Last Name ____________________ First Name & Initial ____________________

Student No. [redacted]

NO AIDS ALLOWED. Answer ALL questions on test paper. Use backs of sheets for scratch work. For any question or part of a question for which you write "I don't know the answer" you will receive 20% of the mark. For any question or part of a question which you leave blank you will receive 10% of the mark.

Total Marks: 50+5 bonus.

[10] 1. **Yes/no questions**
For each of the following, say whether it is true or false. Give a short explanation (1-2 sentences).

(a) The negation of $p \to q$ is $\neg p \to \neg q$.

(b) Any set of logic connectives with at least two elements is complete.

(c) For any p, the formula $p \vee p$ is equivalent to $p \wedge p$.

(d) A truth table of a formula on 3 variables has 6 lines.

(e) Any Boolean function on n variables can be fully described by a CNF formula constructed from its truth table.

[15] 2. **Short answers**
In the following questions, fill in the blank:

(a) By DeMorgan's law, $\neg(p \wedge q) \iff$ ____________________________

(b) By definitions of equivalence and implication,

$$p \leftrightarrow q \iff \underline{\hspace{8cm}}$$

in the language with only $\wedge, \vee$ and $\neg$.

(c) By modus ponens,

If today there is a midterm then today there is no lecture
Today there is a midterm
———————————————————————————————
$\therefore$

(d) Suppose p says "today there is a midterm", q says "today there is a lecture" and r says "I have to be in class". Then $(p \vee q) \rightarrow r$ translates as

____

____

(e) And its negation (of the $(p \vee q) \rightarrow r$ from (d)) translates as:

____

____

[10] 3. **Truth tables and CNFs**

[5] (a) Give a truth table for the following propositional statement: $p \vee \neg(p \to q)$ Write the names of the columns in the first row of the table.

[10] (b) Write a CNF and a DNF for this formula. For the DNF, give both the DNF that is constructed from the truth table and a smaller DNF that you can get by looking at the formula.

CNF: ____________________

DNF: ____________________

Smaller DNF: ____________________

[10] 4. **Circuits**

Draw a circuit computing the following Boolean function on two variables: $Flip(x, y)$ returning y if $x = 1$ and $\neg y$ otherwise.

[15] 5. **Resolution** In this problem, you will start with some facts and show how to derive a conclusion from them and prove that your conclusion is correct using resolution.

(a) Which conclusion about n can you derive from these statements? It does not have to be derivable in one step.

If n is divisible by 6, then n is divisible by 2
If n is divisible by 6, then n is divisible by 3
If n is divisible by 2, then $n+1$ is odd.
$n+1$ is not odd.

$\therefore$

(b) Now, let p be "n is divisible by 6", q be "n is divisible by 2, r be "n is divisible by 3" and s be "$n+1$ is odd". Write a formula using these variables that says that the $\wedge$ of premises implies the conclusion (from part a)): if the argument is valid, this has to be a tautology. Take the negation of this formula and convert this negation into CNF form.

Formula: ________________________________

Its negation (just negate the implication): ________________________________

Negation in CNF: ________________________________

(c) Now show using resolution that the "negation in CNF" formula is a contradiction.

[5] 6. **(Bonus) Knights and knaves again**
Recall that the knights always say truth and knaves always lie. Suppose that you are visiting the Island of Knights and Knaves and want to prove to the locals that you are not from the island (that is, you are neither a knight nor a knave). Can you do it in one sentence (3-4 words)? Can you prove in one sentence that you *are* from the island?