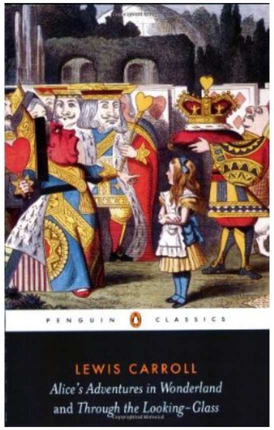
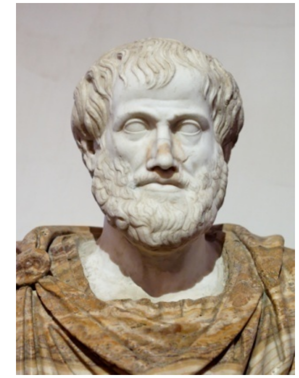


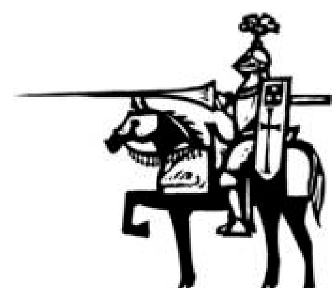


COMP 1002

Intro to Logic for Computer Scientists

Lecture 4

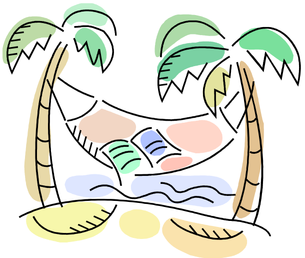
B 5 2 J



Admin stuff

- Labs: Wed 9am.
- First lab Jan 18th (next Wednesday).
-
  - If you do have a time conflict at 11am:
    - Come to EN-1049
  - If you do not have a time conflict at 11am:
    - Come to CS-1019

Knights and knaves



- On a mystical island, there are two kinds of people: knights and knaves. Knights always tell the truth. Knaves always lie.

- Puzzle 2: You see three islanders talking to each other, Arnold, Bob and Charlie. You ask Arnold "Are you a knight?", but can't hear what he answered. Bob pitches in: "Arnold said that he is a knave!" and Charlie interjects "Don't believe Bob, he's lying". Out of Bob and Charlie, who is a knight and who is a knave?


Knights and knaves

- Puzzle 2: You see three islanders talking to each other, Arnold, Bob and Charlie.
  - You ask Arnold "Are you a knight?", but can't hear what he answered.
  - Bob pitches in: "Arnold said that he is a knave!" and
  - Charlie interjects "Don't believe Bob, he's lying".
  - Out of Bob and Charlie, who is a knight and who is a knave?

- Look at the sentence "I am a knave". Who of the knights/ knaves can say this?
- If A is "Arnold is a knight" and S is "I am a knave", when is $S \leftrightarrow A$ (what Arnold said is true if and only if he is a knight).
- But also "I am a knave" is the same as saying $\neg A$
- $A \leftrightarrow \neg A$ is a contradiction: it is false no matter what A is.
- So Bob must be lying: Bob is a knave. And Charlie is a knight.

Logical equivalence

- Two formulas F and G are **logically equivalent** ($F \Leftrightarrow G$ or $F \equiv G$) if they have the same value for every row in the truth table on their variables.
  - $A \wedge \neg A \equiv False$ (same as saying it is a contradiction)
  - $(\neg A \vee B) \equiv (A \rightarrow B)$
  - $(A \leftrightarrow B) \equiv (A \rightarrow B) \wedge (B \rightarrow A)$
    - $\leftrightarrow$ is sometimes called the “biconditional”
    - $\leftrightarrow$ often pronounced as “if and only if”, or “iff”

- Useful fact: proving that $F \equiv G$ can be done by proving that $F \leftrightarrow G$ is a tautology

Double negation

- Negation cancels negation
  - $\neg\neg A \equiv A$
  - “I do not disagree with you” = “I agree with you”

- For a human brain, harder to parse a sentence with multiple negations:
  - Alice says: “I refuse to vote against repealing the ban on smoking in public. “
    - Does Alice like smoking in public or hate it?

De Morgan's Laws

- Simplifying negated formulas
  - For AND: $\neg\,(A \wedge B)$ is equivalent to $(\neg A \vee \neg B)$
  - For OR: $\neg\,(A \vee B) \equiv (\neg A \wedge \neg B)$

- Example:
  - $\neg\,(\neg A \ \vee B)$ is $\neg\neg\, A \ \wedge \neg B,$ same as $A \wedge \neg B$
  - So, since $(A \rightarrow B)$ is equivalent to $(\neg A \vee B)$,

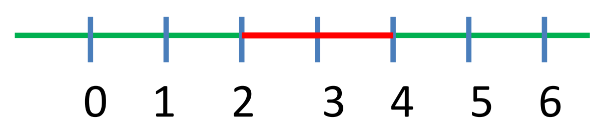
    $\neg(A \rightarrow B)$ is equivalent to $A \wedge \neg B$

De Morgan’s laws: examples

– Let A be “it’s sunny” and B “it’s cold”.
  - “It’s sunny and cold today”! -- No, it’s not!
  - That could mean
    – No, it’s not sunny.
    – No, it’s not cold.
    – No, it’s neither sunny nor cold.

  - In all of these scenarios, “It’s either not sunny or not cold” is true.

– Let A be “x < 2”, B be “x > 4”.



  - “Either $x < 2$ or $x > 4$“ – No, it is not!
  - Then $2 \leq x \leq 4$

More examples

- Let A be "I play" and B "I win".
  - $A \rightarrow B$: "If I play, then I win"
  - Equivalent to $\neg A \vee B$: "Either I do not play, or I win".

- Negation: $\neg(A \rightarrow B)$: "It is not so that if I play then I win".
  - By de Morgan's law: $\neg(\neg A \vee B) \equiv (\neg\neg A \wedge \neg B)$
  - By double negation: $(\neg\neg A \wedge \neg B) \equiv (A \wedge \neg B)$
  - So negation of "If I play then I win" is "I play and I don't win".

Longer example of negation

- Start with the outermost connective and keep applying de Morgan's laws and double negation.
- Stop when all negations are on variables.

- $\neg\,(\,(A \vee \neg B) \rightarrow \ (\neg A \wedge C))$
  - $(A \vee \neg B) \wedge \neg(\neg A \wedge C)$ (negating $\rightarrow$)
  - $(A \vee \neg B) \wedge (\neg\neg A \vee \neg C)$ (de Morgan)
  - $(A \vee \neg B) \wedge (A \vee \neg C)$ (removing $\neg\neg$)

Simplifying formulas

- $A \wedge C \ \rightarrow (\neg B \vee C)$
  - By $(\mathrm{F} \rightarrow G) \equiv \ (\neg F \vee G)$
    - equivalent to $\neg(A \wedge C) \vee (\neg B \vee C)$
  - De Morgan’s law
    - $\neg(A \wedge C)$ is equivalent to $(\neg A \ \vee \neg C)$
  - So the whole formula becomes
    - $\neg A \vee \neg C \vee \neg B \vee C$
    - But $\neg C \vee C$ is always true!
    - So the whole formula is a tautology.

More useful equivalences

- For any formulas A, B, C:
  - $True \vee A \equiv True$. $True \wedge A \equiv A$
  - $False \vee A \equiv A$. $False \wedge A \equiv False$
  - A$\vee A \equiv A \wedge A \equiv A$
- Also, like in arithmetic (with ∨ as +, ∧ as *)
  - $A \vee B \equiv B \vee A \quad and \;\; (A \vee B) \vee C \equiv A \vee (B \vee C)$
  - Same holds for ∧.
  - Also, $(A \vee B) \wedge C \equiv (A \wedge C) \vee (B \wedge C)$
- And unlike arithmetic
  - $(A \wedge B\ ) \vee C \equiv \;\; (A \vee C) \wedge (B \vee C)$

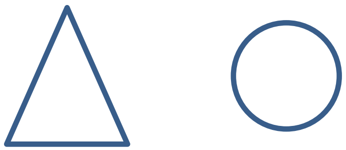
Longer example of negation

- Start with the outermost connective and keep applying de Morgan's laws and double negation. Stop when all negations are on variables.
- $\neg\,(\,(A \vee \neg B) \rightarrow \;\; (\neg A \wedge C)$
  - $(A \vee \neg B) \wedge \neg(\neg A \wedge C)$ (negating →)
  - $(A \vee \neg B) \wedge (\neg\neg A \vee \neg C)$ (de Morgan)
  - $(A \vee \neg B) \wedge (A \vee \neg C)$ (removing ¬¬)

– Can now simplify further, if we want to.
  - $A \vee (\neg B \wedge \neg C)$ (taking A outside the parentheses)

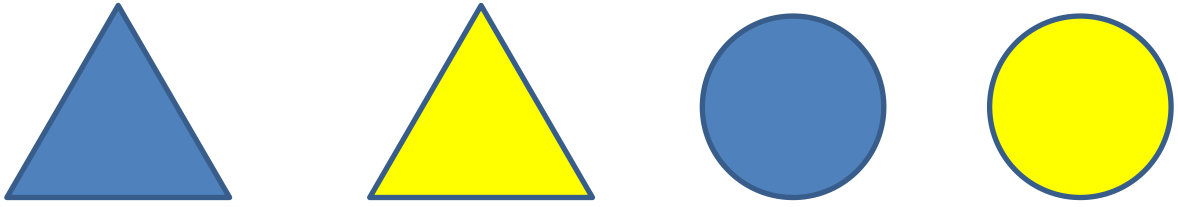
Puzzle 4

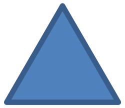
- I like one of the shapes.



- I like one of the colours.

- I like a figure if it has either my favourite shape or my favourite colour.



- I like  . What can you say about the rest?