CS 2742 (Logic in Computer Science) – Lecture 1

Antonina Kolokolova*

Jan 10, 2014

1 What is logic in Computer Science?

Why do we study mathematical logic? Natural languages (such as English) are too ambiguous:

> "Every student knows this" and "Any student knows this" have the same meaning.
>
> "She'd be happy if she passes every course" and "She'd be happy if she passes any course" mean very different things.

Mathematical logic is a kind of language, where everything is designed to have a precise meaning. This is the language of unambiguous reasoning. So this course is somewhat like a foreign language course: there will be a lot of vocabulary to memorize. Also, like in a language course, you need to practice "speaking" logic language for it to become natural.

Logic comes into computer science in many different ways:

- Digital circuit logic
- Proof of correctness of programs, verification
- Automated reasoning in artificial intelligence

*The material in this set of notes came from many sources, in particular "Discrete mathematics with applications" by Susanne Epp, several books by Raymond Smallian, course notes of U. of Toronto CS 238 by Vassos Hadzilacos.

- Database query languages
- ...

We will try to cover, briefly, at least some of these applications.

2 Propositional logic

The basic unit of our reasoning is a sentence that can have a truth value, that is it can be either true or false. We call such a sentence a *proposition*. For example, "it is raining" is a proposition. It is true or false at any particular point in time and space. So is "I am a dolphin" (which happens to be false, as far as I know), or "2 is a prime number". When referring to a proposition, we often will give it a short name (a variable); for example, we can use a variable p to denote the sentence "it is raining". Now, if we say "p is true" we will mean that it is, indeed, raining. Such variables denoting propositions are called *propositional variables*. A propositional variable can have a *truth value* of exactly one of "true" or "false".

Simple propositions can be combined. For example, we can say "if it is raining, then it must be cloudy". Here, we mean that "raining" (that is, our proposition p) implies that it is also "cloudy". If we use a propositional variable q to mean "it is cloudy" then in the language of propositional logic we say that "p implies q". Implication is one of *logical connectives* that allow us to make more complicated statements, *propositional formulas*, out of propositions. The following table lists several common logical connectives.

Meaning	**Name**	**Notation**	**Pronunciation**
both p and q are true	*conjunction*	$p \wedge q$	p and q
at least one of p, q is true	*disjunction*	$p \vee q$	p or q
opposite of p is true	*negation*	$\neg p$	not p
if p is true then q is true	*implication*	$p \to q$	p implies q

Puzzle 1 (Twins puzzle) There are two identical twins, John and Jim. One of them always lies, another always says the truth (that is, every sentence the truth-teller says is true; every sentence the liar utters is false). Suppose you run into one of them and want to find out whether you met John or Jim. Which 3-word question with a yes/no answer could you ask to learn the name of the twin in front of you? You don't know which one of them is the liar.