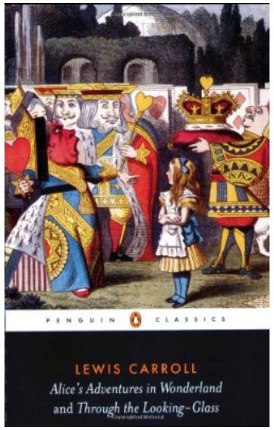
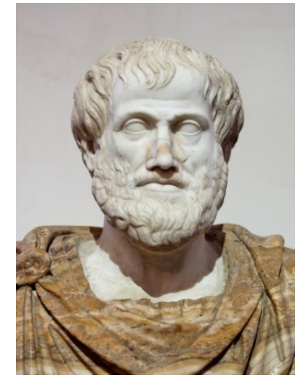
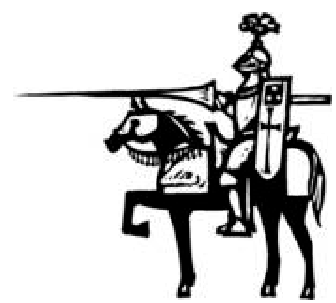




COMP 1002

Intro to Logic for Computer Scientists

Lecture 7



Proof vs. disproof

- To prove that something is (always) true:
  - Make sure it holds in every scenario
    - $\neg B \rightarrow \neg A$ is equivalent to $A \rightarrow B$, because
    $\neg B \rightarrow \neg A \equiv \neg\neg B \vee \neg A \equiv B \vee \neg A \equiv \neg A \vee B \equiv A \rightarrow B$
    - So $(\neg B \rightarrow \neg A) \leftrightarrow (A \rightarrow B)$ is a tautology.

    - I have classes every day that starts with T. I have classes on Tuesday and Thursday (and Monday, but that's irrelevant).

  - Or assume it does not hold, and then get something strange as a consequence:
    - To show A is true, enough to show $\neg A \rightarrow FALSE$.
    - Suppose there are finitely many prime numbers. What divides the number that's a product of all primes +1?

Proof vs. disproof

- To disprove that something is always true, enough to give just one scenario where it is false (find a falsifying assignment).

  – To disprove that $A \to B \equiv B \to A$
    - Take $A = true, B = false,$
    - Then $A \to B$ is false, but $\mathrm{B} \to A$ is true.

  – To disprove that $B \to A \equiv \neg (A \to B)$
    - Take A=true, B=true
    - Then $\mathrm{B} \to A$ is true, but $\neg(A \to B)$ is false.

  – I have classes every day! – No, you don't have classes on Saturday

  – Women don't do Computer Science! – Me?

Treasure hunt

- In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humour and love of logical puzzles. In the note he wrote that he had hidden a treasure somewhere on the property. He listed 5 true statements and challenged the reader to use them to figure out the location of the treasure

Treasure hunt

1. If this house is next to a lake, then a treasure is not in the kitchen
2. If the tree in the font yard is an elm, then the treasure is in the kitchen
3. This house is next to a lake
4. The tree in the front yard is an elm, or the treasure is buried under the flagpole
5. If the tree in the back yard is an oak, then the treasure is in the garage.

Treasure hunt

1. If this house is next to a lake, then a treasure is not in the kitchen
2. If the tree in the font yard is an elm, then the treasure is in the kitchen.
3. This house is next to a lake
4. The tree in the front yard is an elm, or the treasure is buried under the flagpole
5. If the tree in the back yard is an oak, then the treasure is in the garage.

Too many variables for a nice truth table...

- A: this house is next to a lake.
- B: the treasure is in the kitchen
- C: The tree in front is elm
- D: the treasure is under the flagpole.
- E: The tree in the back is oak
- F: The treasure is in the garage

1. If A then not B
2. If C then B
3. A
4. C or D
5. If E then F

1. $A \rightarrow \neg B$
2. $C \rightarrow B$
3. A
4. C V D
5. $E \rightarrow F$

Natural deduction

- A: this house is next to a lake.
- B: the treasure is in the kitchen
- C: The tree in front is elm
- D: the treasure is under the flagpole.
- E: The tree in the back is oak
- F: The treasure is in the garage

1. If A then not B
2. If C then B
3. A
4. C or D
5. If E then F
6. Not B
7. Not C
8. D

- If house is next to the lake then the treasure is not in the kitchen
- The house is next to the lake
- Therefore, the treasure is not in the kitchen.

Arguments and validity

- An **argument**, in logic, is a sequence of propositional statements.
  - Called **argument form** when statements are formulas involving variables.
- The last statement in the sequence is called the **conclusion**. All the rest are **premises**.
- An argument is **valid** if whenever all premises are true, the conclusion is also true.
  - So if premises are $P_1, \dots, P_n$, and conclusion is P_{n+1} ,
  - then the argument is valid

$\Updownarrow$ if and only if

  - $P_1 \wedge P_2 \wedge \cdots P_n \rightarrow P_{n+1}$ is a tautology

Treasure hunt

1. If this house is next to a lake, then a treasure is not in the kitchen
2. If the tree in the font yard is an elm, then the treasure is in the kitchen
3. This house is next to a lake
4. The tree in the front yard is an elm, or the treasure is buried under the flagpole
5. If the tree in the back yard is an oak, then the treasure is in the garage.

Premises

6. The treasure is under the flagpole.

Conclusion

Argument

Arguments and validity

- Arguments are often written in this format:
  - Symbol ∴ is pronounced "therefore"

$$P_1$$
$$P_2$$
$$\vdots$$
$$P_n$$

______

$$\therefore\ P_{n+1}$$

- If house is next to the lake then the treasure is not in the kitchen
- The house is next to the lake

____

∴ the treasure is not in the kitchen

If $x > 3$, then $x > 2$

If $x > 2$, then $x \neq 1$

$x > 3$

____

$\therefore\ x \neq 1$

Arguments and validity

- Valid argument: AND of premises → conclusion is a tautology

- If house is next to the lake then the treasure is not in the kitchen
- The house is next to the lake

____

∴ the treasure is not in the kitchen

Valid argument:
$((p \to q) \wedge p \to q$
is a tautology

If $x > 3$, then $x > 2$

If $x > 2$, then $x \neq 1$

$x > 3$

____

$\therefore \; x \neq 1$

Valid argument:
$(p \to q) \wedge (q \to r) \wedge p \to r$
is a tautology

If $x > 3$, then $x > 2$

If $x > 2$, then $x \neq 1$

$x \neq 1$

____

$\therefore \; x > 3$

Invalid argument!
$(p \to q) \wedge (q \to r) \wedge r \to p$
is a not a tautology!
False when r is true, and p and q are both false.

Natural deduction

- A: this house is next to a lake.
- B: the treasure is in the kitchen
- C: The tree in front is elm
- D: the treasure is under the flagpole.
- E: The tree in the back is oak
- F: The treasure is in the garage

1. If A then not B
2. If C then B
3. A
4. C or D
5. If E then F
6. **Not B**
7. **Not C**
8. D

How do we get the intermediate steps?

- If house is next to the lake then the treasure is not in the kitchen
- The house is next to the lake
- Therefore, the treasure is not in the kitchen.

Rules of inference

- Just like we used equivalences to simplify a formula instead of writing truth tables
- Can apply tautologies of the form $\mathrm{F} \rightarrow G$
  - so that if F is an AND of several formulas derived so far, then we get G, and add G to premises.
  - Such as $((p \rightarrow q) \wedge p) \rightarrow q$
- Keep going until we get the conclusion.

- If house is next to the lake then the treasure is not in the kitchen
- The house is next to the lake
- Therefore, the treasure is not in the kitchen.

- Here, p is "the house is next to the lake", and q is "the treasure is not in the kitchen".

Modus ponens

- The main rule of inference, given by the tautology $(p \to q) \wedge p \to q$, is called **Modus Ponens**.

- If house is next to the lake then the treasure is not in the kitchen
- The house is next to the lake

$\therefore$ the treasure is not in the kitchen

- If Socrates is a man, then Socrates is mortal
- Socrates is a man

$\therefore$ Socrates is mortal

- If $x > 2$, then $x \neq 1$
- $x > 2$

$\therefore\ x \neq 1$

False premises

- An argument can still be valid when some of its premises are false.
  - Remember, false implies anything.
- Bertrand Russell: “If 2+2=5, then I am the pope”

Puzzle 7: can you see how to prove this?