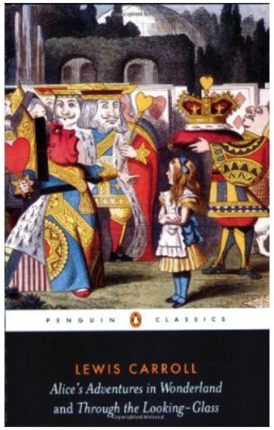
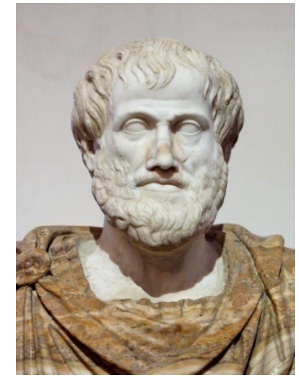


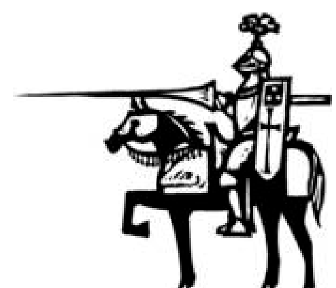


COMP 1002

Intro to Logic for Computer Scientists

Lecture 15

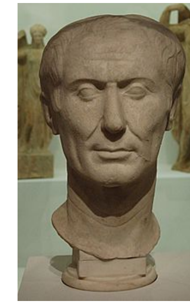
B 5 2 J



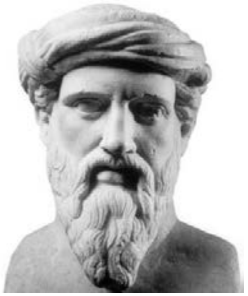
Admin stuff

- Assignments schedule? Split a2 and a3 in two (A2,3,4,5) , 5% each. A2 due Feb 17th.
- Midterm date? March 2nd.
- No office hour on Feb 9th

Puzzle: Caesar cipher

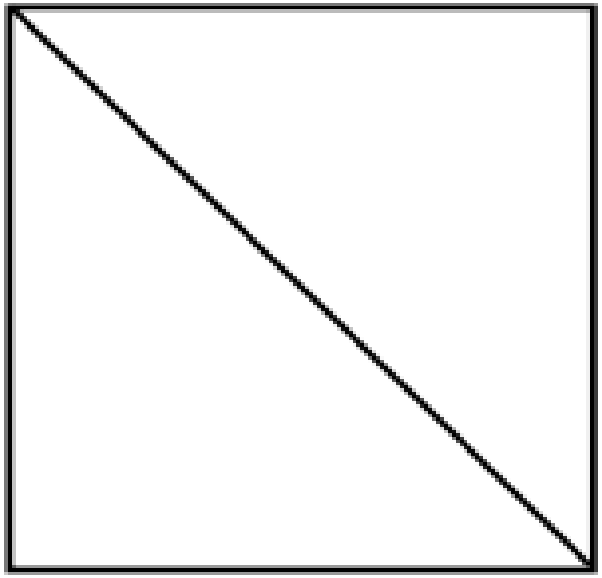


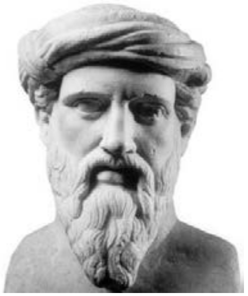
- The Roman dictator Julius Caesar encrypted his personal correspondence using the following code.
  - Number letters of the alphabet: A=0, B=1,… Z=25.
  - To encode a message, replace every letter by a letter three positions before that (wrapping).
    - A letter numbered x by a letter numbered x-3 mod 26.
    - For example, F would be replaced by C, and A by X
- Suppose he sent the following message.
  - QOBXPROB FK QEB ZXSB
- What does it say?



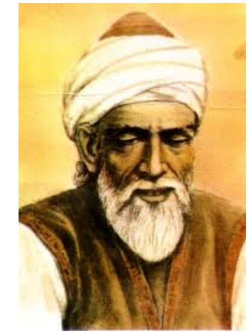
Square root of 2

- Is it possible to have a Pythagorean triple with a=b=1?
- Not quite: $1^2 + 1^2 = 2,$ so the third side would have to be $\sqrt{2}$.
- Is it at least possible to represent $\sqrt{2}$ as a ratio of two integers?...
  - Pythagoras and others tried...

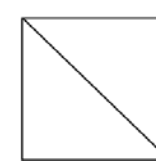


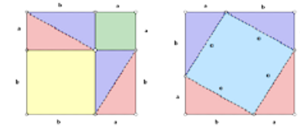


Rational and irrational numbers



- The numbers that are representable as a fraction of two integers are **rational** numbers. Set of all rational numbers is $\mathbb{Q}$.
- Numbers that are not rational are **irrational**.
  - Pythagoras figured out that the diagonal of a square is not comparable to the sides, but did not think of it as a number.
    - More like something weird.
  - It seems that irrational numbers started being treated as numbers in 9th century in the Middle East.
    - Starting with a Persian mathematician and astronomer Abu-Abdullah Muhammad ibn Īsa Māhānī (Al-Mahani).
- Rational and irrational numbers together form the set of all real numbers.
  - Any sequence of digits, potentially infinite after a decimal point, is a real number. Any point on a line.
- Irrationality of $\sqrt{2}$ is a classic proof by contradiction.

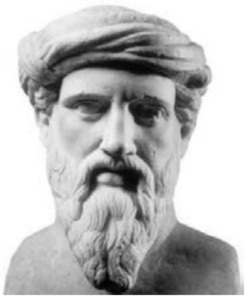




Proof by contradiction

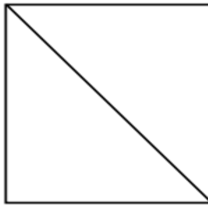
– To prove $\forall x \; F(x)$, prove $\forall x \neg F(x) \rightarrow FALSE$
  - Universal instantiation: “let n be an arbitrary element of the domain S of $\forall x$ ”
  - Suppose that $\neg F(n)$ is true.
  - Derive a contradiction.
  - Conclude that $F(n)$ is true.
  - By universal generalization, $\forall x \, F(x)$ is true.

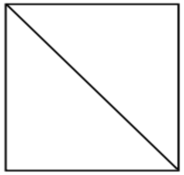
☐ (Done).

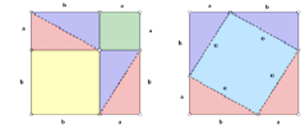


Definition of rational

- We need a slightly more precise definition of rational numbers for our proof that $\sqrt{2}$ is irrational.
- *Definition* (of rational and irrational numbers):
  - A real number r is **rational** iff $\exists m, n \in \mathbb{Z}, n \neq 0 \wedge \gcd(m,n) = 1 \ \wedge \ r = \frac{m}{n}$.
    - Reminder: **greatest common divisor gcd(m,n)** is the largest integer which divides both m and n. When d=1, m and n are **relatively prime**.
    - Any fraction can be simplified until the numerator and denominator are relatively prime, so it is not a restriction
  - A real number which is not rational is called **irrational**.







Proof by contradiction

- *Theorem*: Square root of 2 is irrational.
- *Proof*:
  - Suppose, for the sake of contradiction, that $\sqrt{2}$ is rational. Then there exist relatively prime m, n $\in \mathbb{Z}$, $n \neq 0$ such that $\sqrt{2} = \frac{m}{n}$.
  - By algebra, squaring both sides we get $2 = \frac{m^2}{n^2}$.
  - Thus m^2 is even, and by the theorem we just proved, then m is even. So $m = 2k$ for some k.
  - $2n^2 = 4\,k^2$, so $n^2 = 2k^2$, and by the same argument n is even.
  - This contradicts our assumption that m and n are relatively prime. Therefore, such m and n cannot exist, and so $\sqrt{2}$ is not rational.

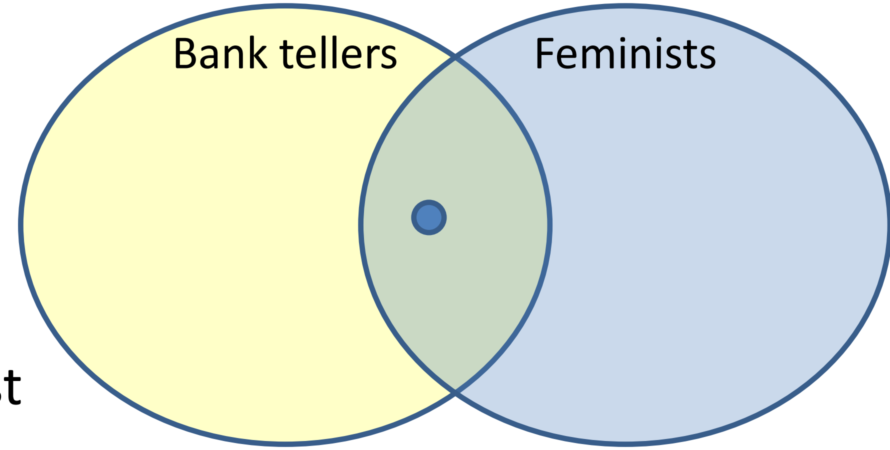
☐ (Done).

Puzzle 9

- Susan is 28 years old, single, outspoken, and very bright. She majored in philosophy. As a student she was deeply concerned with issues of discrimination and social justice and also participated in anti-nuke demonstrations.

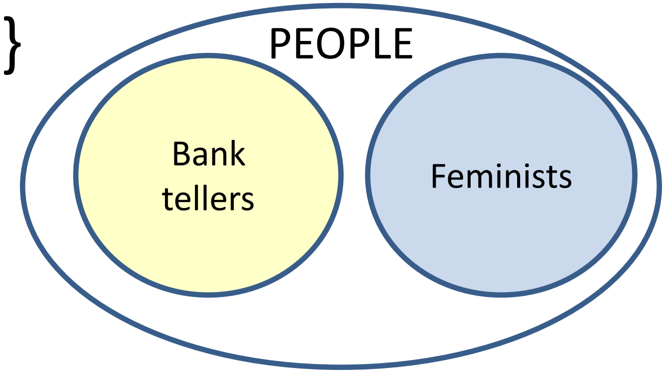
  Please rank the following possibilities by how likely they are. List them from least likely to most likely. Susan is:

1. a kindergarden teacher
2. works in a bookstore and takes yoga classes
3. an active feminist
4. a psychiatric social worker
5. a member of an outdoors club
6. a bank teller
7. an insurance salesperson
8. a bank teller and an active feminist



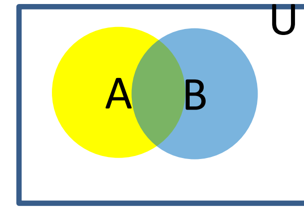
Set inclusion.

- Let A and B be two sets.
  - Such as A={2,3,4} and B= {1,2,3,4,5}
- A is a **subset** of B:
  - $A \subseteq B$ iff $\forall x\ (x \in A \rightarrow x \in B)$
    - $A \subseteq B.\ FEMINISTS \subseteq PEOPLE$
  - A is a **strict subset** of B: $A \subset B$ iff
    $\forall x\ (x \in A \rightarrow x \in B) \wedge \exists\, y\ (y \in B \wedge y \notin A)$
    - $A \subset B.\ FEMINISTS \subset PEOPLE$
  - When both $A \subseteq B$ and $B \subseteq A$, then $A = B$
- A and B are **disjoint** iff $\forall x\ (x \notin A \vee x \notin B)$
  - {1,5} and {2,3,6,9} are disjoint. So are BANKTELLERS and FEMINISTS in the diagram above.

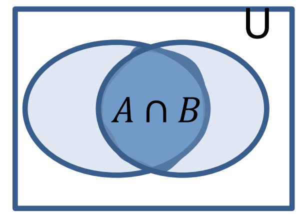


Operations on sets

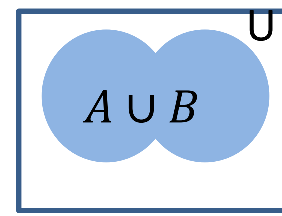
- Let A and B be two sets.
  - Such as A={1,2,3} and B={ 2,3,4}



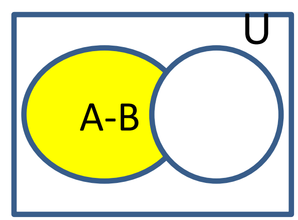
- **Intersection** $A \cap B = \{\, x \mid x \in A \wedge x \in B\}$
  - The green part of the picture above
  - $A \cap B = \{2,3\}$



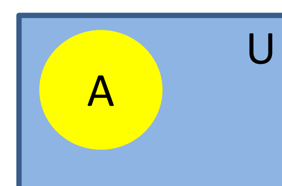
- **Union** $A \cup B = \{\, x \mid x \in A \vee x \in B\}$
  - The coloured part in the top picture.
  - $A \cup B = \{1,2,3,4\}$



- **Difference** $A - B = \{x \mid x \in A \wedge x \notin B\}$
  - The yellow part in the top picture.
  - $\mathrm{A} - B = \{1\}$



- **Complement** $\overline{A} = \{x \in U \mid x \notin A\}$
  - The blue part on the bottom Venn diagram
  - If universe U = ℕ, $\overline{A} = \{x \in \mathbb{N} \mid x \notin \{1,2,3\}\ \}$



Puzzle: the barber

- In a certain village, there is a (male) barber who shaves all and only those men of the village who do not shave themselves.

- *Question: who shaves the barber?*