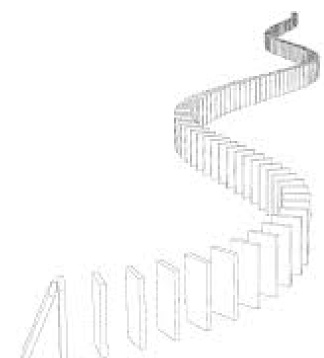
COMP 1002

Logic for Computer Scientists

Lecture 27

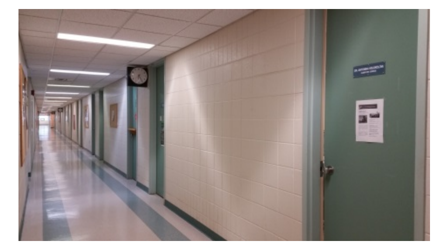


Admin stuff

- Assignment 4 is posted.
  - Due March 23rd.

Permutations

- **Permutations**: number of sequences of objects.
  - Without repetition: each object appears once.
- Example: how many ways to assign offices ER-6030 to ER-6033 to Antonina, Dave, Sharene and Yuanzhu?
  - 4 choices to pick who gets ER-6030. This leaves 3 choices to pick who gets ER-6031. Now 2 remain for ER-6032, and the last one is stuck with ER-6033.
    - By the product rule, get 4*3*2*1 = 4!.



- In general, number of permutations of n elements is n!
  - "Permutations": the difference between choices is only the order of elements.

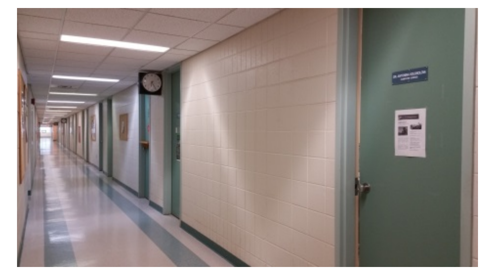
r-Permutations.

- Before, we talked about permutations of all objects in the set. An **r-permutation** P(n,r) involves taking only r out of n objects, and counting the number of possible sequences.
  - That is, the task consists of 1) picking r out of n objects. 2) counting number of sequences out of them.
- Example:
  - How many ways to assign ER offices to 4 out of 25 faculty members?
    - Pick one out of 25 to be in ER-6030. One out of remaining 24 to be in ER-6031, one out of 23 for ER-6032, one out of 22 for ER-6033. By the product rule, get P(25,4) = 25*24*23*22.
  - Alternative way of thinking:
    - There are 25! ways to assign offices to everybody.
    - Out of them, 21! way to assign non-ER offices. We are not interested in what everybody else got – so once the way to assign ER offices is fixed, all sequences with this assignment to ER offices are the same for us.
    - Get $\mathrm{P}(25{,}4) = \frac{25!}{21!}$ =25*24*23*22 possible ways to get 4 people offices in ER.
- Winners example:
  - There are 20 students in a programming competition.
  - How many possible ways are there to choose first, second and third winner?
    - 20*19*18

- General formula: $P(n,r) = \frac{n!}{(n-r)!}$

Combinations.

- In general, how many ways to pick k out of n objects?
  - The number of k-permutations divided by the number of different permutations of the k objects themselves.
- **Combinations**: C(n,k) is number of ways to choose k objects out of n.
  - "n choose k": $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ = C(n,k)
  - How many ways to choose 4 out of 25 faculty members to get an office in Earth Science?
    - 25! ways to assign offices altogether.
    - 4! ways to assign the first 4 offices (ones in Earth Science).
    - 21! ways to assign offices not in Earth Science.
    - Overall, C(25,4)=$\frac{25!}{4!\ 21!}$ ways to pick who gets an Earth Science office.



  - How many ways to select a crew of 6 astronauts out of a team of 30 to go to Mars?
    - C(30,6) = $\binom{30}{6} = \frac{30!}{6!(24!)} = 593{,}775$

FUN FACT: IF YOU SAY "YOU KNOW WHAT'S ACTUALLY REALLY GOOD?" IN THE RIGHT TONE OF VOICE, YOU CAN NAME ANY TWO INDIVIDUALLY-GOOD FOODS HERE AND NO ONE WILL CHALLENGE YOU ON IT.

Permanent link to this comic: https://xkcd.com/1609/

Puzzle: misspelling OSOYOOS

- In the game of Scrabble, players make words out of the pieces they have.
  - Suppose that someone puts the word "OSOYOOS" on the board, using up all her pieces.
  - How many ways could she have had the letters arranged on the rack in front of them?
    - The order of multiple copies of a letter does not matter: switching two S around results in the same sequence, but switching O and S does not.
    - The letters on the rack do not have to form a word.

Puzzle: misspelling OSOYOOS

- Suppose that someone puts the word "OSOYOOS" on the board, using up all her pieces.
- How many ways could she have had the letters arranged on the rack in front of them?
  - There are 7 letters in the word OSOYOOS. If they were all distinct, that would be 7! = 5040 ways.
  - But there are 4 Os, and 2 Ss, order of which does not matter.
  - There are 4! ways to order Os, and 2! ways to order Ss.
  - Therefore, the total number of ways to order the letters ignoring the order of Os and Ss is $^{7!}/_{4!2!}$ = 105

Puzzle: misspelling OSOYOOS

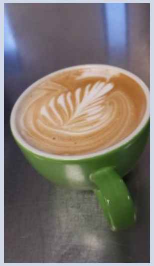
- Suppose that someone puts the word "OSOYOOS" on the board, using up all her pieces.
- How many ways could she have had the letters arranged on the rack in front of them, *such that Ss are not next to each other*?
  - First, let's consider all possible orderings of remaining letters: 5!/4! of them.
  - Now, consider places where S can go: _o_o_y_o_o_ (here, ooyoo are in arbitrary order). There are 6 such places.
  - So there are $\binom{6}{2} = {}^{6!}/_{2!4!}$ ways to place Ss.
  - Therefore, the total number of ways to order the letters ignoring the order of Os and Ss and with Ss not next to each other is ${}^{5!6!}/_{4!4!2!} = 75$
  - Alternatively, consider all orderings with Ss next to each other: there are $\frac{6!}{4!} = 30$ of them (treating the "SS" as a single letter).
  - Now, the total is 105-30 = 75.

Combinations with repetition

- Suppose that 10 of you came to the office hour, and we decided to go to Jumping Bean to get some coffee.
- Jumping Bean sells 6 types of coffee drinks: drip coffee, cappucino, espresso, latte, mocca and americano.
- How many different orders can we place, if each gets one coffee drink?
  - That is, how many ways are there to select 11 items, where each item comes from one of the 6 categories?
    - Let's use one letter for each type:
    - e.g., aacdeeellmm stands for 2 americanos, one cappucino, one drip, three espressos, 2 lattes and 2 moccas.
  - Idea: think of a string of letters with "dividers" between different types of drinks:
    - aa|c|d|eee|ll|mm
  - How many ways are there to position the dividers?
    - 6-1= 5 dividers
    - Number of orders + number of dividers: 11+6-1=16 positions.
    - So $\binom{16}{11} = \binom{16}{5}$ = 4368 possible orders.

- In general, number of ways to select r objects out of n categories with repetition is $\binom{r+n-1}{r}$

Summary

Selecting k out of n objects	Order matters (permutations)	Order ignored (combinations)
With repetitions	n^k 	$\binom{k+n-1}{k}$ 
Without repetitions	$P(n,k) = \frac{n!}{(n-k)!}$ 	$\binom{n}{k}$ 

Puzzle: playing poker

- There are 52 cards in a standard deck; 4 suites of 13 ranks each.
- In poker, some 5-card combinations ("hands") are special:
  - For example, a "three of a kind" consists of three cards with the same rank, together with two arbitrary cards.
- How many ways are there to choose (ignoring the order)
  - a three of a kind hand?
  - A two pairs hand?
  - Other hands?...