1. **Complete set of connectives**

   Show that \{→, ¬\} is a complete set of connectives. That is, use \{→, ¬\} to express some set of connectives from the class that you know is complete.

2. **Predicate logic**

   Let statement 1 be "If any two numbers divide each other, then they must be equal", and statement 2 be "Every person knows at least two other people".

   (a) Write both statements as predicate logic formulas using quantifiers. You can use predicates "x=y", "Knows(x,y)" , but to state that one number divides another, say that there is a third number that, multiplied by the first, is equal to the second. (Hint: "two other people" can be done with variables; make sure to say that they are not the same as the original person).

   (b) For each of the two, give an example of a domain which makes it true (same domain for all quantified variables, a subset of the original domain).

   (c) For each of the two, give a counterexample (a domain and interpretations of predicates)

   (d) Write negations of both statement as formulas (pushing negations inside to predicates)

   (e) Write English sentences corresponding to formulas from the previous subquestion.

3. **Proofs**

   (a) Show that square of any odd natural number is congruent to 1 mod 8.

   (b) Recall that the absolute value of a real number \( |z| \), is equal to \( z \) if \( z \geq 0 \), and equal to \( -z \) if \( z < 0 \). Using this definition, and the proof by cases technique, show that for all pairs of real numbers \( x, y \), the \( \min(x, y) = (x + y - |x - y|)/2 \).