CS 2742 (Logic in Computer Science) – Winter 2014
Lecture 10

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How to describe a set? List elements, or list a property that elements have.

$S = \{John, Bob, Mary, George, Alex\}$. Another set is $S = \{2, 3, 4\}$, which is the same as $S = \{x \mid 2 \leq x \leq 4\}$. If a set is infinite, we cannot list its elements, so we have to list the property: $S = \{x \mid x \in \mathbb{N} \wedge x|2\}$ is the set of all even natural numbers

5.1 Operations on sets

Suppose S_1 and S_2 are two sets (with the universe U). Then the following are set operations:

$$\begin{aligned}
\text{Union:} \quad & S_1 \cup S_2 = \{x|x \in S_1 \vee x \in S_2\} \\
\text{Intersection:} \quad & S_1 \cap S_2 = \{x|x \in S_1 \wedge x \in S_2\} \\
\text{Difference:} \quad & S_1 - S_2 = \{x|x \in S_1 \wedge x \notin S_2\} \\
\text{Complement:} \quad & \bar{S_1} = \{x|x \notin S_1\}
\end{aligned}$$

Note that the complement of an empty set is the universe, and the complement of the universe is the empty set.

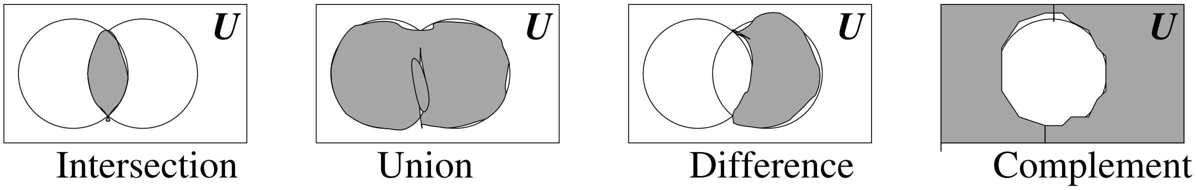


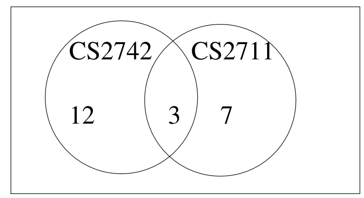
Figure 1: Set operations

Two sets are equal of they are subsets of each other: $A = B$ is defined to be $\forall x(x \in A \iff x \in B)$, or, alternatively, iff $A \subseteq B \wedge B \subseteq A$.

One way of representing sets is Venn diagrams. Here, each set is drawn as a circle, intersecting of the sets have common elements. Venn diagrams are useful for visualizing sets containing common elements.

Example 1. Suppose 15 students take CS2742 and 10 students take CS2711. Provided that 3 students take both, how many are in either one of these two courses?

To solve the problem, draw a Venn diagram (two intersecting circles) for the sets of students in CS2742 and CS2711, respectively. Put the number of students taking both (3) in the intersection. Now, put the remaining number of students in the corresponding sets outside of the intersection: 15-3=12 in the CS2742 set, and 10-3=7 in the CS2711 set.



Now, the total number of students, that is students that are in either of these two courses, is the sum of these three numbers: $|A \cup B| = (|A| - |A \cap B|) + (|B| - |A \cap B|) + |A \cap B| = 12 + 7 + 3 = 22$. Equivalently, $|A \cup B| = |A| + |B| - |A \cap B| = 15 + 10 - 3$ (since we subtracted $|A \cap B|$ twice and added once).

In general, this type of argument is called *the rule of inclusion/exclusion* and it can be generalized for an arbitrary number of sets. Using the notation $|S|$ to denote the number of elements in the set,

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Later in the course we will develop techniques that will allow us to prove a general inclusion/exclusion rule.

Puzzle 1. [1] What is wrong with the following argument?
"Nothing is better than an eternal bliss. A hamburger is better than nothing. Therefore,a hamburger is better than eternal bliss".

[1] This puzzle is for the material that will come later in the course, but also for the definition of the empty set.