1. CS course structure

Consider the following propositions:

- $p$: Algorithm correctness proofs need logic
- $q$: Logic is studied in COMP 1002
- $r$: COMP 2002 covers algorithm correctness
- $s$: COMP 1002 is a prerequisite to COMP 2002

Using these propositions as definitions of $p$, $q$, $r$, and $s$, answer the following questions.

(a) Using propositions above, write an English sentence corresponding to $(r \rightarrow \neg p)$

(b) Write, in English, a contrapositive of the statement in (a).

(c) Write, in English, a negation of the statement in (a).

(d) Write the following statement in the logic notation using $p, q, r, s$ above: If COMP 2002 does not cover algorithm correctness and algorithm correctness proofs need logic, then both COMP 1002 is not a prerequisite for COMP 2002 and logic is studied in COMP 1002.

(e) Write in logic notation the negation of your formula from (d). Then move all negations to variables using DeMorgan’s law, definition of $\rightarrow$ and double negation.

(f) Write an English sentence corresponding to the resulting (with negations on variables) formula from your previous subquestion.

(g) Suppose that the statement in (d) is false. Suppose also that COMP 1002 is a prerequisite for COMP 2002. Is this enough information to figure out whether logic is studied in COMP 1002? Explain your answer.

(h) Give an English sentence with these propositions that is a tautology.

2. More knights and knaves

(a) Suppose you meet two islanders and ask who they are. One of them says “if I am a knight then so is my friend”. Can you tell who is who? (you can use a truth table).

(b) You come back to your ship just to find that it disappeared. Two islanders, call them Charlile and Donald, were seen near it before. You find another islander, call him Edward, and ask who got the ship. Edward makes the following two statements: 1) Charlie is guilty 2) Charlie and Donald cannot both be guilty.

Can you tell who got your ship? Is Edward a knave or a knight? (For the solution, start by writing Edward’s statements in logic notation, both for the case when Edward is telling the truth (then both his statements are true), and for the case when he is lying (both are false). Then use logic identities.)

3. Treasure hunt

In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humor and love of logical puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements and challenged the reader to use them to figure out the location of the treasure.
(a) If there is an old shipwreck near the beach, then the treasure is buried under a coconut palm tree.
(b) There is a coconut palm tree growing either at the far end of the island or near the cave.
(c) Either there is a shipwreck near the beach, or the treasure is hidden in a cave.
(d) If there is a coconut palm tree at the far end of the island, then there is no shipwreck on the beach.
(e) There is no coconut palm tree near the cave.

In this problem you will solve this puzzle using natural deduction.

(a) Identify the propositional statements used in this puzzle and state how the five facts translate into propositional formulas using these statements. Additionally, include a formula stating that there is only one treasure.

(b) Now find the solution to the problem by applying inference rules such as modus ponens to these formulas. You will need several applications of rules before reaching the conclusion.

4. Resolution

(a) Give a resolution refutation of the following formula: \((x \lor y) \land (y \lor \neg z \lor \neg x) \land \neg x \land \neg (y \lor x) \land \neg (z \lor y)\)
(b) Convert the formula into a CNF and give a resolution refutation for it:
\[x_1 \land (x_1 \rightarrow x_2) \land (x_2 \rightarrow x_3) \land (x_3 \rightarrow x_4) \land (x_4 \rightarrow \neg x_1)\]

5. From argument to proof

Given the following information about a computer program, find a mistake in the program.

(a) There is an undeclared variable or there is a syntax error in the first five lines.
(b) If there is a syntax error in the first five lines, then there is a missing semicolon or a variable name is misspelled.
(c) There is no missing semicolon unless there is a misspelled variable name.
(d) There is no misspelled variable name.

(a) Let us name the propositions occurring in this problem as follows:
\[p: \text{There is an undeclared variable.}\]
\[q: \text{There is a syntax error in the first five lines.}\]
\[r: \text{There is a missing semicolon}\]
\[s: \text{A variable name is misspelled.}\]

Now, using these propositions, rewrite each of the four statements above. For example, ”there is no missing semicolon unless there is a misspelled variable name” will become \(r \rightarrow s\).

(b) Using modus ponens on the formulas from the previous subquestion, find a mistake in the program.

(c) Now you want to check that your argument is valid; you will use a resolution proof for that. Start by writing it in the argument form with four statements above as premises, and the formula describing the mistake you found as a conclusion.

(d) Now, write one formula that states that the argument from the previous subquestion is not valid. Your goal will be to show that this formula is a contradiction.

(e) Convert the formula you just wrote into CNF form.

(f) Now, give a resolution refutation of the CNF from the previous subquestion.