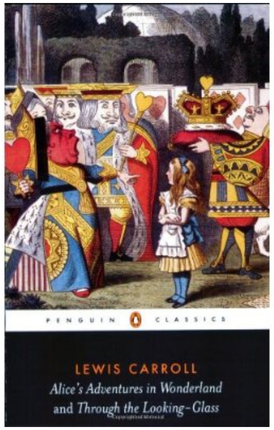
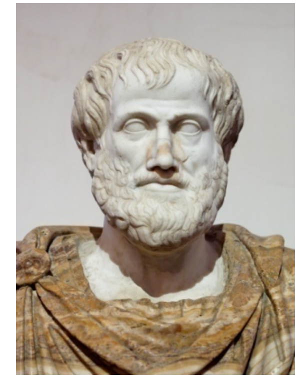


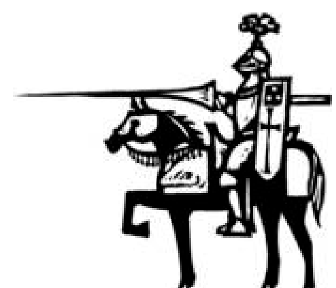


COMP 1002

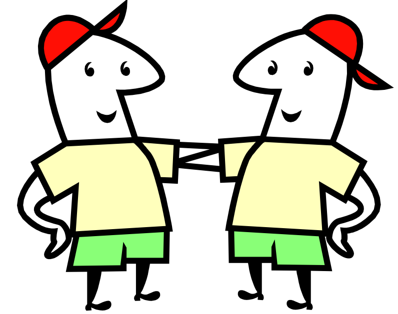
Intro to Logic for Computer Scientists

Lecture 2

B 5 2 J

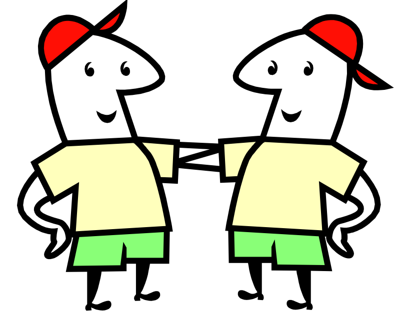



Twins puzzle



- There are two identical twin brothers, Dave and Jim.
- One of them always lies; another always tells the truth.
- Suppose you see one of them and you want to find out his name.
- How can you learn if you met Dave or Jim by asking just one short yes-no question? You don’t know which one of them is the liar.


Twins puzzle



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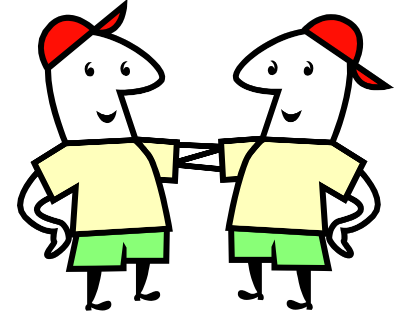
This is Jim	Jim is a liar				
Yes	Yes				
Yes	No				
No	Yes				
No	No				

Twins puzzle



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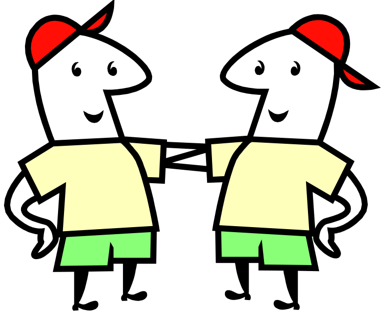
This is Jim	Jim is a liar	This is a liar			
Yes	**Yes**	Yes			
Yes	**No**	No			
No	**Yes**	No			
No	**No**	Yes			

Twins puzzle



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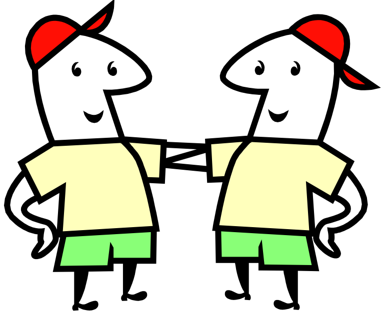
- How can you learn if you met Dave or Jim by asking just one short yes-no question? You don’t know which one of them is the liar.

This is Jim	Jim is a liar	This is a liar	Are you Jim?		
Yes	**Yes**	No	No		
Yes	**No**	No	Yes		
No	**Yes**	Yes	No		
No	**No**	Yes	Yes		

Twins puzzle



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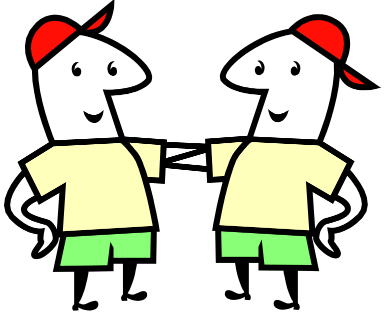
- How can you learn if you met Dave or Jim by asking just one short yes-no question? You don’t know which one of them is the liar.

This is Jim	Jim is a liar	This is a liar	Are you Jim?	Is 2+2=4?	
Yes	**Yes**	Yes	No	No	
Yes	**No**	No	Yes	Yes	
No	**Yes**	No	No	Yes	
No	**No**	Yes	Yes	No	

Twins puzzle



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- How can you learn if you met Dave or Jim by asking just one short yes-no question? You don't know which one of them is the liar.

This is Jim	Jim is a liar	This is a liar	Are you Jim?	Is 2+2=4?	Is Dave a liar?
Yes	**Yes**	Yes	No	No	Yes
Yes	**No**	No	Yes	Yes	Yes
No	**Yes**	No	No	Yes	No
No	**No**	Yes	Yes	No	No

Language of logic: building blocks

- **Proposition**: A sentence that can be *true* or *false.*
  - A: “It is raining in St. John’s right now”.
  - B: “2+2=7”
  - But not “Hi!” or “x is an even number”
- **Propositional variables:**
  - A, B, C (or p, q, r)
  - It is a shorthand to denote propositions:
    - “B is true”, for the B above, means “2+2=7” is true.

Language of logic: connectives

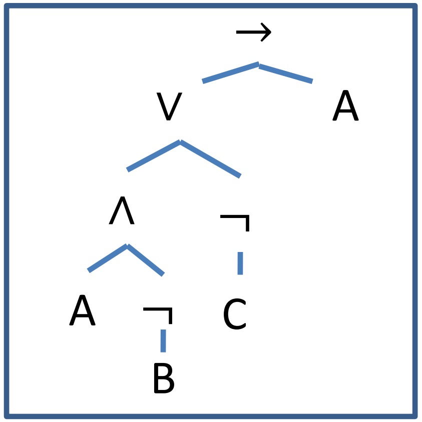
Pronunciation	Notation	Meaning
A and B (conjunction)	$A \wedge B$	True if both A and B are true
A or B (disjunction)	$A \vee B$	True if either A or B are true (or both)
If A then B (implication)	$A \rightarrow B$	True whenever if A is true, then B is also true
Not A (negation)	$\neg A$	Opposite of A is true, $\neg A$ is true when A is false

- Let A be "It is sunny" and B be "it is cold"
  - $A \wedge B$: It is sunny and cold
  - $A \vee B$: It is either sunny or cold
  - $A \rightarrow B$: If it is sunny, then it is cold
  - $\neg A$: It is not sunny

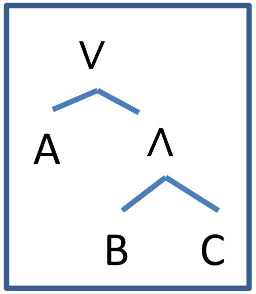
Language of logic

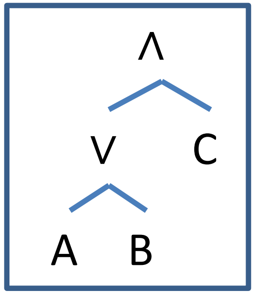
Pronunciation	Notation	True when
A and B	$A \wedge B$	Both A and B must be true
A or B	$A \vee B$	Either A or B must be true (or both)
If A then B	$A \to B$	if A is true, then B is also true
Not A	$\neg A$	Opposite of A is true

- Now we can combine these operations to make longer formulas
- Precedence: ¬ first, then ∧, then ∨, → last
  - ¬ is like a unary minus, ∧ like * and ∨ like +
- $A \wedge \neg B \vee \neg C \to A$ is $\big((A \wedge (\neg B)) \vee (\neg C)\big) \to A$



  - When in doubt or need a different order, use parentheses
  - $A \vee B \wedge C$ is not the same as $(A \vee B) \wedge C$
    - Check the scenario when A is true, but both B and C are false.





Language of logic

Pronunciation	Notation	True when
A and B	A /\ B	Both A and B must be true
A or B	A \/ B	Either A or B must be true (or both)
If A then B	A -> B	if A is true, then B is also true
Not A	~A	Opposite of A is true

- Let
  - A be "It is sunny",
  - B be "it is cold",
  - C be "It's snowing"

- What are the translations of:
  - $B \land C \rightarrow \neg A$ IF (AND) THEN NOT
    - If it is cold and snowing, then it is not sunny
  - $B \rightarrow (C \lor A)$ IF THEN (OR)
    - If it is cold, then it is either snowing or sunny
  - $\neg A \land A \rightarrow C$ IF (NOT AND) THEN
    - If it is sunny and not sunny, then it is snowing.

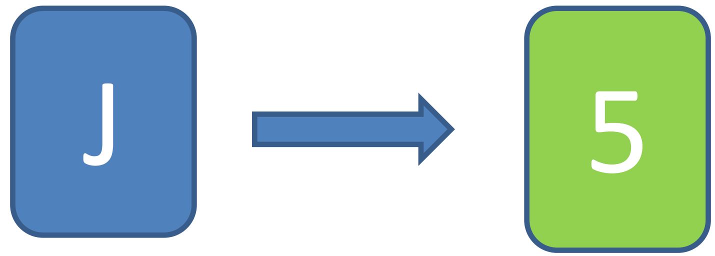
The truth

- We talk about a sentence being true or false when the values of the variables are known.
  - If we didn't know whether it is sunny, we would not know whether $A \wedge B \rightarrow C$ is true or false.
- **Truth assignment:** setting values of variables to true/false.
  - A=true, B=false, C=false.
- **Satisfying assignment** for a sentence: assignment that makes it true.
  - (Otherwise, **falsifying** assignment).
  - A=true, B=false, C= false satisfies $A \wedge B \rightarrow C$
  - A=true, B=true, C=false falsifies $A \wedge B \rightarrow C$

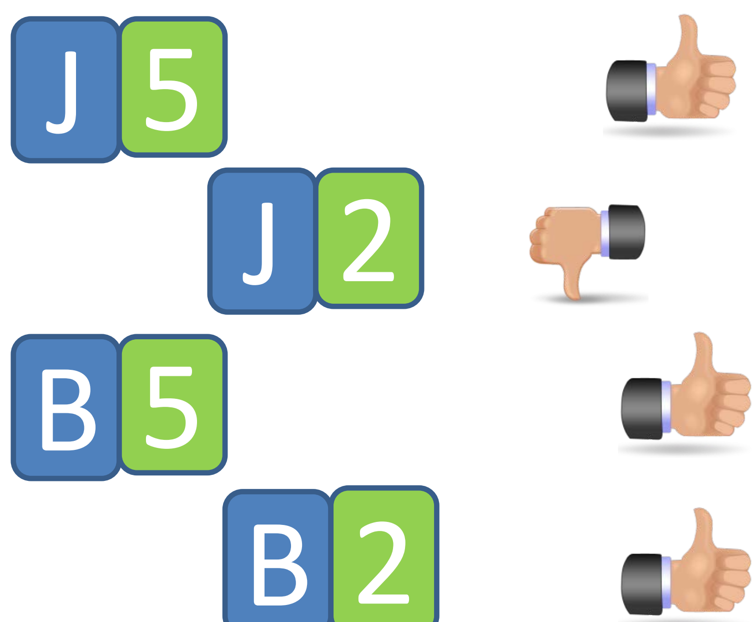
“if … then” in logic

- Last class’ puzzle has a logical structure:

“if A then B”



- What circumstances make this true?
  - A is true and B is true
  - A is true and B is false
  - A is false and B is true
  - A is false and B is false



Truth tables

A	B	not A	A and B	A or B	if A then B
True	*True*	False	True	True	True
True	*False*	False	False	True	False
False	*True*	True	False	True	True
False	*False*	True	False	False	True

A	B
True	*True*
True	*False*
False	*True*
False	*False*

- Let
  - A be “It is sunny”
  - B be “it is cold”
- It is sunny and cold.
- It is sunny and not cold
- It is not sunny and cold
- It is neither sunny nor cold

Truth tables

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  - A be “It is sunny”
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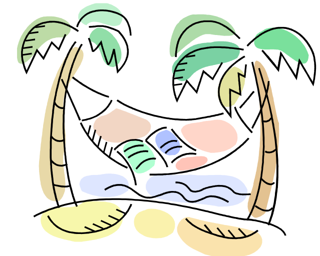
A	B	not A	A and B	A or B	if A then B
True	*True*	False	True	True	True
True	*False*	False	False	True	False
False	*True*	True	False	True	True
False	*False*	True	False	False	True

- It is sunny and cold.
- It is sunny and not cold
- It is not sunny and cold
- It is neither sunny nor cold

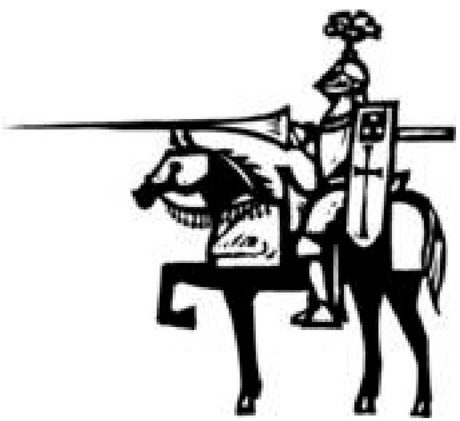
- Now, $\neg A \vee B$ is:
  - Same as $\mathrm{A} \rightarrow \mathrm{B}$
  - So $\neg A \vee B$ and $\mathrm{A} \rightarrow \mathrm{B}$ are **equivalent.**

A	B	(Not A) or B
True	*True*	*True*
True	*False*	*False*
False	*True*	*True*
False	*False*	*True*

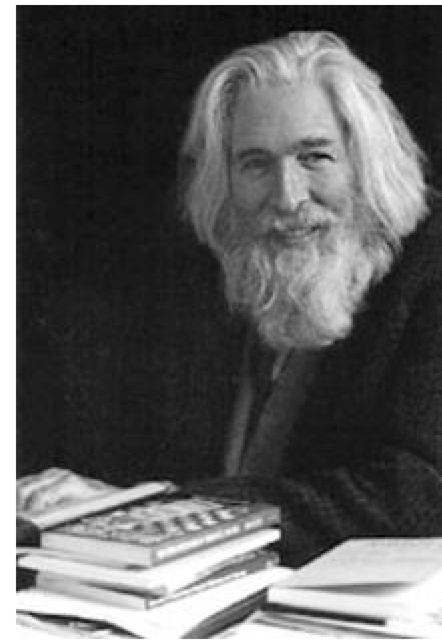
Knights and knaves



- On a mystical island, there are two kinds of people: knights and knaves.



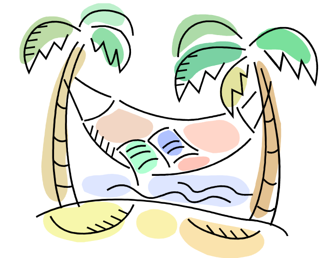
- Knights always tell the truth.



Raymond Smullyan

- Knaves always lie.

Knights and knaves



- On a mystical island, there are two kinds of people: knights and knaves. Knights always tell the truth. Knaves always lie.

- Puzzle 1: You meet two people on the island, Arnold and Bob. Arnold says “Either I am a knave, or Bob is a knight”.
  - Is Arnold a knight or a knave?
  - What about Bob?