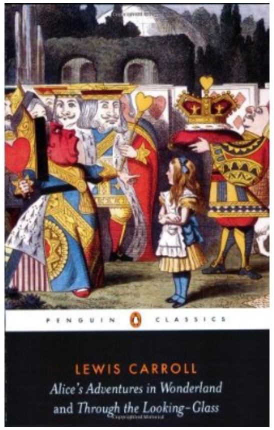
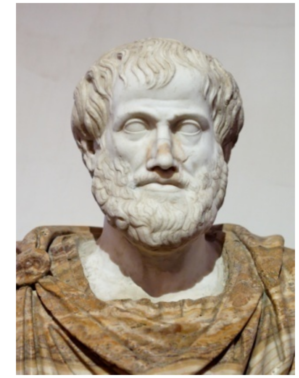
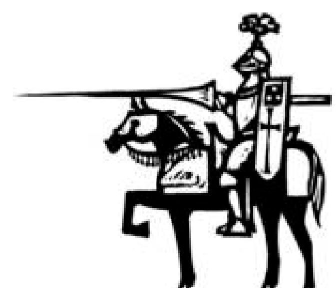




COMP 1002

Intro to Logic for Computer Scientists

Lecture 9



Puzzle 8

- Suppose that nobody in our class carries more than 10 pens.
- There are 70 students in our class.

- Prove that there are at least 2 students in our class who carry the same number of pens.
  - In fact, there are at least 7 who do.

Pigeonhole Principle

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- The Pigeonhole Principle:
  - If there are n pigeons
  - And n-1 pigeonholes
  - Then if every pigeon is in a pigeonhole
  - At least two pigeons sit in the same hole

Pigeonhole Principle

- Suppose that nobody in our class carries more than 10 pens. There are 70 students in our class. Prove that there are at least 2 students in our class who carry the same number of pens.
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- The Pigeonhole Principle:
  - If there are n pigeons and n-1 pigeonholes
  - Then if every pigeon is in a pigeonhole
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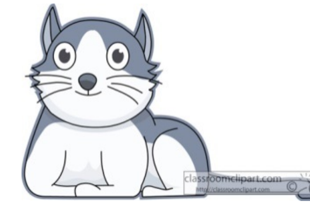
- Applying to our problem:
  - n-1 = 11 possible numbers of pens (from 0 to 10)
  - Even with n=12 people, there would be 2 who have the same number.
  - If there were less than 7, say 6 for each scenario, total would be 66.
  - Note that it does not tell us which number or who these people are!

Resolution and Pigeons

- It is not that hard to write the Pigeonhole Principle as a tautology
- But we can prove that resolution has trouble with this kind of reasoning
  - the smallest resolution proof of this tautology is exponential size!
- By contrast, natural deduction (and you!) can figure it out fairly quickly
  - though it is not straightforward.
- The problem is that resolution cannot count.
  - But ability to count makes things harder...


Meow-stery

- One evening there was a cat fight in a family consisting of a mother cat, a father cat, and their son and daughter kittens.
- One of these four cats attacked and bit another!
- One of the cats watched the fight.
- The other one hissed at the fighters.

- These are the things we know for sure:
  - 1. The watcher and the hisser were not of the same sex.
  - 2. The oldest cat and the watcher were not of the same sex.
  - 3. The youngest cat and the victim were not of the same sex.
  - 4. The hissing cat was older than the victim.
  - 5. The father was the oldest of the four.
  - 6. The attacker was not the youngest of the four.

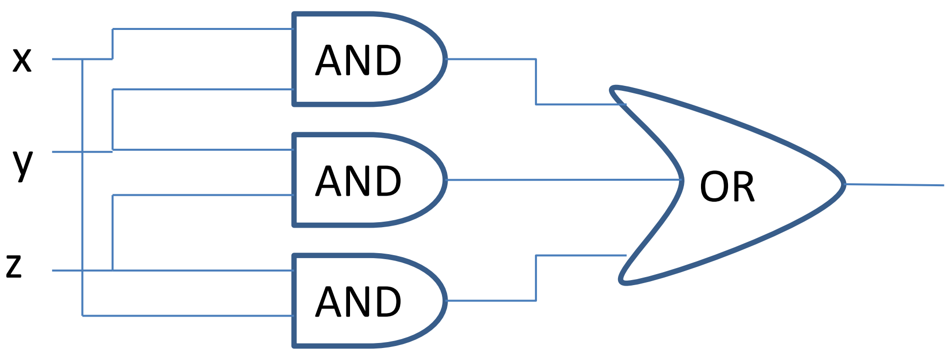
- Which nasty cat was the attacker?

Boolean functions and circuits

- What is the relation between propositional logic we studied and logic circuits?
  - View a formula as computing a function (called a Boolean function),
    - inputs are values of variables,
    - output is either *true (1)* or *false (0)*.
  - For example, $Majority(x, y, z) = true$ when at least two out of x, y, z are true, and false otherwise.
  - Such a function is fully described by a truth table of its formula (or its circuit: circuits have truth tables too).

Boolean functions and circuits

- What is the relation between propositional logic and logic circuits?
  - So both formulas and circuits "compute" Boolean functions – that is, truth tables.
  - In a circuit, can "reuse" a piece in several places, so a circuit can be smaller than a formula.
    - Still, most circuits are big!
  - $Majority(x, y, z)$ is $(x \wedge y) \vee (x \wedge z) \vee (y \wedge z)$



Canonical CNF

- Every truth table (Boolean function) can be written as a CNF:
  - Take every falsifying assignment
    - Say, A = False, B = True, C=False.
  - Write it as a formula which is true only on this assignment:
    - $\neg A \wedge B \wedge \neg C$
  - To say that this assignment does not happen, write its negation:
    - $\neg(\neg A \wedge B \wedge \neg C) \equiv (A \vee \neg B \vee C)$
  - Take an AND of these for all falsifying assignments
    - It is equivalent to the original formula.
    - And it is a CNF! Called the **canonical CNF** of this formula.

Canonical DNF

- So for every formula, there is a unique canonical CNF (and a truth table, and a Boolean function).
- And for every possible truth table (a Boolean function), there is a formula (the canonical CNF).

- A negation of a CNF is an OR of ANDs of literals. It is called a **DNF (disjunctive normal form).**
  - To make a canonical DNF from a truth table:
  - take all satisfying assignments.
  - Write each as an AND of literals, as before.
  - Then take an OR of these ANDs.

Complete set of connectives

- CNFs only have ¬,∨,∧, yet any formula can be converted into a CNF
  - Any truth table can be coded as a CNF
- Call a set of connectives which can be used to express any formula **a complete set** of connectives.
  - In fact, ¬,∨ is already complete. So is ¬,∧ .
    - By DeMorgan, $(A \lor B) \equiv \neg(\neg A \land \neg B)$ No need for ∨!
  - But ∧,∨ is not: cannot do ¬ with just ∧,∨.
    - Because when both inputs have the same value, both ∧,∨ leave them unchanged.

Complete set of connectives

- How many connectives is enough?
  - Just one: NAND (NotAND), also called the Sheffer stroke, written as |
  - $\neg A \equiv A \mid A$
  - $A \vee B \equiv \neg(\neg A \wedge \neg B)$
    $\equiv (\neg A \mid \neg B)$
    $\equiv ((A \mid A) \mid (B \mid B))$
  - In practice, most often stick to $\wedge, \vee, \neg$

A	B	A | B
True	*True*	*False*
True	*False*	*True*
False	*True*	*True*
False	*False*	*True*

Puzzle 9

- Susan is 28 years old, single, outspoken, and very bright. She majored in philosophy. As a student she was deeply concerned with issues of discrimination and social justice and also participated in anti-nuke demonstrations.

  Please rank the following possibilities by how likely they are. List them from least likely to most likely. Susan is:

1. a kindergarden teacher
2. works in a bookstore and takes yoga classes
3. an active feminist
4. a psychiatric social worker
5. a member of an outdoors club
6. a bank teller
7. an insurance salesperson
8. a bank teller and an active feminist