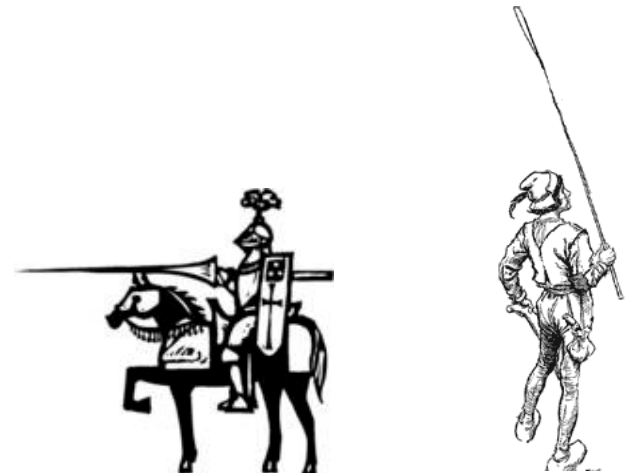


# COMP 1002

## Intro to Logic for Computer Scientists

### Lecture 5



# Puzzle 4

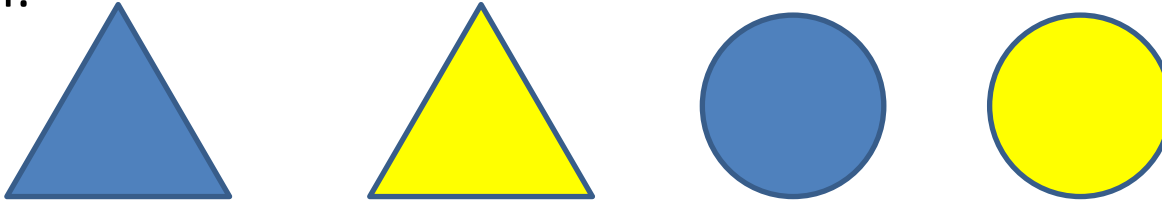
- I like one of the shapes.





- I like one of the colours.



- I like a figure if it has either my favourite shape or my favourite colour.

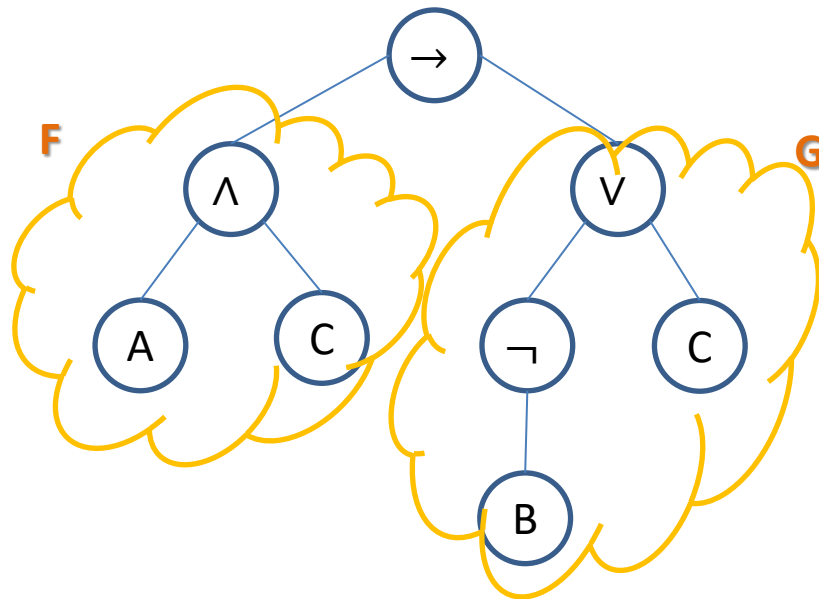


- I like . What can you say about the rest?

- I might like triangles, or blue things, or both.
- There is one figure I don't like, but not enough information to say which one it is. I might still like 

# Simplifying formulas

- $A \wedge C \rightarrow (\neg B \vee C)$ 
  - Order of precedence:  $\rightarrow$  is the outermost, that is, the formula is of the form  $F \rightarrow G$ , where F is  $(A \wedge C)$ , and G is  $(\neg B \vee C)$ .



# Simplifying formulas

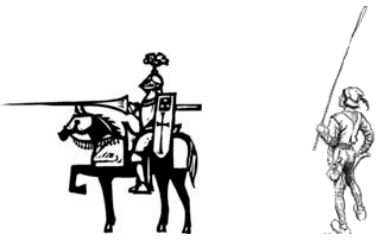
- $A \wedge C \rightarrow (\neg B \vee C)$ 
  - $\text{By}(F \rightarrow G) \equiv (\neg F \vee G)$ 
    - equivalent to  $\neg(A \wedge C) \vee (\neg B \vee C)$
  - De Morgan's law
    - $\neg(A \wedge C)$  is equivalent to  $(\neg A \vee \neg C)$
  - So the whole formula becomes
    - $\neg A \vee \neg C \vee \neg B \vee C$
    - But  $\neg C \vee C$  is always true!
    - So the whole formula is a tautology.

# More useful equivalences

- For any formulas  $A, B, C$ :
  - $TRUE \vee A \equiv TRUE.$                        $TRUE \wedge A \equiv A$
  - $FALSE \vee A \equiv A.$                                $FALSE \wedge A \equiv FALSE$
  - $A \vee A \equiv A \wedge A \equiv A$
- Also, like in arithmetic (with  $\vee$  as  $+$ ,  $\wedge$  as  $*$ )
  - $A \vee B \equiv B \vee A$     *and*     $(A \vee B) \vee C \equiv A \vee (B \vee C)$
  - Same holds for  $\wedge$ .
  - Also,  $(A \vee B) \wedge C \equiv (A \wedge C) \vee (B \wedge C)$
- And unlike arithmetic
  - $(A \wedge B) \vee C \equiv (A \vee C) \wedge (B \vee C)$

# Longer example of negation

- Start with the outermost connective and keep applying de Morgan's laws and double negation. Stop when all negations are on variables.
- $\neg ((A \vee \neg B) \rightarrow (\neg A \wedge C))$ 
  - $(A \vee \neg B) \wedge \neg(\neg A \wedge C)$  (negating  $\rightarrow$ )
  - $(A \vee \neg B) \wedge (\neg\neg A \vee \neg C)$  (de Morgan)
  - $(A \vee \neg B) \wedge (A \vee \neg C)$  (removing  $\neg\neg$ )
- Can now simplify further, if we want to.
  - $A \vee (\neg B \wedge \neg C)$  (taking A outside the parentheses)



# Knights and knaves



- On a mystical island, there are two kinds of people: knights and knaves. Knights always tell the truth. Knaves always lie.
- Puzzle 5: You hear a person from the island of knights and knaves say “if I am a knight, then it will rain tomorrow”. What can you conclude from this?