

1. Evaluate each limit or explain why it does not exist. Assign ∞ or $-\infty$ when appropriate.

(a) $\lim_{x \rightarrow 5} \frac{25 - x^2}{x^2 - 2x - 15}$

(b) $\lim_{x \rightarrow -1} \frac{3x}{(x + 1)^2}$

(c) $\lim_{x \rightarrow 0} \frac{\sin x}{1 - \sqrt{1 + x}}$

2. (a) Use the definition of derivative to find $f'(x)$ given $f(x) = \sqrt{2x + 1}$.

(b) Find the equation of the tangent line to the graph of $f(x) = \sqrt{2x + 1}$ at $x = 4$.

3. Let $f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & , x \geq 0 \\ \frac{4}{x + 2} & , x < 0. \end{cases}$

(a) Use the definition of continuity at a point to show that f is continuous at $x = 0$.

(b) Determine any discontinuities of f and show whether each discontinuity is removable or non-removable.

4. An airplane flying at an altitude of 10 km passed over a radar antenna. When the distance from the radar antenna to the plane was 26 km, the radar detected that the distance to the airplane was changing at a rate of 200 km per hour. What was the speed of the plane?

5. Find the derivative $\frac{dy}{dx}$. Show all work and simplify your answer where possible.

(a) $y = \frac{x^2 + c^2}{x^2 - c^2}$ (c is a constant).

(b) $y = e^{x^2}(x^2 + 1)$

(c) $y - x = \tan y$

(d) $y = \frac{(x^2 + 1)^x}{x}$

(e) $y = \sin \sqrt{x} + \sqrt{\sin x}$

6. The velocity of a particle moving along a straight line is given by $v(t) = t^3 - 2t + 3$, where the position is measured in centimeters and the time t in seconds. If $s(0) = 1$ find the position of the particle at time $t = 1$ seconds.

7. Determine the largest possible area for a right triangle whose hypotenuse is 6 cm.

8. Given that $f(x) = \frac{2x^2 + x + 6}{x^2 + 3}$

(a) Find any vertical and horizontal asymptotes to the graph of f .

(b) Given that $f'(x) = \frac{3 - x^2}{(x^2 + 3)^2}$, find any intervals on which f is increasing or decreasing.

(c) Given that $f''(x) = \frac{2x(x^2 - 9)}{(x^2 + 3)^3}$, find any intervals on which the graph of f is concave up or concave down.

(d) Find the exact coordinates of any relative extreme points and inflection points on the graph of f .

9. Find each of the following:

(a) $\int (\tan^2(x) + 1) dx$

(b) $\int \frac{3}{5 - 2u} du$

(c) $\int_{-3}^1 (x + 3)^{-1/2} dx$

10. Sketch the graph of a function f that satisfies all of the given conditions:

$$f(-3) = f(3) = 0 \quad \text{and} \quad f(0) = 27$$

$$f'(-3) = f'(1) = 0$$

$$f'(x) < 0 \quad \text{if} \quad -3 < x < 1$$

$$f'(x) > 0 \quad \text{if} \quad x < -3 \quad \text{and} \quad x > 1$$

$$f''(x) < 0 \quad \text{if} \quad x < -1 \quad \text{and} \quad f''(x) > 0 \quad \text{if} \quad x > -1$$

$$f''(-1) = 0$$

11. Answer ONE of the following:

(a) Find the area of the region in the xy -plane bounded by the graphs of $y = 4 - x^2$ and $y = x^2 + 6x + 4$.

(b) Find the equation of the parabola $y = ax^2 + bx + c$ that passes through $(0, 1)$ and is tangent to the line $y = x - 1$ at $(1, 0)$.