1. **Translations in predicate logic**
   
   (a) Let \( \text{Parent}(x, y) \) be a relation stating that \( x \) is a parent of \( y \). Write a predicate logic formula stating that \( x \) is an uncle or an aunt of \( y \). Use only predicate \( \text{Parent} \). Hint: variables can take the same value, unless you explicitly tell them not to.
   
   (b) Now write a formula stating that \( x \) has a sibling. Use predicates \( \text{Parent} \) and \( = \).

2. **Domains and predicates**
   
   (a) Consider a sentence \( \forall x \exists y \exists z ( P(x, y) \land P(x, z) \rightarrow Q(y, z)) \). Let all quantifiers have the same domain, consisting of two elements \( a, b \). Now, give interpretations of \( P \) and \( Q \) over this domain that make this formula true. Finally, give another pair of interpretations of \( P \) and \( Q \) that makes this formula false.
   
   (b) Suppose that the formula is \( \forall x \exists y \exists z ( P(x, y) \land P(x, z) \rightarrow P(y, z)) \), that is, there is no \( Q \). What is the value of this formula on the empty domain? On the domain consisting of only one element (for any interpretation of \( P \))? 

3. **Predicate logic reasoning**
   
   Let the domain be a group of three cats, Tiger, Ashes and Smokey. Consider the following premises:
   
   - \( \forall x \forall y (\text{Kitten}(x) \land \neg \text{Kitten}(y) \rightarrow \text{Parent}(y, x)) \)
   - \( \forall x \forall y (\text{Parent}(x, y) \rightarrow \neg \text{Kitten}(x)) \)
   - \( \forall x \forall y (\text{Parent}(x, y) \rightarrow \text{Kitten}(y)) \)
   - \( \text{Parent}(\text{Smokey}, \text{Ashes}) \)
   - \( \neg \text{Kitten}(\text{Tiger}) \)
   
   (a) Use predicate logic natural deduction to figure out the relationship between Tiger and Ashes. Hint: if you have two (or more) universal quantifiers in a row, you can treat them as one quantifier for a pair of elements, and instantiate them together in universal modus ponens. Also, use the rule that from \( A \) and \( B \) you can derive \( A \land B \) in one step.
   
   (b) Use resolution with unification to confirm that your conclusion follows from premises. Hint: if you run out of letters, you can use \( xx, w_1 \) and such.

4. **Proofs**
   
   Use direct proof technique to show that product of two odd numbers is odd. Use the definition of an integer \( n \) being odd if there is another integer \( k \) such that \( n = 2k + 1 \).