x_i) - 2\prod_{i=1}^{n} \cos(x_i) \\
\text{where} \quad \prod_{i=1}^{n} x_i \geq 0.75, \sum_{i=1}^{n} x_i \leq 7.5, 0 \leq x_i \leq 10.1 \quad i = 1, \ldots, n.
\]

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**Penalize Constraints Violation**

- To distinguish individuals who violate constrains from those who do not, a penalty term \( \phi \) is introduced into the fitness function.
  - weighted sum approach: \( f(x) = o(x) \pm w \phi(x) \)
- However, balance between constraint penalty \( \phi \) and objective function \( o \) to guide evolutionary search to find the optimum is not always easy:
  - Small penalty: the infeasible solutions may not be penalized enough, hence the evolutionary search delivers an infeasible solution.
  - Large penalty: discourage the exploration of infeasible region, hence may evolve a feasible but sub-optimum solution.

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**Penalty Method**

- At the beginning of the evolutionary search, low constraint penalty is preferred to help discovering a good region which may contain both feasible and infeasible individuals.
- Toward the end of the search, high constraint penalty is preferred to locate a good feasible individuals.
- One way to achieve this balance effectively and efficiently is to adjust it directly and explicitly.

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**Stochastic Ranking**

- Introduce a probability \( Pf \) of using only the objective function (not constraint penalty) for comparisons in ranking in the infeasible regions of the search space.

<table>
<thead>
<tr>
<th>( x ), ( \phi(x) = 0 )</th>
<th>( y ), ( \phi(y) = 0 )</th>
<th>compare</th>
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</thead>
<tbody>
<tr>
<td>( f(x), \phi(x) = 0 )</td>
<td>( f(y), \phi(y) = 0 )</td>
<td>( f )</td>
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<tr>
<td>( f(x), \phi(x) &gt; 0 )</td>
<td>( f(y), \phi(y) = 0 )</td>
<td>( \text{if } (U &lt; Pf) \text{ f else } \phi )</td>
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Stochastic Ranking Algorithm

A stochastic bubble-sort

1. $i = j \in [1, \ldots, A]$
2. for $i = 1$ to $N$
3. if $j \in 1$ to $\lambda - 1$
4. if $(f_i) = f_{i+1} = 0$ then
5. if $(f_i) > f_{i+1} = 0$ then
6. else
7. enddo
8. $f$
9. $N$ = number of sweeps;
10. $\lambda$ = pop_size

$\phi$: penalty function

Rank 1 is the best.

Minimization problem:
smaller $f$ and smaller $\phi$ are better.

Example

N=4, $\lambda=6$

The procedure is halted when no change in the rank ordering occurs within a complete sweep or the number of sweep $N$ is reached.

Does the initial ranking have impact on the final ranking result?

How about if $N=\infty$?

What is a good Pf?

- Pf=0, the ranking is an over-penalization;
- Pf=1, the ranking is an under-penalization.
- Pf=0.5, there will be an equal probability that the comparison will be based on objective function and the penalty function.
- Since we are only interested in feasible individuals as final solutions, Pf should be less than 1/2 so that there is a bias against infeasible solutions.

Experimental Results

Pareto Ranking

- Solution A is better (Pareto-dominates) than solution B if:
  - $f(A) < f(B)$ & $\phi(A) \leq \phi(B)$, or
  - $f(A) \leq f(B)$ & $\phi(A) < \phi(B)$
  - Under minimization problems.
- Treat both objective and penalty functions equally.
- Two solutions are with the same rank if they don’t dominate each other.
Iteration 1

- Identify the 1st Pareto Front
- All individuals in this front has the same front label.

Pareto Ranking Results

\[ \text{rank}(x_i, t) = 1 + \sum_{j=1}^{t} \text{dominated}(x_i) \]

Number of dominating individuals (solutions better than \(x_i\)) at time \(t\)

Lexicographic Ranking

- Solution A is better than solution B if:
  - \(f(A) > f(B)\)
  - \(f(A) = f(B) \& \phi(A) < \phi(B)\)
- Treat objective function as the primary optimization target.
- Only when there is a tie, penalty function is used to break the tie

Exercise

- Identify the 2nd Pareto Front
- All individuals in this front have worse objective function and penalty than all individuals in the 1st front.

Exercise