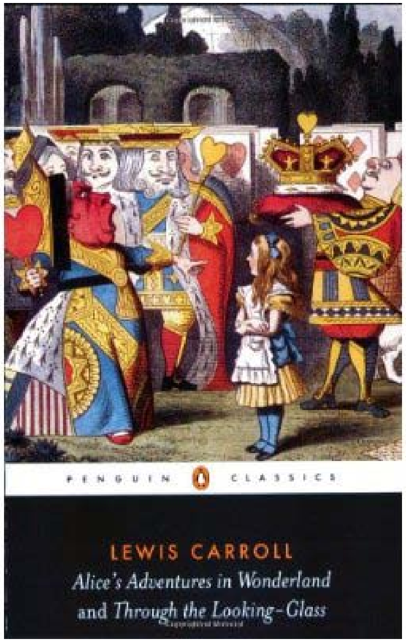
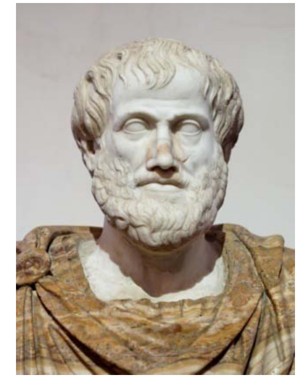


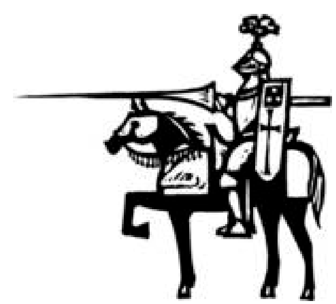


COMP2000

Lecture 3

Logic: puzzles, truth and human fallacies

B 5 2 J



Do we think logically?

- You see the following cards. Each has a letter on one side and a number on the other.

- Which cards do you need to turn to check that if a card has a J on it then it has a 5 on the other side?

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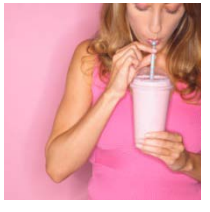
Another puzzle

- You are one of the organizers at a mixer; many people are drinking, some are not.
- You need to make sure that nobody underage is drinking: that is, if somebody is drinking, then they are over 19 years old.

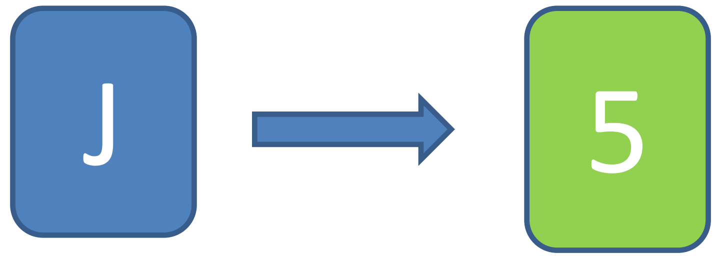



- Which category of people will you need to check?

“if … then” in logic

- Both puzzles have the same structure:

“if A then B”



- What circumstances make this true?
  - A is true and B is true

  - A is true and B is false

  - A is false and B is true

  - A is false and B is false

$A \rightarrow B$

- We make logical conclusions all the time
- But do we always make them “logically”?
- Sometimes people think that “if ... then” goes both ways...
  – If you live in NL, you must pay HST. John lives in BC. Does he pay HST?
  – If today it Tuesday, then there is a COMP2000 lecture. Today is Thursday. Is there a lecture?

Natural vs. Logic language

- Natural languages are ambiguous.
- For example, the word “any” can have different meanings depending on the context:

- Any = some
  - She will be happy if she can solve any question.
  - She will be happy if she can solve every question.

- Any = all
  - Any student knows this.
  - Every student knows this.

Language of logic

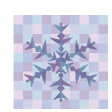
Pronunciation	Notation	Meaning
A and B	A /\ B	True if both A and B are true
A or B	A \/ B	True if either A or B are true (or both)
If A then B	A -> B	True whenever if A is true, then B is also true
Not A	~A	Opposite of A is true, so true if A is false

- Let A be “It is sunny” and B be “it is cold”
  - A /\ B: It is sunny and cold
  - A \/ B: It is either sunny or cold
  - A -> B: If it is sunny, then it is cold
  - ~A: It is not sunny

Language of logic

Pronunciation	Notation	True when
A and B	A /\ B	Both A and B must be true
A or B	A \/ B	Either A or B must be true (or both)
If A then B	A -> B	if A is true, then B is also true
Not A	~A	Opposite of A is true

- Now we can combine these operations to make longer formulas in the language of logic

- Let
  - A be “It is sunny”, 
  - B be “it is cold”, 
  - C be “It’s snowing” 
- Let’s make some sentences out of A, B, C

 Language of logic 

Pronunciation	Notation	True when
A and B	A /\ B	Both A and B must be true
A or B	A \/ B	Either A or B must be true (or both)
If A then B	A -> B	if A is true, then B is also true
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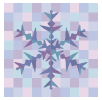
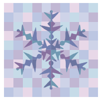
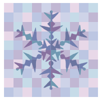
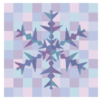
  - C be “It’s snowing” 

- What are the translations of:
  - B /\ C -> ~A   IF ( AND ) THEN NOT 
    - If it is cold and snowing, then it is not sunny
  - B -> (C \/ A)   IF  THEN ( OR )
    - If it is cold, then it is either snowing or sunny
  - ~A /\ A -> C   IF (NOT  AND ) THEN 
    - If it is sunny and not sunny, then it is snowing.

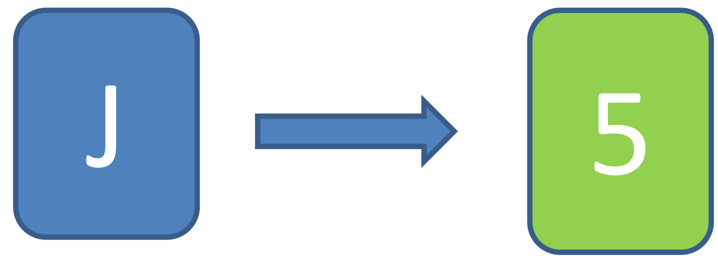
The truth

- We talk about a sentence being true or false when the values of the variables are known.
- If we didn't know whether it is sunny, we would not know whether A /\ B -> C is true or false.
- If a sentence is true for every possible combinations of variable truth assignments, we call it a "tautology"

“if … then” in logic

- Both puzzles have the same structure:

“if A then B”



- What circumstances make this true?
  - A is true and B is true
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  - A is false and B is true
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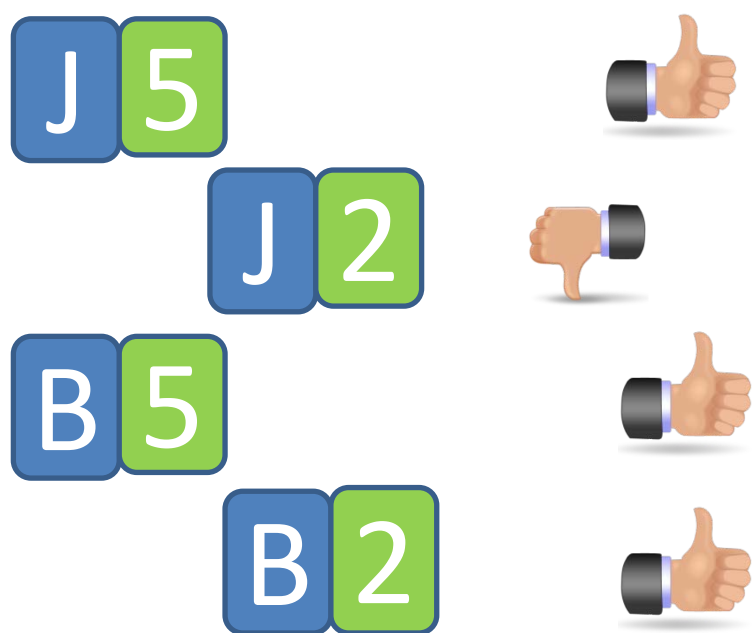


Truth tables

A	B	not A	A and B	A or B	if A then B
True	*True*	False	True	True	True
True	*False*	False	False	True	False
False	*True*	True	False	True	True
False	*False*	True	False	False	True

A	B
True	*True*
True	*False*
False	*True*
False	*False*

- Let
  - A be “It is sunny”
  - B be “it is cold”
- It is sunny and cold.
- It is sunny and not cold
- It is not sunny and cold
- It is neither sunny nor cold

Truth tables

- Let
  - A be “It is sunny”
  - B be “it is cold”
- It is sunny and cold.
- It is sunny and not cold
- It is not sunny and cold
- It is neither sunny nor cold
- Now, ~ A $\bigvee$ B is:

A	B	not A	A and B	A or B	if A then B
True	*True*	False	True	True	True
True	*False*	False	False	True	False
False	*True*	True	False	True	True
False	*False*	True	False	False	True

A	B	(Not A) or B
True	*True*	*True*
True	*False*	*False*
False	*True*	*True*
False	*False*	*True*

Or: the law of excluded middle

- In classical logic, the law of excluded middle say that either a statement or its opposite must be true.
- But here by the opposite we really mean a negation

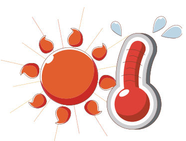
  - A: It is sunny.
    - ~A: It is not sunny
  - A: Today is Tuesday.
    - ~A: Today is not Tuesday
  - A: John votes for NDP.
    - ~A: John does not vote for NDP
  - A: You are with us
    - ~A: You are not with us.

Negating composite statements

- What is the negation (opposite) of a longer logic statement? Take a truth table column and flip all the values.
  - The negation of "A and B", not(A and B), is true whenever either A or B is false (check the truth table. That is not(A and B) is the same as (not A or not B).
  - The negation of "A or B" is true whenever both A and B are false: not(A or B) is the same as (not A and not B).
  - Since "if A then B" is the same as "not A or B", its negation is "A and not B". Remember that "if A then B" is only false when A is true, and B is false; check that "A and not B" is true in exactly the same scenario.

NOT’ing longer sentences

- For a longer combination, start with the connective applied last when computing a truth table:
  - “not (if (A or not B) then (A and C))” becomes
  - (A or not B) and not(A and C) by negating “if... Then..”
  - (A or not B) and (not A or not C) by negating the last “and”
- Let A be “it’s sunny” and B “it’s cold”.
  - “It’s sunny and cold today”! -- No, it’s not!
  - That could mean
    - No, it’s not sunny.
    - No, it’s not cold.
    - No, it’s neither sunny nor cold.
  - In all of these scenarios, “It’s either not sunny or not cold” is true.

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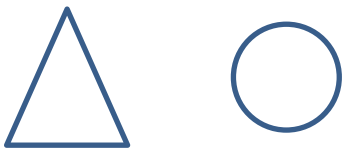
More on “not if.. then”

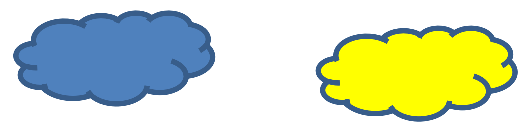
- Remember that for “if A then B” there is only one scenario when it is false: it is when A is true, and B is false.
- Let A be “it’s raining” and B “it is cloudy”. Then “if A then B” means if it is raining it must be cloudy.
- “not (if A then B)” means “it’s not true that when it is raining it must be cloudy”. Or, equivalently, “it’s raining, and it is not cloudy” (there is probably a rainbow then, too) .
- “not (if A then B) is definitely not a negation of “if B then A”!

Or: elusive, not exclusive.

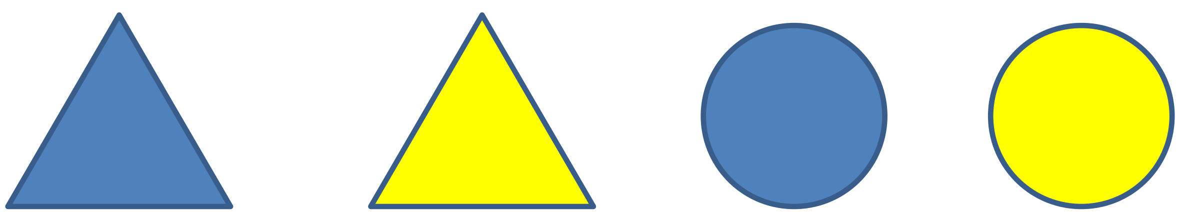
- I like one of the shapes.



- I like one of the colours.



- I like a figure if it has either my favourite shape or my favourite colour.



- I like the blue triangle. What can you say about the rest?

Proof vs. disproof

- To prove that something is (always) true:
  - Make sure it holds in every case.
    - I have classes on every day that starts with T. I have classes on Tuesday and Thursday (and Friday, but that's irrelevant).
  - Or assume it does not hold, and then get something strange as a consequence
    - Suppose there are finitely many prime numbers. What divides the number that's a product of all primes +1?
- To disprove that something is always true:
  - Give just one example where it breaks down.
    - I have classes every day! – No, you don't have classes on Saturday!
    - All girls hate math! – No, I love math and I am a girl :)

Proof vs. disproof

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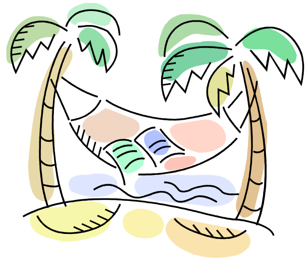
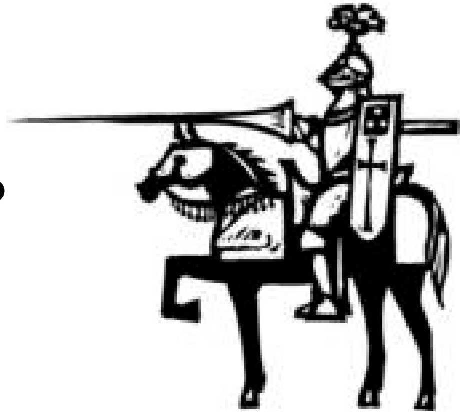
Knights and knaves

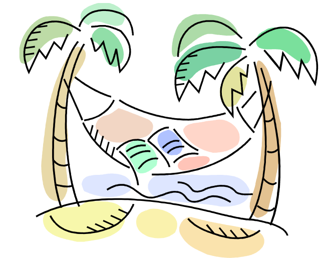


- On a mystical island, there are two kinds of people: knights and knaves.

-  Knights always say the truth.

- Knaves always lie.

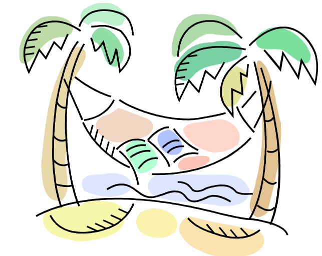

Knights and knaves



- On a mystical island, there are two kinds of people: knights and knaves. Knights always tell the truth. Knaves always lie.

- Puzzle 1: You meet two people on the island, Arnold and Bob. Arnold says “at least one of us is a knave”. Is Arnold a knight or a knave? What about Bob?


Knights and knaves

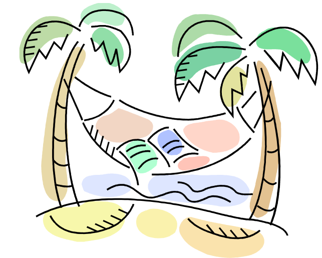


- On a mystical island, there are two kinds of people: knights and knaves. Knights always tell the truth. Knaves always lie.

- Puzzle 2: You meet two people on the island, Arnold and Bob. Arnold says “Either I am NOT a knight, or Bob is a knave” Is Arnold a knight or a knave? What about Bob?

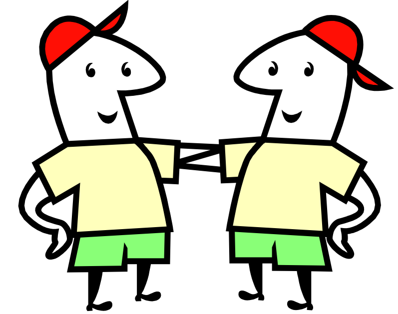
Knights and knaves



- On a mystical island, there are two kinds of people: knights and knaves. Knights always tell the truth. Knaves always lie.

- Puzzle 3: You see three islanders talking to each other, Arnold, Bob and Charlie. You ask Arnold “Are you a knight?”, but can’t hear what he answered. Bob pitches in: “Arnold said that he is a knave!” and Charlie interjects “Don’t believe Bob, he’s lying”. Out of Bob and Charlie, who is a knight a who is a knave?


Twins puzzle



- There are two identical twin brothers, Dave and Jim.
- One of them always lies; another always tells the truth (like knights and knaves).
- Suppose you see one of them and you want to find out his name.
- How can you learn if you met Dave or Jim by asking just one short yes-no question? You don't know which one of them is the liar.