Homework Assignment #1
Due: Jan 31, 2019

[50]   1. **Finite automata and regular languages**

   (a) Construct a DFA and a regular expression for the language $L = \{x \in \{0,1\}^* \mid x$ contains a substring "100" $\}$.

   (b) Construct a DFA and a regular expression for the language $L = \{x \in \{0,1\}^* \mid$ length of x is not divisible by 3$\}$.

   (c) Construct a 6-state NFA and a regular expression for the language $L = \{x \in \{a,b\}^* \mid x$ ends in "ba" or has exactly two a's $\}$, such that your NFA has at least one ϵ-arrow and also multiple transitions on the same symbol.

   (d) Construct an NFA from a regular expression $((a\cup(ab)^*)c)^*$ over the alphabet $\Sigma = \{a,b,c\}$, using the general construction described in the proof that NFAs can simulate regular expressions.

   (e) Given the following NFA, construct a DFA using the construction from the class/notes. Describe in words the language this NFA accepts.
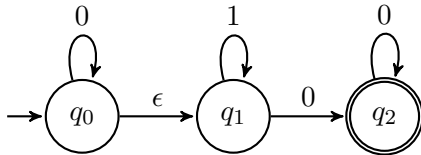
[20]   2. **Pumping lemmas**

   (a) Prove that the language $\{wtttw \mid w, t \in \Sigma^*\}$ is not regular.

   (b) Prove that the language $\{a^n b a^{3n} b a^{2n}\}$ is not context-free.

[30]   3. **Context-free grammars and pushdown automata**
   For each of the following languages, give a grammar and a PDA. State if your grammar is ambiguous.

   (a) $\{ba, ab, c\}$

   (b) $\{w \in \{a,b\}^* \mid w$ is an odd-length palindrome starting with $a\}$.

   (c) $\{w \in \{0,1,2\}^* \mid$ the number of 0's is exactly 3 times the number of 1's $\}$