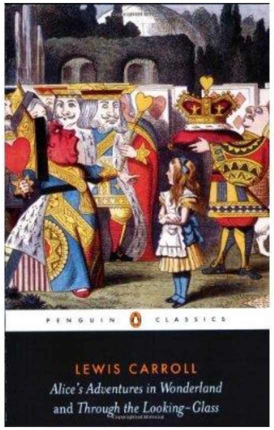
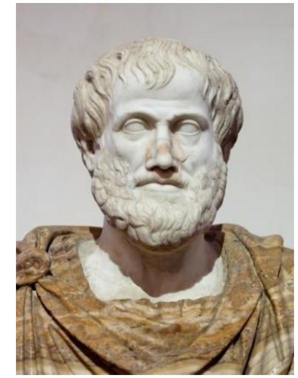




COMP 1002

Intro to Logic for Computer Scientists

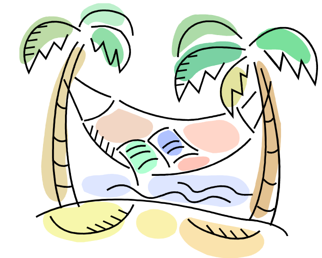
Lecture 4-5

B 5 2 J

Admin stuff

- Labs starting next week:
  - Section 1: Wednesday 9am, CS-1019
  - Section 2: Monday 9am, CS-1019
-


Knights and knaves

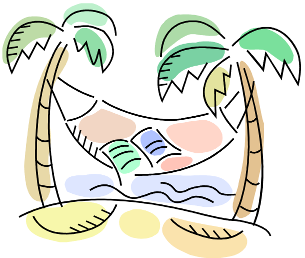


- On a mystical island, there are two kinds of people: knights and knaves. Knights always tell the truth. Knaves always lie.

- Puzzle 2: You see three islanders talking to each other, Arnold, Bob and Charlie. You ask Arnold “Are you a knight?”, but can’t hear what he answered. Bob pitches in: “Arnold said that he is a knave!” and Charlie interjects “Don’t believe Bob, he’s lying”. Out of Bob and Charlie, who is a knight and who is a knave?

Knights and knaves

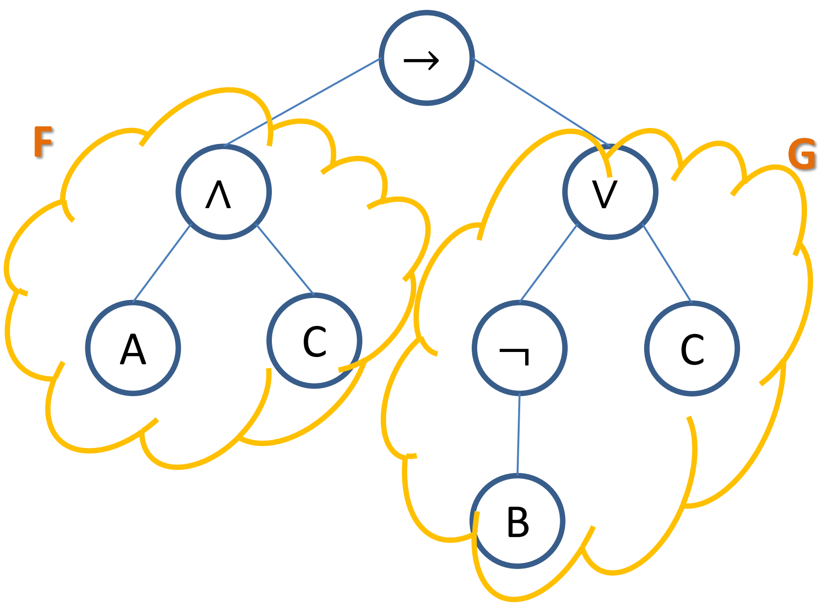


- Puzzle 2: You see three islanders talking to each other, Arnold, Bob and Charlie.
  - You ask Arnold "Are you a knight?", but can't hear what he answered.
  - Bob pitches in: "Arnold said that he is a knave!" and
  - Charlie interjects "Don't believe Bob, he's lying".
  - Out of Bob and Charlie, who is a knight and who is a knave?

- Look at the sentence "I am a knave". Who of the knights/ knaves can say this?
- If A is "Arnold is a knight" and S is "I am a knave", when is $S \leftrightarrow A$ (what Arnold said is true if and only if he is a knight).
- But also "I am a knave" is the same as saying $\neg A$
- $A \leftrightarrow \neg A$ is a contradiction: it is false no matter what A is.
- So Bob must be lying: Bob is a knave. And Charlie is a knight.

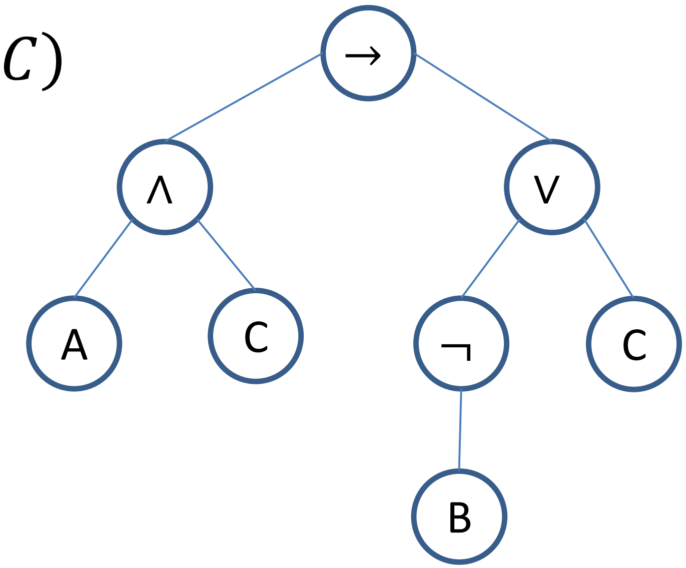
Simplifying formulas

- $A \wedge C \ \rightarrow \neg B \vee C$
  - Order of precedence: $\rightarrow$ is the outermost, that is, the formula is of the form $F \rightarrow G$, where F is $(A \wedge C)$, and G is $(\neg B \vee C)$.



Simplifying formulas

- $A \wedge C \rightarrow \neg B \vee C$
  - By $(F \rightarrow G) \equiv (\neg F \vee G)$
    - equivalent to $\neg(A \wedge C) \vee (\neg B \vee C)$
  - De Morgan's law
    - $\neg(A \wedge C)$ is equivalent to $(\neg A \vee \neg C)$
  - So the whole formula becomes
    - $\neg A \vee \neg C \vee \neg B \vee C$
    - But $\neg C \vee C$ is always true!
    - So the whole formula is a tautology.



More useful equivalences

- For any formulas A, B, C:
  - $TRUE \vee A \equiv TRUE.$ $TRUE \wedge A \equiv A$
  - $FALSE \ \vee A \equiv A.$ $FALSE \wedge A \equiv FALSE$
  - A∨ $A \equiv A \wedge A \equiv A$
- Also, like in arithmetic (with ∨ as +, ∧ as *)
  - $A \vee B \equiv B \vee A \quad and \ (A \vee B) \vee C \equiv A \vee (B \vee C)$
  - Same holds for ∧.
  - Also, $(A \vee B) \wedge C \equiv (A \wedge C) \vee (B \wedge C)$
- And unlike arithmetic
  - $(A \wedge B\ ) \vee C \equiv \ (A \vee C) \wedge (B \vee C)$

Longer example of negation

- Start with the outermost connective and keep applying de Morgan's laws and double negation. Stop when all negations are on variables.
- $\neg\left((A \vee \neg B) \rightarrow (\neg A \wedge C)\right)$
  - $(A \vee \neg B) \wedge \neg(\neg A \wedge C)$ (negating →)
  - $(A \vee \neg B) \wedge (\neg\neg A \vee \neg C)$ (de Morgan)
  - $(A \vee \neg B) \wedge (A \vee \neg C)$ (removing ¬¬)

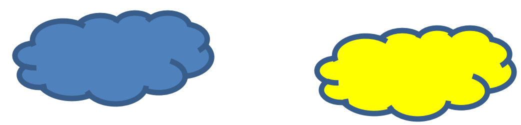
– Can now simplify further, if we want to.

  - $A \vee (\neg B \wedge \neg C)$ (taking A outside the parentheses)

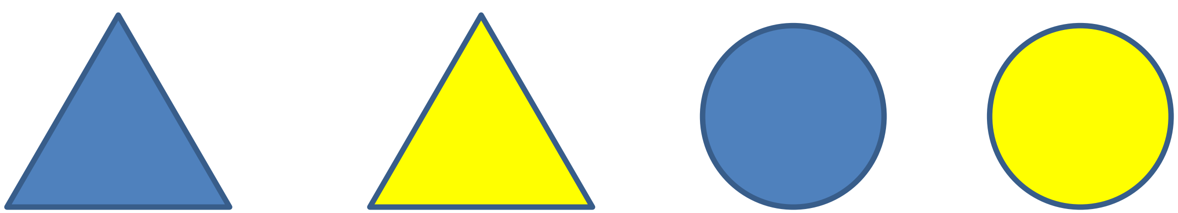
Puzzle 4

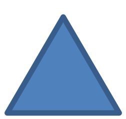
- I like one of the shapes.

- I like one of the colours.



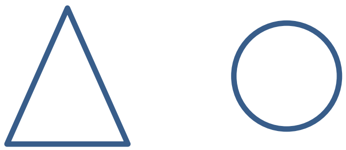
- I like a figure if it has either my favourite shape or my favourite colour.



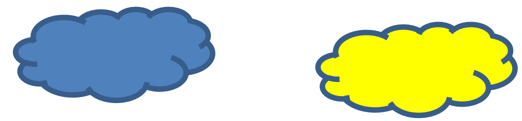
- I like  . What can you say about the rest?

Puzzle 4

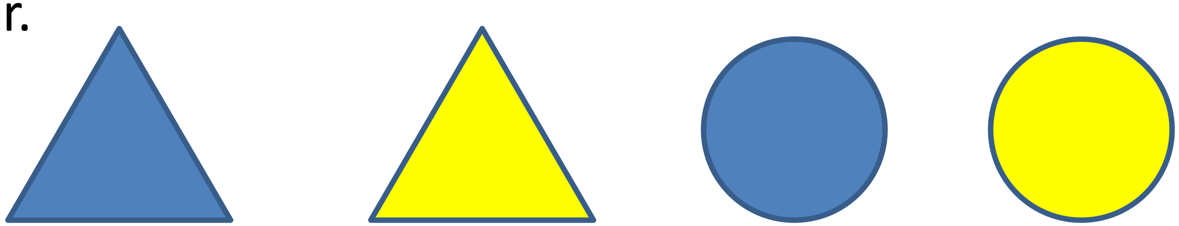
- I like one of the shapes.

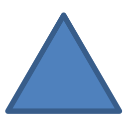


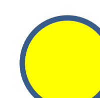
- I like one of the colours.



- I like a figure if it has either my favourite shape or my favourite colour.



- I like  . What can you say about the rest?

- I might like triangles, or blue things, or both.
- There is one figure I don't like, but not enough information to say which one it is. I might still like 

More on if..then..

- You see the following cards. Each has a letter on one side and a number on the other.

- Which cards do you need to turn to check that **if** a card **has a J** on it **then** it **has a 5** on the other side?

Contrapositive

- Let $A \to B$ be an **implication** (if A then B).
  - If a card has a J on one side, it has 5 on the other.
- Its **contrapositive** is $\neg B \to \neg A$.
  - If a card does not have 5 on one side, then it cannot have J on the other.
- Contrapositive is equivalent to the original implication: $A \to B \equiv \neg B \to \neg A$.
  - This is why we need to check cards with numbers other than 5!
  - $\neg B \to \neg A \equiv \neg\neg B \vee \neg A$
    $\equiv B \vee \neg A \equiv \neg A \vee B$

Proof vs. disproof

- To prove that something is (always) true:
  - Make sure it holds in every scenario
    - $\neg B \rightarrow \neg A$ is equivalent to $A \rightarrow B$, because

    $\neg B \rightarrow \neg A \equiv \neg\neg B \vee \neg A \equiv B \vee \neg A \equiv \neg A \vee B \equiv A \rightarrow B$
    - So $(\neg B \rightarrow \neg A) \leftrightarrow (A \rightarrow B)$ is a tautology.

    - I have classes every day that starts with T. I have classes on Tuesday and Thursday (and Monday, but that's irrelevant).

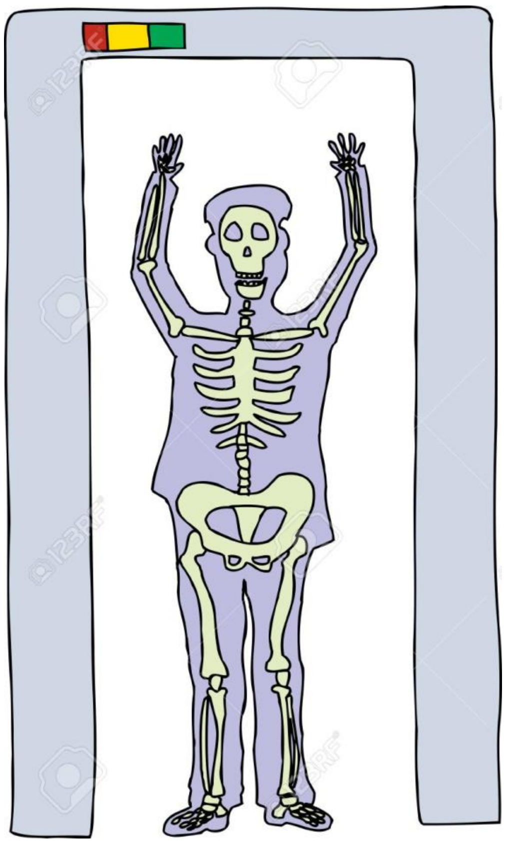
  - Or assume it does not hold, and then get something strange as a consequence:
    - To show A is true, enough to show $\neg A \rightarrow FALSE$.
    - Suppose there are finitely many prime numbers. What divides the number that's a product of all primes +1?

Converse and inverse

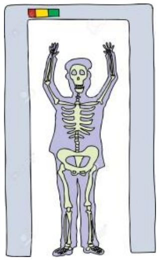
- Let $A \rightarrow B$ be an **implication** (if A then B).
- Its **converse** is $B \rightarrow A$
  - If a card has 5 on one side, then it has J on the other.
- Converse is **not equivalent** to the original implication!
  - For $A = true, B = false, A \rightarrow B$ is false, $\mathrm{B} \rightarrow A$ is true.
- Converse is **not equivalent** to the negation of $A \rightarrow B$
  - $\neg(A \rightarrow B) \equiv \ A \wedge \neg B$.
  - For A=true, B=true, $\mathrm{B} \rightarrow A$ is true, but $\neg(A \rightarrow B)$ is false.
- Converse is equivalent to the **inverse** $\neg A \rightarrow \neg\, B$ of $A \rightarrow B$
  - If a card does not have J on one side, it cannot have 5 on the other.

Contrapositive vs. Converse

- "If a person is carrying a weapon, then airport metal detector will ring".
  - Same as "If the airport metal detector does not ring, then the person is not carrying a weapon".
  - Not the same as: "If the airport metal detector rings, then the person is carrying a weapon."
- "If the person is sick, then the test is positive".
- "If he is a murderer, his fingerprints are on the knife".



Contrapositive vs. Converse



- Let A =“person carries a weapon”, B =“metal detector rings.
- In statistics, talk about **sensitivity** vs. **specificity**:
  - **Sensitivity**: percentage of correct positives
    - probability that $A \rightarrow B$
    - that if a person has a weapon, then detector rings:
    - that if the person is sick, then the test is positive
    - 100% sensitive test: catches all weapons/sick (maybe some innocent/healthy, too)
  - **Specificity**: percentage of correct negatives
    - Probability that $B \rightarrow A$
    - that if the detector rings, then the person has a weapon
    - that if the person is not sick, then the test is negative
    - 100% specific test: catches only weapons/sick (no innocent/healthy, but maybe not all weapons/sick)

If and only if

- $A \leftrightarrow B$ ("A if and only if B") is true exactly when both the implication $A \to B$ and its converse $\mathrm{B} \to A$ (equivalently, inverse $\neg A \to \neg B$) are true
  – Come to the lab on Monday **if and only if** you are in section 2.
  – If you are in section 2, then come to lab on Monday
  – If you came to lab on Monday, you are in section 2
    - Equivalently, if you are not in section 2, do not come to Monday lab: come to Wednesday lab instead.

Proof vs. disproof

- To disprove that something is always true, enough to give just one scenario where it is false (find a falsifying assignment).

  – To disprove that $A \to B \equiv B \to A$
    - Take $A = true, B = false,$
    - Then $A \to B$ is false, but $B \to A$ is true.

  – To disprove that $B \to A \equiv \neg (A \to B)$
    - Take A=true, B=true
    - Then $B \to A$ is true, but $\neg(A \to B)$ is false.

  – I have classes every day! – No, you don’t have classes on Saturday

  – Women don’t do Computer Science! – Me?

Treasure hunt

- In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humor and love of logical puzzles. In the note he wrote that he had hidden a treasure somewhere on the property. He listed 5 true statements and challenged the reader to use them to figure out the location of the treasure

Treasure hunt

1. If this house is next to a lake, then a treasure is not in the kitchen
2. If the tree in the font yard is an elm, then the treasure is in the kitchen
3. This house is next to a lake
4. The tree in the front yard is an elm, or the treasure is buried under the flagpole
5. If the tree in the back yard is an oak, then the treasure is in the garage.