MATH 1000

Logarithmic Identities

1. Definition of the logarithmic function: $\log_a x = b$ is equivalent to $a^b = x$, where $a, b > 0$. For example,

$$10^3 = 1000 \qquad \text{hence} \qquad \log_{10} 1000 = 3.$$

2. Natural log:

$$\log_e x = \ln x \qquad \text{for example} \qquad \ln e^5 = 5.$$

3. Algebraic identities: Log brings a product to a sum

$$\log_a(xy) = \log_a x + \log_a y$$

and a power to a multiple

$$\log_a(x^n) = n \log_a x$$

In particular,

$$\ln(x^2) = 2\ln x, \qquad \ln\sqrt{x} = \frac{1}{2}\ln x, \qquad \ln\frac{1}{x} = -\ln x, \qquad \ln\frac{1}{\sqrt{x}} = -\frac{1}{2}\ln x.$$

4. Change of base:

$$\log_a x = \frac{\log_b x}{\log_b a}, \qquad \text{in particular,} \qquad \log_a x = C\ln x, \qquad \text{where } C = \frac{1}{\ln a} = \ln_a e.$$

5. Differentiation: $(\ln x)' = \frac{1}{x}$, moreover,

$$(\ln|x|)' = \frac{1}{x}, \qquad \text{and}(\ln|u(x)|)' = \frac{u'(x)}{u(x)} \qquad \text{whenever} \quad u(x) \neq 0.$$

6. "Logarithmic differentiation"

$$(\ln f(x)g(x))' = (\ln f(x))' + (\ln g(x))' = \frac{f'(x)}{f(x)} + \frac{g'(x)}{g(x)}$$

$$(\ln\frac{f(x)}{g(x)})' = (\ln f(x))' - (\ln g(x))' = \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}$$

To find the derivative of $f(x)^{g(x)}$ use the identity

$$\ln f(x)^{g(x)} = g(x)\ln f(x).$$

and the Logarithmic Differentiation. A special case worth to to memorize: If a is a constant, then

$$(a^x)' = a^x \ln a, \qquad \text{for example} \qquad (3^x)' = 3^x \ln 3.$$

(Not to be confused with the Power Rule $(x^a)' = ax^{a-1}$.)