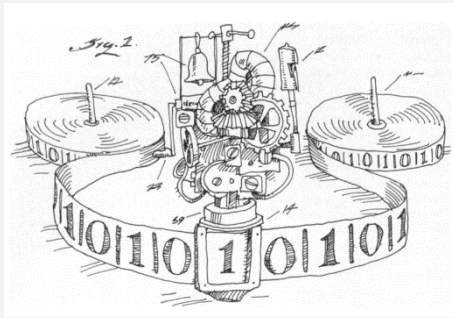
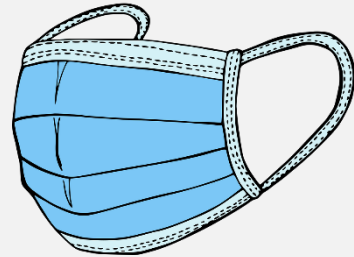
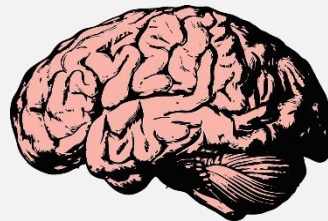
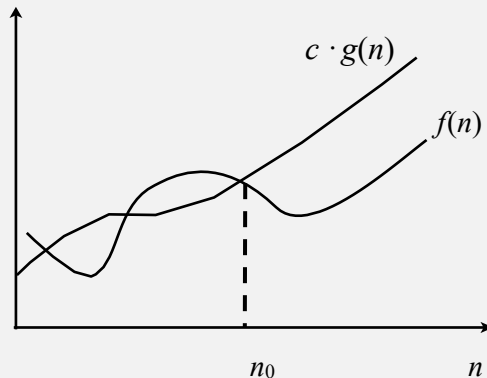


COMP 3602

Intro to Theory of Computation



Lecture 20



Announcements



- Complexity drill due Thu Mar 20
- Two more tests left:
 - Oral exam on NP-completeness: Mon Mar 23
 - Practice problems are posted
 - Sign up by Friday before
 - Written exam on all the material we covered in the course.
 - Fri Mar 27th In class
- Presentations in class:
 - Mar 30 and Apr 1, please sign up for your preferred day
 - All slides due Sun Mar 29th

1. Show that NP is closed under Kleene star, by showing that if L is in NP, then L^* is in NP as well, where every string in L^* is a concatenation of 0 or more strings from L , that is, $L^* = \{ x \mid x = x_1 \dots x_k \text{ for some } k \geq 0 \text{ and strings } x_1, \dots, x_k \text{ such that } \forall i \leq k \ x_i \in L \}$. Use the quantifier/verifier definition of NP.

2. Prove that the problems below are NP-complete. You can assume that the following problems are NP-complete:

3SAT, SubsetSum, Partition, HamPath (LongestPath with $k=|V|$), IndSet and Clique.

- a. DoubleSAT = $\{ \langle \phi \rangle \mid \phi \text{ has at least two satisfying assignments} \}$.
- b. SixOccurrenceSAT = $\{ \langle \phi \rangle \mid \phi \text{ is a satisfiable 3CNF with at most 6 occurrences of every variable} \}$
- c. kClique-kIndSet = $\{ \langle G, k \rangle \mid G \text{ contains both a clique of size } k \text{ and an independent set of size } k \}$
- d. Half-Clique = $\{ \langle G \rangle \mid G \text{ has a clique of size } \lceil |V|/2 \rceil \}$
- e. HamCycle = $\{ \langle G \rangle \mid G \text{ contains a cycle which goes through all vertices exactly once} \}$
- f. FourPartition = $\{ \langle (a_1, \dots, a_n) \rangle \mid \forall i \leq n, a_i \geq 0, \text{ and } 1 \dots n \text{ can be partitioned into four disjoint sets } S_1, S_2, S_3 \text{ and } S_4 \text{ such that } \sum_{i \in S_1} a_i = \sum_{i \in S_2} a_i = \sum_{i \in S_3} a_i = \sum_{i \in S_4} a_i \text{ and } S_1 \cup S_2 \cup S_3 \cup S_4 = \{1, \dots, n\} \}$

- **HamPath, HamCycle and TravelingSalesmanProblem:**

- A Hamiltonian path in a graph is a path which visits every vertex exactly once.
 - That is, a longest path in G is of length $|V|=n$
- A Hamiltonian cycle is a simple cycle which visits every vertex exactly once
- $HamPath(G) = \{ \langle G \rangle \mid G \text{ is a directed graph that has a Hamiltonian path} \}$
- $HamCycle(G) = \{ \langle G \rangle \mid G \text{ is a directed graph that has a Hamiltonian cycle} \}$
- $TSP(G) = \{ \langle G, t \rangle \mid \text{a weighted directed } G \text{ has a Hamiltonian cycle of weight } \leq t \}$
- All three problems above are NP-hard for undirected graphs too.

Proving NP-completeness: template.

Goal: show that a language L' is NP-complete.

1. Show that $L' \in NP$

- Explain what potential solutions are and how to check them efficiently.
- That is, explain what y and R are in $L = \{x | \exists y |y| < poly(|x|) \wedge R(x, y)\}$,
- and why y is short and R is polytime computable.

2. Show that L' is NP-hard

- Pick a problem L we have showed to be NP-complete already.

- Show $L \leq_p L'$

a) Construct a polytime reduction f from L to L' ,

- that is, describe how to get $x' = f(x)$ for each x

» **Important: f cannot determine whether $x \in L$!**

b) Show correctness: two directions.

i. $\Rightarrow$ Given a solution y to x , construct a solution y' to $f(x)$

ii. $\Leftarrow$ Given a solution y' to $f(x)$, reconstruct a solution y to x

» **This applies only to $x' = f(x)$, not arbitrary element of L' !**

c) Explain briefly why f is polynomial-time computable

