

Homework Assignment #5
 Due: Mar 21, 2019, by 10:00pm
 (Type it up and upload on D2L)

[60] 1. **Induction examples**

Prove the following by math. induction. The format of the proof should be as follows: 1) state what statement you are proving (it should have a variable on which you do induction as a parameter). 2) state the base case(s) and prove them. 3) state the induction hypothesis 4) state and prove the induction step.

- (a) Show that for all $n \geq 2$, $5^n + 9 \leq 6^n$.
- (b) Show that $\forall n \geq 0$, the number $7^{n+2} + 8^{2n+1}$ is divisible by 57.
- (c) Consider a sequence defined as follows: $s_0 = 1$, $s_1 = 2$ and for $n \geq 2$, $s_n = 1 + \max\{s_{\lfloor n/2 \rfloor}, s_{\lceil n/2 \rceil}\}$ (Recall that $\lfloor x \rfloor$ is the floor of x , that is, largest integer $y \leq x$, and $\lceil x \rceil$ is the ceiling of x , smallest integer $y \geq x$.)
 Prove by **strong induction** that $\forall n > 0$, $s_n \geq s_{n-1}$. Hint: compare $s_{\lfloor (n+1)/2 \rfloor}$, $s_{\lceil (n+1)/2 \rceil}$, $s_{\lfloor n/2 \rfloor}$ and $s_{\lceil n/2 \rceil}$. Which of them are the same and when? What is the relationship among ones that are not the same?

[40] 2. **Structural induction**

- (a) Give a recursive definition of a set S of all binary that have the same number of 0s and 1s.
- (b) Now show, using structural induction on your recursive definition, that every string in your set has even number of symbols.