COMP 6902 Spring, 2018

Homework Assignment #2
Due: June 15, 2018, by 7:00am

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[10] 1. Suppose that $L_1 \leq_p L_2$ and $L_2 \leq_p L_3$. Also suppose that L_3 is in NP. Explain how to solve L_1 deterministically in *exponential* time using polynomial space.

[75] 2. Prove that the problems below are NP-complete. You can assume that the following problems are NP-complete: 3SAT, HamPath, Partition, Clique, and $\neq$SAT. Make sure you show that the problems are in NP, then give respective reductions and give an "if and only if" argument for reduction correctness, as well as say why your reduction runs in polynomial time.

(a) **Two-Long-Paths:** $TLP = \{\langle G \rangle | G$ has two paths on at least $\lfloor n/2 \rfloor$ vertices each$\}$

(b) **Quarter Partition** $QP = \{\langle a_1, \ldots, a_n \rangle | \forall i, a_i \geq 0$, and $a_1 \ldots a_n$ can be partitioned into two disjoint sets S and $\bar{S}$ and such that $\Sigma_{a_i \in S} a_i = \Sigma_{a_i \in \bar{S}} a_i$ and, moreover, $|S| = \lfloor n/4 \rfloor\}$

(c) **QuadSat:** $QuadSat = \{\langle \phi \rangle | \phi$ is a satisfiable 3CNF formula with at least 4 satisfying assignments$\}$.

(d) **k-Clique HamPath:** $kCHP = \{\langle G, k \rangle | G$ is an undirected graph and $k \in \mathbb{N}$, and G contains both a clique of size k and a Hamiltonian path$\}$.

(e) **Representatives:** $Rep = \{\langle S_1, \ldots, S_m, k \rangle | S_1, \ldots, S_m$ are subsets of some set U with $|U| = n$, $k \in \mathbb{N}$, and there is a set $S \subseteq U$, $|S| \leq k$, which contains at least one element from each of $S_i\}$.

[15] 3. Prove that the following problems are solvable in polynomial time by describing (in words) algorithms that solve them. State the complexity of your algorithms in O-notation.

(a) **5-clique:** $5\text{-}Clique = \{\langle G \rangle | G$ is an undirected graph G containing a clique on 5 vertices $\}$.

(b) **Small number partition:** $SNP = \{\langle a_1, \ldots, a_n \rangle | a_i \in \mathbb{N}$ and $\forall i, a_i = 1$ or $a_i = 2$ and $\exists S \subseteq \{1, .., n\} \Sigma_{i \in S} a_i = \Sigma_{j \notin S} a_j\}$