Homework Assignment #2, part 2
Due: March 5, 2019

[50] 1. **Undecidability**.

Out of the four problems listed below, one is decidable, one semi-decidable, one co-semi-decidable and one is higher in the arithmetic hierarchy.

To get a sense which one is which, first rewrite the problem descriptions using quantifiers and verifiers (use V_A, V_R, V_H for checking that a computation is accepting, rejecting and halting, respectively.) Then prove that your intuition is correct; remember that to prove that a problem is in a complexity class (decidable, semi-decidable, co-semi-decidable) you need to give an algorithm, and to prove that the problem is not in the class, give a reduction. You can use A_{TM} for most reductions, except use a single reduction from a harder problem for the one higher in the arithmetic hierarchy (you can do two reductions there if you find it easier). Use reductions for all hardness proofs, not the argument by contradiction as in the proof that A_{TM} is undecidable!

(a) $A_{powers} = \{< M >|$ the language of M contains *only* binary numbers that are powers of 2 $\}$.

(b) $A_{loop2} = \{< M >| M, |M$ loops on some string of length at least 2 $\}$

(c) $Halt_{01} = \{< M >| M$ rejects string "1" and halts on some string ending with $0\}$.

(d) $A_{in} = \{< M, w > |M$ accepts w without moving to a blank cell$\}$.