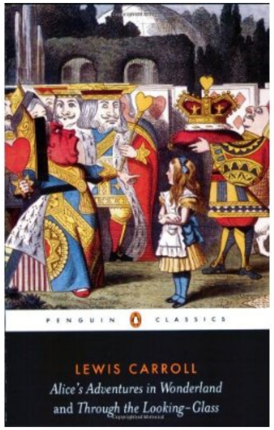
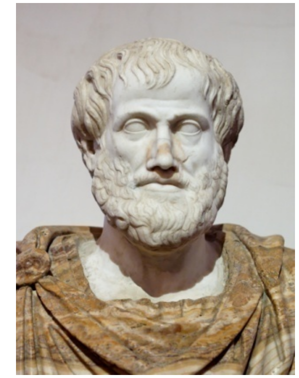


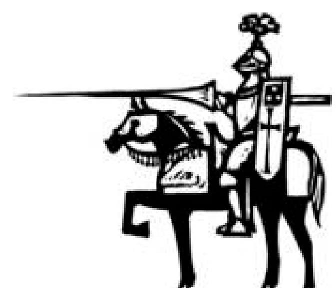


COMP 1002

Intro to Logic for Computer Scientists

Lecture 5

B 5 2 J



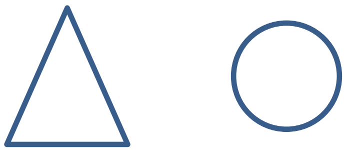
Admin stuff

- First lab Jan 18th (this Wednesday).
- Lab is posted: see the webpage.

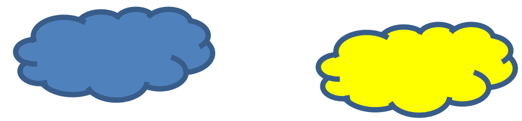
  - If you do have a time conflict at 11am:
    - Come to EN-1049
  - If you do not have a time conflict at 11am:
    - Come to CS-1019
  - Lab quizzes count as 25% part of your mark!

Puzzle 4

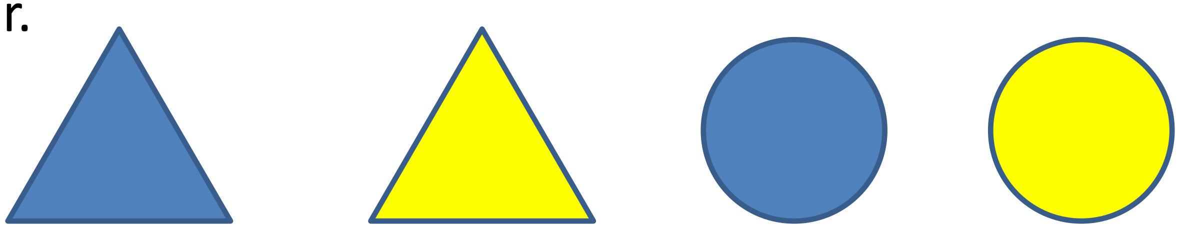
- I like one of the shapes.

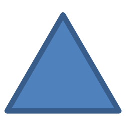


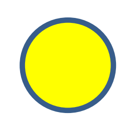
- I like one of the colours.



- I like a figure if it has either my favourite shape or my favourite colour.

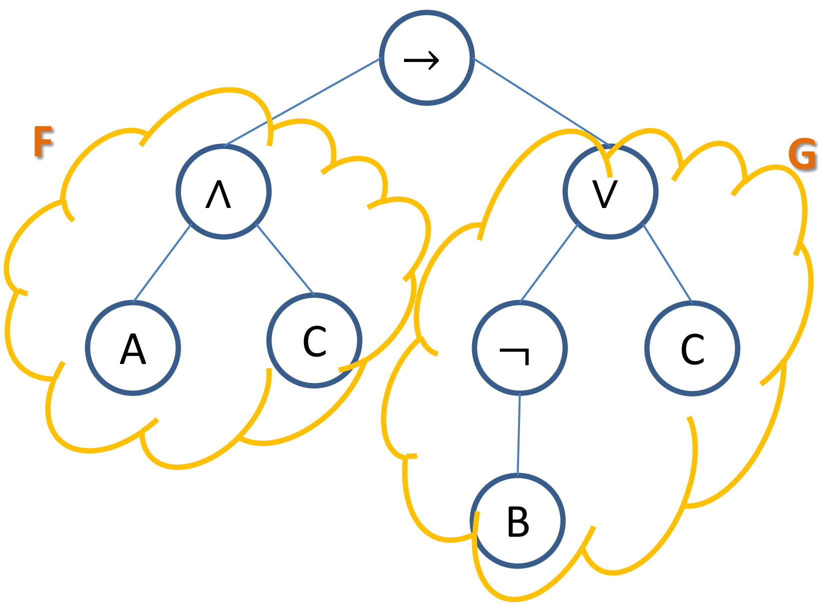


- I like . What can you say about the rest?

- I might like triangles, or blue things, or both.
- There is one figure I don't like, but not enough information to say which one it is. I might still like 

Simplifying formulas

- $A \wedge C \ \rightarrow (\neg B \vee C)$
  - Order of precedence: $\rightarrow$ is the outermost, that is, the formula is of the form $F \rightarrow G$, where F is $(A \wedge C)$, and G is $(\neg B \vee C)$.



Simplifying formulas

- $A \wedge C \ \rightarrow (\neg B \vee C)$
  - By$(\mathrm{F} \rightarrow G) \equiv \ (\neg F \vee G)$
    - equivalent to $\neg(A \wedge C) \vee (\neg B \vee C)$
  - De Morgan's law
    - $\neg(A \wedge C)$ is equivalent to $(\neg A \ \vee \neg C)$
  - So the whole formula becomes
    - $\neg A \vee \neg C \vee \neg B \vee C$
    - But $\neg C \vee C$ is always true!
    - So the whole formula is a tautology.

More useful equivalences

- For any formulas A, B, C:
  - $TRUE \vee A \equiv TRUE.$ $TRUE \wedge A \equiv A$
  - $FALSE \ \vee A \equiv A.$ $FALSE \wedge A \equiv FALSE$
  - A$\vee$ $A \equiv A \wedge A \equiv A$
- Also, like in arithmetic (with ∨ as +, ∧ as *)
  - $A \vee B \equiv B \vee A \quad and \ (A \vee B) \vee C \equiv A \vee (B \vee C)$
  - Same holds for ∧.
  - Also, $(A \vee B) \wedge C \equiv (A \wedge C) \vee (B \wedge C)$
- And unlike arithmetic
  - $(A \wedge B\ ) \vee C \equiv \ (A \vee C) \wedge (B \vee C)$

Longer example of negation

- Start with the outermost connective and keep applying de Morgan's laws and double negation. Stop when all negations are on variables.
- $\neg\,(\,(A \vee \neg B) \rightarrow \;\; (\neg A \wedge C))$
  - $(A \vee \neg B) \wedge \neg(\neg A \wedge C)$ (negating $\rightarrow$)
  - $(A \vee \neg B) \wedge (\neg\neg A \vee \neg C)$ (de Morgan)
  - $(A \vee \neg B) \wedge (A \vee \neg C)$ (removing $\neg\neg$)

– Can now simplify further, if we want to.
  - $A \vee (\neg B \wedge \neg C)$ (taking A outside the parentheses)


Knights and knaves

- On a mystical island, there are two kinds of people: knights and knaves. Knights always tell the truth. Knaves always lie.

- Puzzle 5: You hear a person from the island of knights and knaves say “if I am a knight, then it will rain tomorrow”. What can you conclude from this?