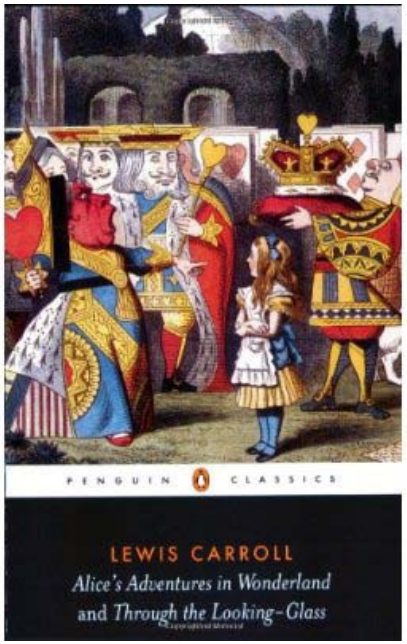
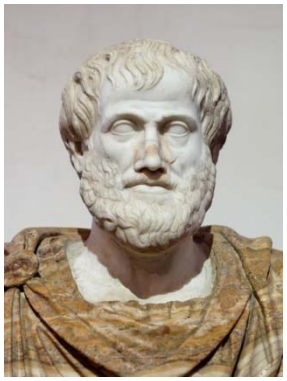
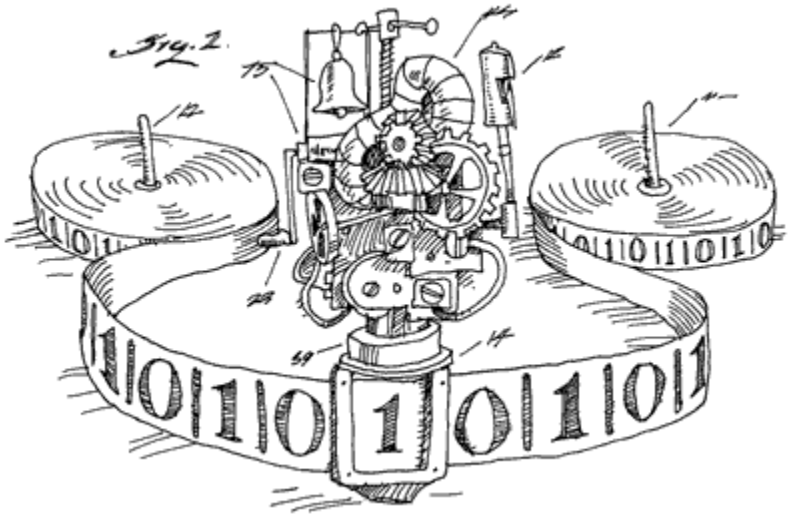
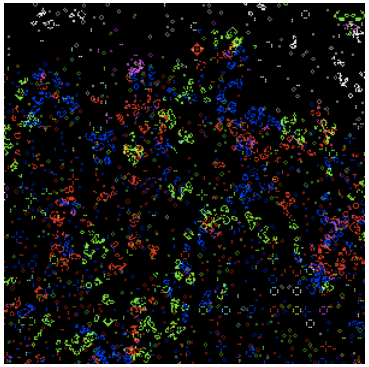


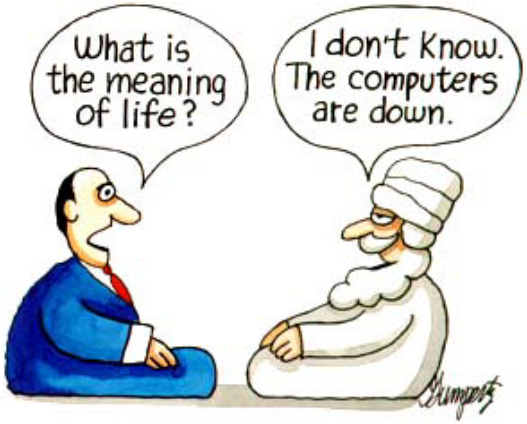


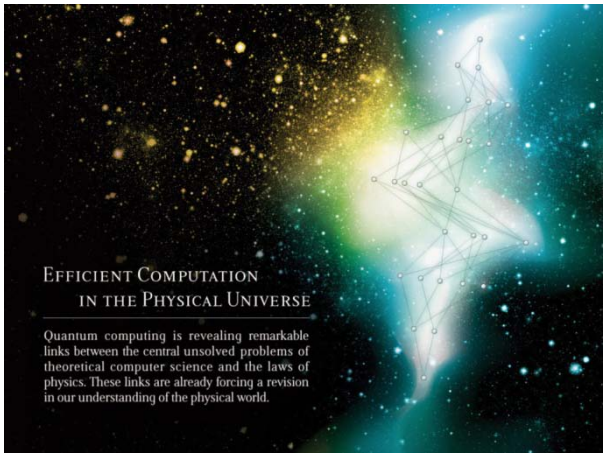


COMP2000

Computation, logic and meaning

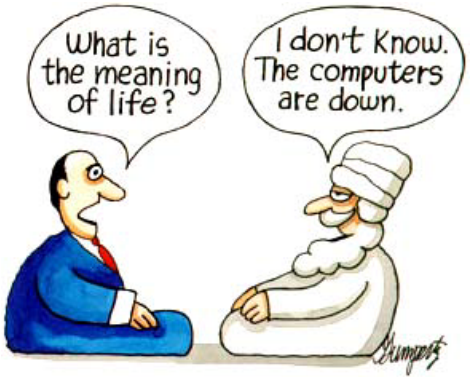






- What is computation?

- What is information?



- What is meaning?

- What is (logical) thinking?

What is information?

- What do you mean when you say that a certain lecture, conversation or a TV program was “informative”?
- Does it have something to do with learning something you have not known before?
- Exact definition of information is related to entropy: see the textbook, chapter 3, for more detail.

What is information?

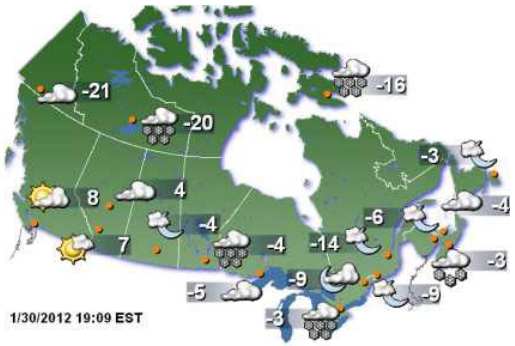
- Does string 11111111111 contain more information than the string 10010110100?
- How much information does a coin toss give you for a fair coin? For a coin with two heads?
- Do you learn more from a coin toss of a fair coin or a coin toss of a coin with two heads? How about a roll of dice?

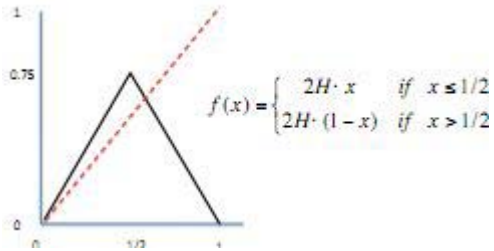
What is information?

The less you can predict an outcome

The more you learn from it:

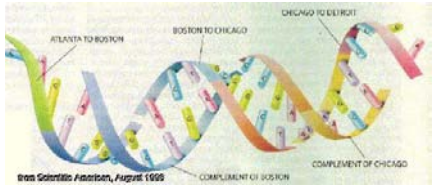
The more information you get.





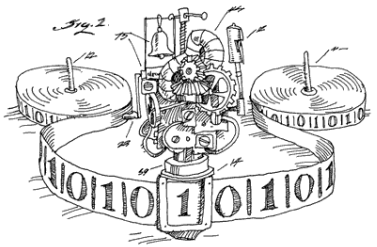
The science of information

- In many languages the word for “Computer Science” is derived from the word for information
  - French: Informatique
  - German: Informatik
  - Russian: Информатика



- The information comes in and we process it.
- So do computers. So do living cells, etc, etc.

What is computation?

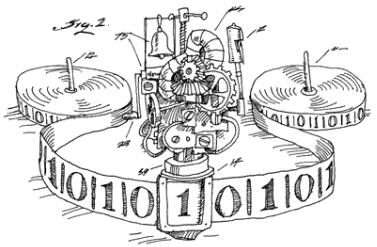


- We process information by doing a “computation on it”. Changing it from one representation to another.

- But what is computation?
  - What does your smartphone compute when you are playing Angry Birds?
  - How does DNA “compute”?

- Is there a limit to what can be computed?

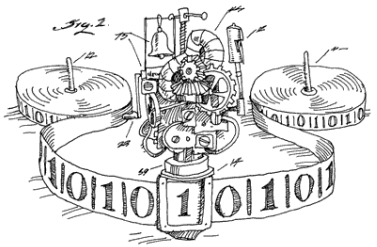
What is computation?



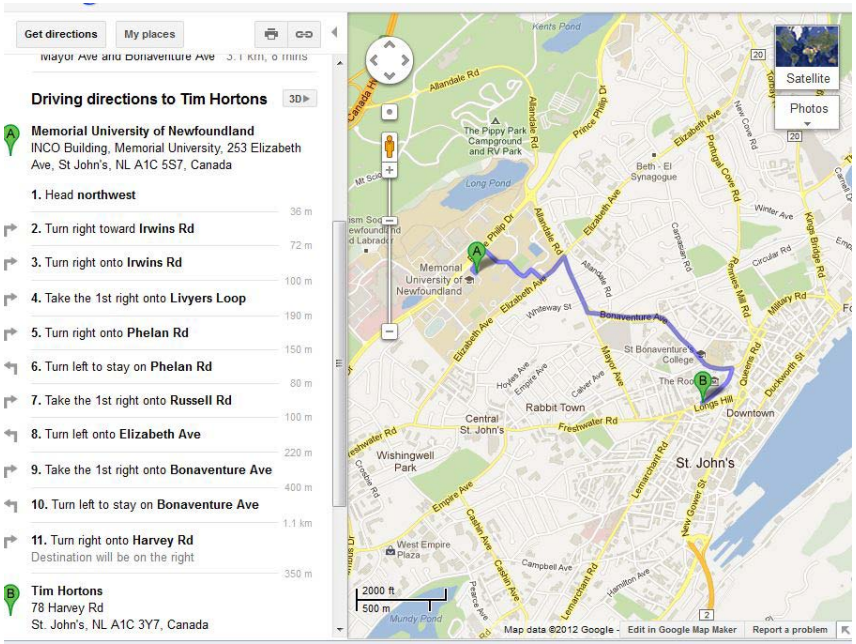
- During the World War II, hundreds of people were employed as "computers" to calculate ballistic trajectories.

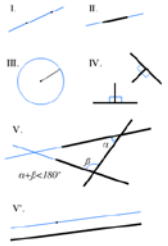
- This is the same kind of calculation as in the "Angry Birds".

What is computation?

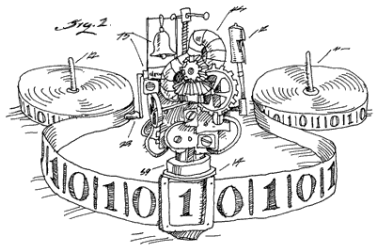


- Computation as executing a list of instructions:
  - Drive straight until you see the Basilica
  - Then turn right, and drive till the next light.
  - Then turn right, and enter Tim Hortons parking lot.



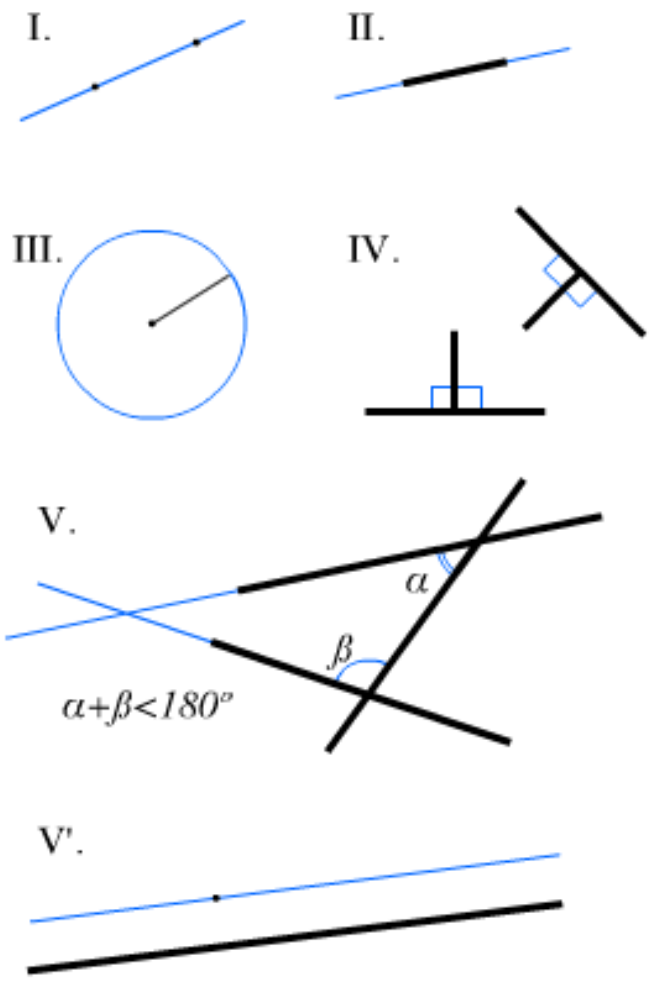


Limits of computation

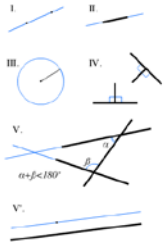


- In 1900, at the International Congress of Mathematicians in Paris, David Hilbert posed a list of 23 problems. Problem 2 asked to prove that mathematics contains no self-contradictions.
- In 1920, Hilbert extended it to what is now known as “Hilbert’s program”

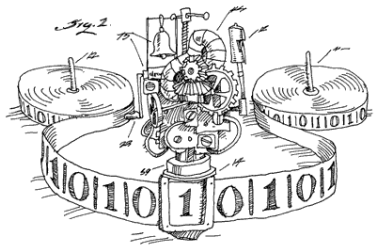
Axioms example: Euclid's postulates



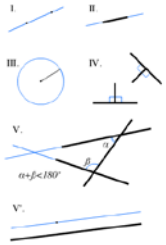
I. Through 2 points a line segment can be drawn
II. A line segment can be extended to a straight line indefinitely
III. Given a line segment, a circle can be drawn with it as a radius and one endpoint as a centre
IV. All right angles are congruent
V. Parallel postulate



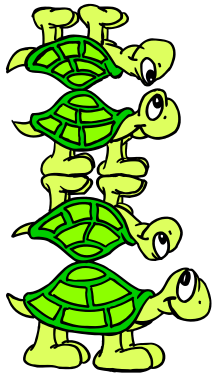
Hilbert's program



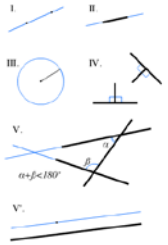
- Express all mathematics in a precise way
- Allowing a formal proof of all true statements
- With a proof, inside mathematics, that there is no self-contradiction
- And a procedure (an algorithm) for deciding, for any given mathematical statement, whether it is true or false.



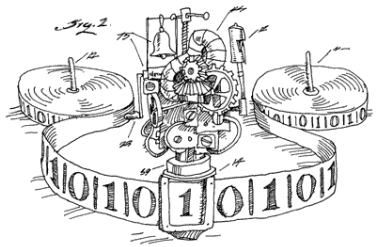
Gödel Incompleteness Theorem



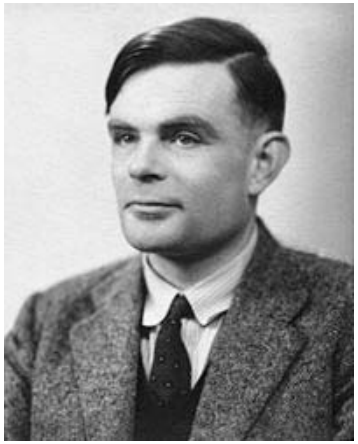
- If mathematics is not self-contradictory...
- Then there are statements that can't be proven!
- Such as "I am not provable"
- Like with dynamical systems, self-reference leads to something strange, a paradox!



Church and Turing:



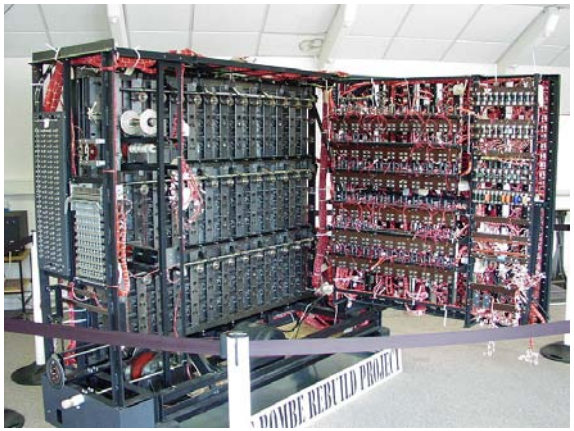
- Moreover,
- there is no procedure
- to decide if a given statement is true or false!
- And to decide many other things...



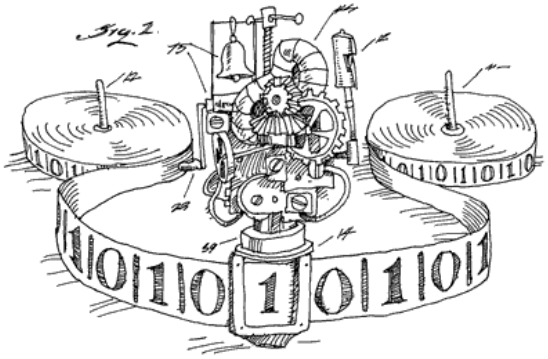
- But what do we mean by a “procedure”?

Digression: a bit about Alan Turing

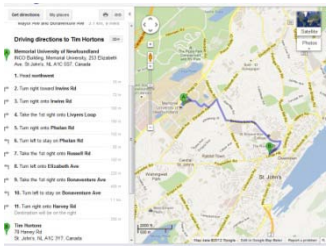
- This year marks 100 years from Alan Turing's birth
- He is known for
  - The Turing machine
  - Breaking German's codes (Enigma machine) during the World War II
  - The Turing test (Artificial intelligence)
- He was prosecuted for homosexuality and died from suicide in 1954...



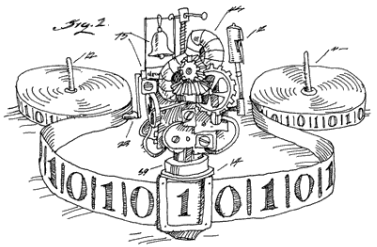
Turing machine



- A Turing machine has an (unlimited) memory, visualized as a tape
- Or a stack of paper
- And takes very simple instructions:
  - Read a symbol
  - Write a symbol
  - Move one step left or right on the tape
  - Change internal state.



Executing instructions



- Drive straight until you see the Basilica

  Internal state: looking for Basilica

  Go straight. Check for Basilica. Repeat.

- Then turn right, and drive till the next light.

  Turn right.

  Change state to “Look for traffic light”

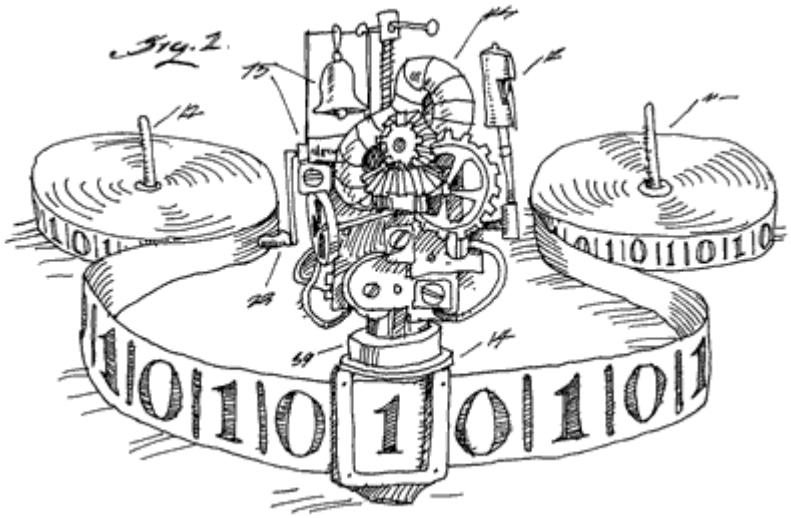
  Go straight. Check for traffic light. Repeat.

- Then turn right, and enter Tim Hortons parking lot.

  Change state to “Look for Tim Hortons”

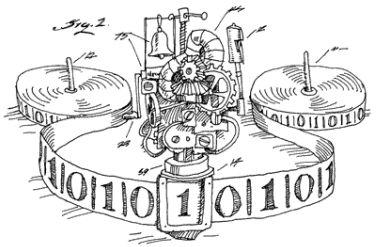
  When see Tim Hortons, turn right into the parking lot

Church-Turing thesis



- Everything we can call “computable” in any sense of this word is computable by a Turing machine.

Church-Turing thesis



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