MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 2 (Sample) **MATH 1000-5** **Solutions**

1. (a) $y = 3^x + \log_3 x + \ln|3x|$

$y' = 3^x \ln 3\, 3^x + \dfrac{1}{x \ln 3} + \dfrac{1}{x}.$ (Notice: $ln|3x| = \ln 3 + \ln|x| = \text{const} + \ln x$)

(b) $y = x^2(5x^3 - 1)^6$ (Factorize the answer.)

(c) $y = \ln(\csc x) + \tan(\ln x)$

(d) $y = 5\sin^4\sqrt{1 - 3x}$

(e) $2x^2 + 3y^3 = 3x^2y^3$

2. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ for the following:

(a) $y = (4x + 1)^{\sqrt{3x+2}}$

(b) $y = \dfrac{(x^3 + 1)^5}{6^x\sqrt{x^3 - x}}$

3. The length of a rectangle is increasing at a rate of 5 cm/sec while the width is decreasing at a rate of 2 cm/sec.

(a) Find the rate at which the area of the rectangle is changing when the length is 50 cm and the width is 40 cm.

(b) Find the rate at which the length of the diagonal is changing when the length is 12 cm and the width is 9 cm.

4. Given $f(x) = 3x^4 - 2x^2$, use differentiation to determine the intervals over which the function is increasing, decreasing, concave up or concave down. Find the exact values of both coordinates of all extreme points, inflection points and intercepts.

5. Sketch a graph of a function that satisfies all the given conditions. Clearly label any extreme points, inflection points and asymptotes.

Domain $= \{x \mid x \neq 1\}$

$f(0) = 1,\ f(2) = 0,\ f(3) = 1$

$f'(2) = 0$

$f''(x) < 0$ when $x < -1$ and when $x > 3$

$f''(x) > 0$ when $-1 < x < 3$

$\lim_{x\to -1^-} f(x) = -\infty,\quad \lim_{x\to -1^+} f(x) = +\infty$

$\lim_{x\to -\infty} f(x) = 0,\quad \lim_{x\to \infty} f(x) = 3.$