Admin stuff

• Labs starting next week:
  – Section 1: Thursday 9am, CS-1019
  – Section 2: Tuesday 9am, CS-1019
  – If you are on a waiting list, come to Tuesday lab.

• Assignment 1 is posted, due Sep 25\textsuperscript{th}, 10pm
  – Type it up and submit on D2L
  – If you’d like to try setting it in LaTeX, the source is available
Knights and knaves

• On a mystical island, there are two kinds of people: knights and knaves. Knights always tell the truth. Knaves always lie.

• Puzzle 2: You see three islanders talking to each other, Arnold, Bob and Charlie. You ask Arnold “Are you a knight?”, but can’t hear what he answered. Bob pitches in: “Arnold said that he is a knave!” and Charlie interjects “Don’t believe Bob, he’s lying”. Out of Bob and Charlie, who is a knight and who is a knave?
Knights and knaves

• Puzzle 2: You see three islanders talking to each other, Arnold, Bob and Charlie.
  – You ask Arnold “Are you a knight?”, but can’t hear what he answered.
  – Bob pitches in: “Arnold said that he is a knave!” and
  – Charlie interjects “Don’t believe Bob, he’s lying”.
  – Out of Bob and Charlie, who is a knight and who is a knave?

• Look at the sentence “I am a knave”. Who of the knights/knaves can say this?
• If A is “Arnold is a knight” and S is “I am a knave”, when is $S \leftrightarrow A$ (what Arnold said is true if and only if he is a knight).
• But also “I am a knave” is the same as saying $\neg A$
• $A \leftrightarrow \neg A$ is a contradiction: it is false no matter what A is.
• So Bob must be lying: Bob is a knave. And Charlie is a knight.
Logical equivalence

• Two formulas F and G are **logically equivalent** \((F \iff G)\) if they have the same value for every row in the truth table on their variables.
  
  – \(A \land \neg A \equiv False\) (same as saying it is a contradiction)
  
  – \((\neg A \lor B) \equiv (A \rightarrow B)\)
  
  – \((A \leftrightarrow B) \equiv (A \rightarrow B) \land (B \rightarrow A)\)
    
    • \(\leftrightarrow\) is sometimes called the “biconditional”
    
    • \(\leftrightarrow\) often pronounced as “if and only if”, or “iff”

• Useful fact: proving that \(F \equiv G\) can be done by proving that \(F \leftrightarrow G\) is a tautology
Double negation

- Negation cancels negation
  \[ \neg \neg A \equiv A \]
  - “I do not disagree with you” = “I agree with you”

- For a human brain, harder to parse a sentence with multiple negations:
  - Alice says: “I refuse to vote against repealing the ban on smoking in public. “
    - Does Alice like smoking in public or hate it?
De Morgan’s Laws

• Simplifying negated formulas
  – For AND: \( \neg (A \land B) \) is equivalent to \( \neg A \lor \neg B \)
  – For OR: \( \neg (A \lor B) \equiv (\neg A \land \neg B) \)

• Example:
  – \( \neg (\neg A \lor B) \) is \( \neg \neg A \land \neg B \), same as \( A \land \neg B \)
  – So, since \( (A \to B) \) is equivalent to \( (\neg A \lor B) \),
    \( \neg (A \to B) \) is equivalent to \( A \land \neg B \)
De Morgan’s laws: examples

– Let A be “it’s sunny” and B “it’s cold”.
  • “It’s sunny and cold today”! -- No, it’s not!
  • That could mean
    – No, it’s not sunny.
    – No, it’s not cold.
    – No, it’s neither sunny nor cold.
  • In all of these scenarios, “It’s either not sunny or not cold” is true.

– Let A be “x < 2”, B be “x > 4”.
  • “Either x < 2 or x > 4” – No, it is not!
  • Then 2 ≤ x ≤ 4
More examples

– Let A be “I play” and B “I win”.
  • $A \rightarrow B$: “If I play, then I win”
  • Equivalent to $\neg A \lor B$: “Either I do not play, or I win”.

– Negation: $\neg(A \rightarrow B)$: “It is not so that if I play then I win”.
  • By de Morgan’s law: $\neg(\neg A \lor B) \equiv (\neg \neg A \land \neg B)$
  • By double negation: $(\neg \neg A \land \neg B) \equiv (A \land \neg B)$
  • So negation of “If I play then I win” is “I play and I don’t win”.

 Longer example of negation

• Start with the outermost connective and keep applying de Morgan’s laws and double negation.
• Stop when all negations are on variables.

• \( \neg \left( (A \lor \neg B) \rightarrow (\neg A \land C) \right) \)
  • \((A \lor \neg B) \land \neg (\neg A \land C)\) (negating \(\rightarrow\))
  • \((A \lor \neg B) \land (\neg \neg A \lor \neg C)\) (de Morgan)
  • \((A \lor \neg B) \land (A \lor \neg C)\) (removing \(\neg \neg\))
Simplifying formulas

- \( A \land C \rightarrow (\neg B \lor C) \)
  - By \((F \rightarrow G) \equiv (\neg F \lor G)\)
    - equivalent to \(\neg (A \land C) \lor (\neg B \lor C)\)
  - De Morgan’s law
    - \(\neg (A \land C)\) is equivalent to \((\neg A \lor \neg C)\)
  - So the whole formula becomes
    - \(\neg A \lor \neg C \lor \neg B \lor C\)
  - But \(\neg C \lor C\) is always true!
  - So the whole formula is a tautology.
More useful equivalences

• For any formulas $A$, $B$, $C$:
  
  – $A \lor \neg A \equiv True$ \hspace{1cm} $A \land \neg A \equiv False$
  
  – $True \lor A \equiv True$. \hspace{1cm} $True \land A \equiv A$
  
  – $False \lor A \equiv A$. \hspace{1cm} $False \land A \equiv False$
  
  – $A \lor A \equiv A \land A \equiv A$

• Also, like in arithmetic (with $\lor$ as $+$, $\land$ as $\ast$)
  
  – $A \lor B \equiv B \lor A$ and $(A \lor B) \lor C \equiv A \lor (B \lor C)$
  
  – Same holds for $\land$.
  
  – Also, $(A \lor B) \land C \equiv (A \land C) \lor (B \land C)$

• And unlike arithmetic
  
  – $(A \land B) \lor C \equiv (A \lor C) \land (B \lor C)$
Longer example of negation

• Start with the outermost connective and keep applying de Morgan’s laws and double negation. Stop when all negations are on variables.

• \( \neg ( (A \lor \neg B) \rightarrow (\neg A \land C) ) \)
  • \( (A \lor \neg B) \land \neg (\neg A \land C) \) (negating \( \rightarrow \))
  • \( (A \lor \neg B) \land (\neg \neg A \lor \neg C) \) (de Morgan)
  • \( (A \lor \neg B) \land (A \lor \neg C) \) (removing \( \neg \neg \))

– Can now simplify further, if we want to.
  • \( A \lor (\neg B \land \neg C) \) (taking A outside the parentheses)
Puzzle 4

• I like one of the shapes.

• I like one of the colours.

• I like a figure if it has either my favourite shape or my favourite colour.

• I like ▲ . What can you say about the rest?