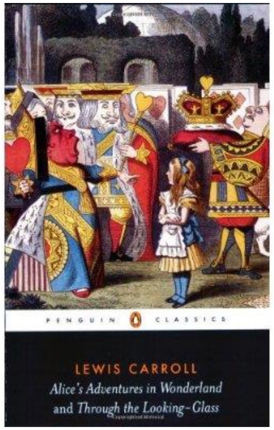
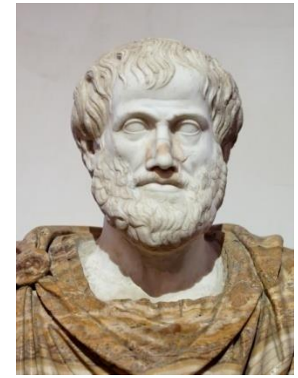




COMP 1002

Intro to Logic for Computer Scientists

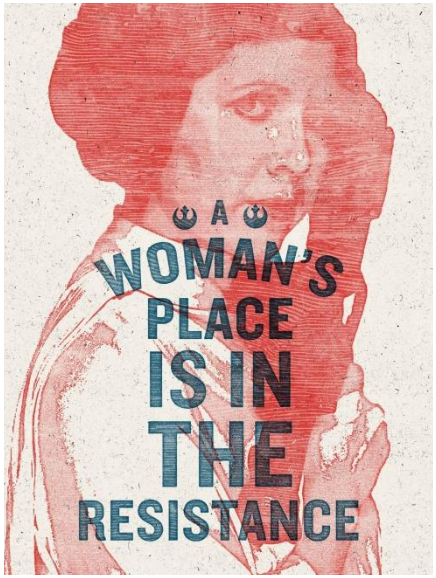
Lecture 10

Puzzle 9

- Susan is 28 years old, single, outspoken, and very bright. She majored in philosophy. As a student she was deeply concerned with issues of discrimination and social justice and also participated in anti-nuke demonstrations.

  Please rank the following possibilities by how likely they are. List them from least likely to most likely. Susan is:

1. a kindergarden teacher
2. works in a bookstore and takes yoga classes
3. an active feminist
4. a psychiatric social worker
5. a member of an outdoors club
6. a bank teller
7. an insurance salesperson
8. a bank teller and an active feminist



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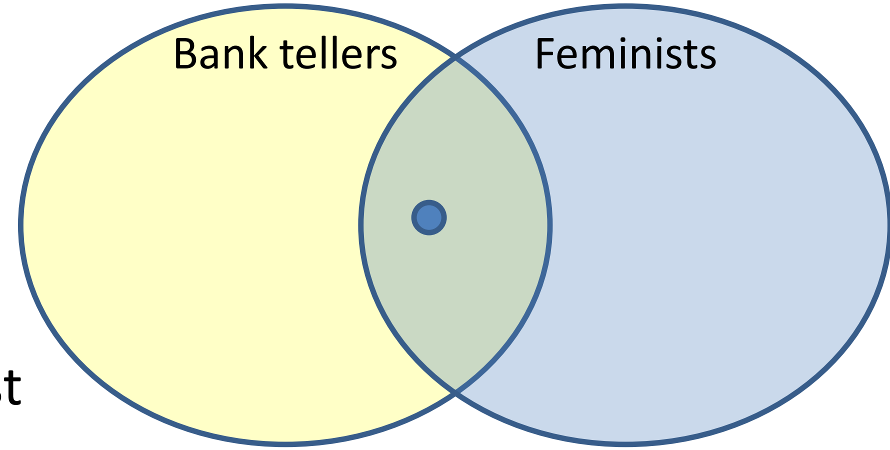


Scenarios and sets

- Want to reason about more general scenarios
- Rather than just true/false, vary over objects:
  - even numbers, integers, primes
  - people that are and are not bank tellers,
  - pairs of animals in the same ecosystem...
- Want multiple properties of these objects:
  - an even number that is divisible by 4 and > 10,
  - a person that is also a bank teller...

Sets

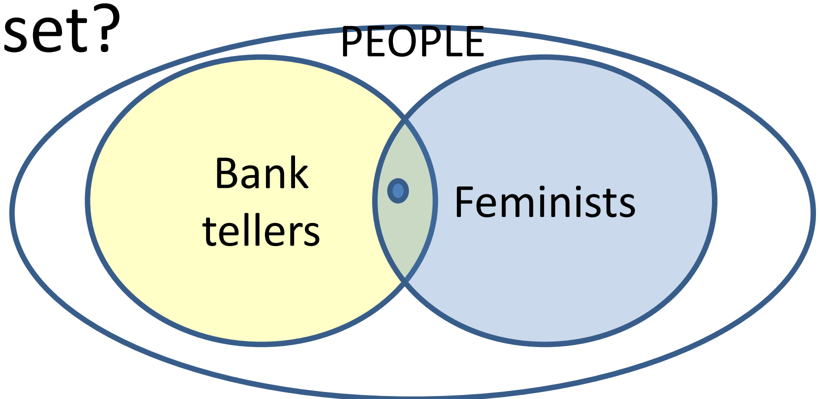
- A **set** is a collection of objects.
  - S_1={1, 2, 3}, S_2 = {Cathy, Alaa, Keiko, Daniela}
  - S_3 = [-1, 2] (real numbers from -1 to 2, inclusive)
  - PEOPLE = {x | x is a person living on Earth now}
    - {x | such that x   ... } is called **set builder notation**
  - S_4 = { (x,y) | x and y are people, and x is a parent of y}
  - BANKTELLERS = { x | x is a person who is a bank teller}
- The order of elements does not matter.
- There are no duplicates.

Special sets

- Notation for some special sets (much of which you are likely to have seen):
  - Empty set $\emptyset$
  - Natural numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$ (sometimes with 0)
  - Integers $\mathbb{Z} = \{\ldots - 2, -1, 0, 1, 2, \ldots\}$
  - Rational numbers $\mathbb{Q} = \left\{\frac{m}{n} \,\middle|\, m, n \; in \; \mathbb{Z}, n \neq 0\right\}$
  - Real numbers $\mathbb{R}$
  - complex numbers $\mathbb{C}$

Set elements

- $a \in S$ means that an element a is in a set S, and $a \notin S$ that a is not in S. That is, $a \in S \equiv \neg\,(a \notin S)$
  - Susan ∈ PEOPLE. Susan ∉ BANKTELLERS
  - $0.23 \in [-1,2]$. $3.54 \notin [-1,2]$
- Also, write $x \in S$ for a variable x.
  - BANKTELLERS = { $x \in PEOPLE$ | x is a bank teller}
- How do we generalize sentences like “x is a bank teller”, where x is an element of some set?



Predicates

- A **predicate** $P(x_1, \ldots, x_n)$ is a "proposition with variables", where values of the variables $x_1, \ldots, x_n$ come from some sets $S_1, \ldots, S_n$, called their **domains** or **universes.**
  - $P(x)$ is true for some values of $x \in S$, and false for the rest.
    - Even(x) for $x \in \mathbb{Z}$, Feminist(y) for y$\in PEOPLE$...
    - Here, domain of x is $\mathbb{Z}$, and domain of y is $PEOPLE$
    - Even(y) is not defined for y$\in PEOPLE$, only for elements of $\mathbb{Z}$.
  - A predicate can have several variables:
    - $x > y$, for $x, y \in \mathbb{R}$
    - $\text{Divides}(x, y)$, which is true for $x, y \in \mathbb{Z}$ such that x divides y.
- When all variables in a predicate are replaced with specific elements (**instantiated**), the predicate becomes a proposition.
  - "Even(3)" is false. "Feminist(Susan)" is true.

Predicates

- We can make formulas out of predicates the same way as we did for propositions, but now our formulas have **free variables**:
  - $Even(x) \vee Divides(3, x) \rightarrow \neg Prime(x)$
  - $Feminist(x) \wedge Bankteller(x)$
  - Now scenarios can correspond to values of x.
    - The first formula is false for x=2, because $Even(2) = true$, but $\neg Prime(2) = false$.
- This is called **predicate logic (**or **first-order logic),** as opposed to propositional logic we did so far.

Quantifiers: universal (∀)

- Theorems often look like this: "For all x , the following is true", and then a formula with x as a free variable.
  - For all $x \in \mathbb{Z},\ Divides(6, x) \to Divides(3, x)$
  - For all $n \in \mathbb{N},\ \ n > 4,\ 2^n\ > n^2$

- We write this in predicate logic using a **universal quantifier** (written as ∀):
  - $\forall x \in \mathbb{Z},\ Divides(6, x) \to Divides(3, x)$
  - $\forall n \in \mathbb{N},\ \ n > 4 \to\ 2^n\ > n^2$

Quantifiers: universal (∀)

- In general, for every formula F of predicate logic with a free variable x, we can write $\forall x \in S,\ F(x)$
  - The formula "$\forall x \in S,\ F(x)$" is true if and only if $F(a)$ is true for every $a \in S$.
  - That is, if $a_1, a_2, \ldots, a_n, \ldots$ is a list of all elements of S, then $\forall x \in S,\ F(x)$ is true if and only if $F(a_1) \wedge F(a_2) \wedge \cdots \wedge F(a_n) \wedge \cdots$ is true.
  - If there are no more free variables or quantifiers in F, then $\forall x \in S,\ F(x)$ is true if and only if $F(a_1) \wedge F(a_2) \wedge \cdots \wedge F(a_n) \wedge \cdots$ is a tautology.

Negating the universal

- What is the negation of “All”? When would a statement “$\forall x \in S,\ F(x)$“ be false?
  - All girls hate math.
  - No!
    - All girls love math?
    - Some girls do not hate math!
  - Everybody in O’Brian family is tall
    - No, Jenny is O’Brian and she is quite short.
  - It is foggy all the time, every day in St. John’s
    - No, sometimes it is not foggy (like today).

Quantifiers: existential (∃)

- To prove that something is not always true, we give a counterexample. In predicate logic, use **existential quantifier** ∃.

- $\exists x \in S, F(x)$ is true if and only if there exist some a $\in S$ such that $F(a)$ is true (and we don't care for the rest). That is, when $F(a_1) \vee F(a_2) \vee \cdots \vee F(a_n) \vee$ … is true.
  - $\exists t \in TIMESLOTS, Scheduled(COMP1002, t) \wedge Scheduled(COMP1000, t)$
  - $\exists x \in \mathbb{N},\ Even(x) \wedge Prime(x)$.

- $\neg \forall x \in S, F(x) \equiv \exists x \in S,\ \neg F(x)$

- Once a variable is quantified, it is no longer free.
  - x is free in $Even(x) \wedge Prime(x)$,
  - But $\exists x \in \mathbb{N},\ Even(x) \wedge Prime(x)$ has no free variables.

Quantifiers in English

- Universal quantifier: usually “every”, “all”, “each”, “any”.
  - Every day it is foggy. Each number is divisible by 1.

- Existential quantifier: “some”, “a”, “exists”
  - Some students got 100% on both labs.
  - There exists a prime number greater than 100.

- The word “any” can mean either!

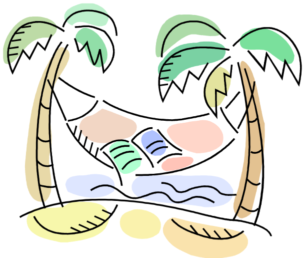
Quantifiers in English: “any”

- “Any” can mean “every”:

  - Any student in our class knows logic 
  - Every student in our class knows logic. 

- But “any” can also mean “some”!

  - I will be happy if I do well on every quiz. 
  - I will be happy if I do well on any quiz. 



Puzzle 10

- The first formulation of the famous liar's paradox, attributed to a Cretan philosopher Epimenides, stated

"All Cretans are liars".

Is this really a paradox?