Homework Assignment #2, part 1
Due: Feb 13, 2019

[25] 1. **Programming Turing machines**

(a) Let $L = \{x \in \{0,1\}^* \mid x \text{ ends with } 1\}$. First describe in English, and then give a full description of a deterministic Turing machine $M = \langle Q, \Sigma, \Gamma, q_0, q_a, q_r, \delta \rangle$ deciding L. Your English description should explain what your Turing machine is doing, and your full description should include a table for δ. Use only states q_0, q_1, q_a and q_r, and take $\Gamma = \{0, 1, \sqcup\}$.

(b) List a sequence of configurations describing the computation of your Turing machine from the previous subquestion on the string 010. Now do the same for the string 11. Finally, what sequence of configurations would your Turing machine go through on blank input?

(c) Describe in English a Turing machine that, on an input of the form 1^k0w, where $w \in \{0,1\}^*$, inserts a special symbol # after k characters of w and then erases prefix 1^k0. That is, when started on an input of the form $1^k0w_1 \dots w_n$, your Turing machine should end in configuration $q_a w_1 \dots w_k \# w_{k+1} \dots w_n$, for example on input 110010101 it should end in q_a01#0101. If the input is not of the form $1^k0w_1 \dots w_n$ for $k \geq 0$, or $|w| < k$, your TM should reject (end in rejecting state, any tape content/head position). Hint: add new symbols to Γ.

[25] 2. **Double-right triple-left Turing machine**

Consider the following simple variant of a Turing machine: a double-right triple-left (DRTL) Turing machine is a two-way infinite tape Turing machine in which every Right (RR) move moves 2 cells to the right, and every Left (LLL) move moves three cells to the left. Here we will show one direction of the proof that DRTL is equivalent to the "normal" two-way infinite tape Turing machine.

(a) Show how, given a normal Turing machine M, to construct a DRTL Turing machine M' recognizing the same language, by showing how to simulate transitions $(q_i, a) \to (q_j, b, R)$ and $(q_i, a) \to (q_j, b, L)$ of a normal Turing machine on DRTL TM. Make sure to say explicitly which and how many states and transitions M' would have, if M has $|Q| = n$ states and $|\Gamma| = d$. This will show that DRTL TMs are at least as powerful as normal TMs.

(b) Now, consider a generalization of DRTL to k-right m-left Turing machines. What should be the condition on k and m for your proofs to still go through? And for which values of k and m you do not get an equivalent Turing machine? Assume the input is written sequentially on the tape.