COMP2000 Quiz Notes

Computation and logic
Computation: Turing machine

• Turing machine:
  – Can compute anything we know to be computable by other means (Church-Turing thesis)
  – Has an infinite tape with a read/write head, which starts with an input written on it. Has finitely many states in its description.
  – Simple rules: in a state, read a symbol then change a state (maybe), overwrite the symbol with another one (maybe) and move Left or Right
  – Invented by Alan Turing to show that some problems are not computable.

<table>
<thead>
<tr>
<th>Current state</th>
<th>Reads</th>
<th>Writes</th>
<th>Moves</th>
<th>New state</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start_state</td>
<td>_</td>
<td>Y</td>
<td>Right</td>
<td>Halt</td>
</tr>
<tr>
<td>Start_state</td>
<td>0</td>
<td>_</td>
<td>Right</td>
<td>Start_state</td>
</tr>
<tr>
<td>Start_state</td>
<td>1</td>
<td>_</td>
<td>Right</td>
<td>Start_state</td>
</tr>
<tr>
<td>Start_state</td>
<td>_</td>
<td>N</td>
<td>Right</td>
<td>Halt</td>
</tr>
</tbody>
</table>
Game of Life:

- As powerful as a Turing machine: a Turing machine can simulate it moves, and it can simulate a Turing machine (non-trivial!)
- Start with a board; some of the cells on the board are marked “live”. Every cell has 8 neighbours (above, below, sides and diagonal).
- Rules:
  - If a live cell has fewer than 2, or more than 3 neighbours, it dies.
  - If a live cell has 2 or 3 neighbours, it stays alive.
  - If a dead cell has exactly 3 neighbours, it becomes live.
- Many patterns: static, oscillating, moving
Logic: propositions

- **Propositions**: sentences that can be true or false
  - “It is Tuesday today” is true on a Tuesday and false on all other days.
  - “5 is a prime number” is true, “6 is prime” is false
  - Non-example: “x is a prime number”. This is a predicate, it would become a proposition if x is set to a specific number. “Come here” is not a proposition either.

- **Logic connectives**: connect several propositions into a composite sentence that can be true or false, depending on the propositions. A sentence that is always true is called a “tautology”, always false “contradiction”.

- **Truth table**: a table listing all possible combinations of truth values of propositions, together with columns for composite statements. Can be used to find out when a composite statement is true and when it is false.

- Below is a truth table for the logic connectives we used:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>not A</th>
<th>A and B</th>
<th>A or B</th>
<th>if A then B</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>True</td>
<td>False</td>
<td>True</td>
<td>True</td>
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<tr>
<td>True</td>
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<td>False</td>
<td>True</td>
</tr>
</tbody>
</table>
Logic: quantifiers

• A *predicate* is a “proposition with parameters”, which becomes a proposition if the parameters are set to a specific value.
  – “x is a prime number”. Prime(x)
  – “X loves Y”

• One can also make composite statements with predicates stating that for *every* or for *some* element of the kind the predicate is talking about something is true. The “every” (∀) and “some” or “exists”, (∃) are called quantifiers, universal and existential.
  – Every man is mortal
  – There exists a number that is prime (some numbers are prime)
  – It is foggy every day in St. John’s.

• Now, both propositional logic connectives and quantifiers can be combined to make composite statements of first-order logic.
  – Every man is an animal, and some animals have fur
  – Everybody loves somebody, and that somebody loves Elvis Presley.
    • ∀ X ∃ Y (X loves Y) and (Y loves Elvis)
Modus ponens

• Modus ponens is a rule used in proofs in natural deduction. Here is the general form:
  – If A then B
  – A holds
  – Therefore, B holds

• With universal quantifier, becomes universal modus ponens; second line substitutes specific value.
  – All man are mortal
  – Socrates is a man
  – Therefore, Socrates is mortal.

• Modus ponens allows solving some logic problems and puzzles faster than truth tables. But nobody knows if it is always better (there is a million dollar prize for that).
Negations

• To disprove a universal ("every") statement, give a counterexample ("exists" statement).
  – It is foggy every day in St. John’s
    • No, on Monday it was sunny.
  – So “not (every day is foggy)” is “exists a day that was not foggy”.
  – But it’s definitely does not mean that every day is not foggy
  – “not (exists a day that is foggy)” is “every day is not foggy”.

• Similarly, negating an existential get universal of the “not”s:
  – There exists a natural number less than 1
    • No, every natural number is not less than 1.

• For logic connectives, notice that “if.. Then” is similar to “every”, and “and” is similar to “exists”:
  – If Socrates is a man, then Socrates is mortal
    • No, Socrates is a man, and he is immortal
  – Number 0.5 is a natural number and it is less than than 1
    • No, either 0.5 is not a natural number, or it is not less than 1.
  – Either today is Tuesday or today is Thursday
    • No, today is not Tuesday and today is not Thursday.
For the quiz:

• Be able to say what a transition of a Turing machine such as “in state A on reading 0 go to state B, write 1 and move right” does to the tape content.

• Be able to do a few generations of a game of life starting with a given pattern.

• There will be a few yes/no questions (e.g., is it true that everything we know to be computable is computable by a Turing machine?)
For the quiz

• Practice solving knight and knave puzzles and (simple) treasure hunts. Use truth tables and modus ponens, where appropriate.

• Make sure that you can recognize those fallacies we played with (which card to turn over? If I like blue triangle, what about yellow circle? Is Susan more likely to be a banker or a banker-activist?)

• Practice translating between logic and English, and identifying propositions and quantifiers in English sentences.

• Practice doing negations. See some new slides in Lecture 3, and last lecture.

• Email me if you have any questions! I will be around on Wednesday afternoon, if you’d like to stop by.