CS 3719 Winter, 2019

Homework Assignment #3
Due: March 26, 2019, by 10:00pm

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[60] 1. Prove that the problems below are NP-complete. You can assume that the following problems are NP-complete: 3SAT, Partition, IndSet and Clique.

   (a) **m-Clique k-IndSet:** $mCkIS = \{\langle G, m, k\rangle | G$ is an undirected graph and $m, k \in \mathbb{N}$, and G contains both a clique of size m and an independent set of size $k\}$.

   (b) **QuadSat:** $QuadSat = \{\langle \phi \rangle | \phi$ is a satisfiable 3CNF formula with at least 4 satisfying assignments$\}$.

   (c) **Quarter Partition** $QP = \{\langle a_1, \ldots, a_n\rangle | \forall i, a_i \geq 0$, and $a_1 \ldots a_n$ can be partitioned into two disjoint sets S and $\bar{S}$ and such that $\Sigma_{a_i \in S} a_i = \Sigma_{a_i \in \bar{S}} a_i$ and, moreover, $|S| = \lfloor n/4 \rfloor\}$

[15] 2. Show that if P=NP, then every language in P other than $A = \emptyset$ and $A = \Sigma^*$ is NP-complete (with respect to polytime reductions).

[25] 3. Prove that the following problems are solvable in polynomial time by giving an algorithm that solves them. State the complexity of your algorithm in O-notation.

   (a) **15-cycle:** $15\text{-}Cycle = \{\langle G \rangle | G$ is an undirected graph G containing a cycle on 15 vertices $\}$.

   (b) **Splitting the pairs:** $StP = \{\langle n, (a_1, b_1), \ldots, (a_m, b_m)\rangle | \forall i, a_i, b_i \in \{1, \ldots, n\}$ the elements can be split into 2 groups so that no elements from the same pair are in the same group. $\}$.