Science 1000: Lecture #5 (Wareham):

How We Move:
Robot Motion Planning

Moving in
crowded space – is a
puzzlement

Dealing with Intractability (Take II)

- What the NP-completeness contract *really* means:

  if problem $\mathcal{A}$ is NP-complete then
  There is no poly-time algorithm for $\mathcal{A}$ that
  is deterministic and
  computes the best outputs
  for all inputs
  unless $P = NP$.

- This contract only holds for algorithms that satisfy **all** of the listed conditions $\Rightarrow$ **practical algorithms that break one or more of these conditions are still possible, *e.g.*, randomized, approximation**!
- Focus on what happens when we break the "poly-time" and "for all inputs" conditions.

Fixed-Parameter Tractability

- Let's relax our notion of tractability:
    1. Focus on a set P of one or more problem-aspects (**parameters**) whose values are small in practice.
    2. Only consider inputs with small values for P.
    3. Relax poly-time to fixed-parameter (fp-)time, *i.e.*, run-time $f(P)n^c$ for some function f.
- When the parameters in P are small, fp-time is effectively poly-time, *e.g.*, when $P = \{k\}$ and $k = 5$,

$$2^k n^2 \Rightarrow 2^5 n^2 \Rightarrow 32n^2 \Rightarrow O(n^2)$$

- Can prove fp-intractability with appropriate reductions and classes.

Fixed-Parameter Tractability (Cont'd)
The *Cole's Notes* Version

	good	bad
classical complexity	poly-time solvable (Best)	NP-Complete
parameterized complexity	fp-tractable (Still OK)	fp-intractable

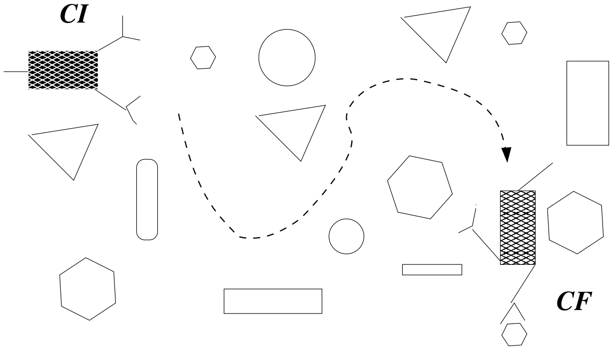
Complexity Analysis of Important Problems

> **The Tractable Computation Thesis**:
> WHERE POSSIBLE, IMPORTANT PROBLEMS
> SHOULD BE SOLVED QUICKLY.

- Two conceptions of "quickly":
  - quick in general (poly-time solvability)
  - quick under restrictions (fp-tractability relative to P)
- If a problem is intractable, look for restrictions to make it tractable.
- One way to do this is to look for parameters whose values are small in practice and then see if these restrictions yield fp-tractability.

Robot Motion Planning (Take I)

- Consider 3D motion planning in an obstacle-filled environment where we have to totally plan out a collisionless path from some initial robot-configuration c_I to a final robot-configuration c_F, *e.g.*,



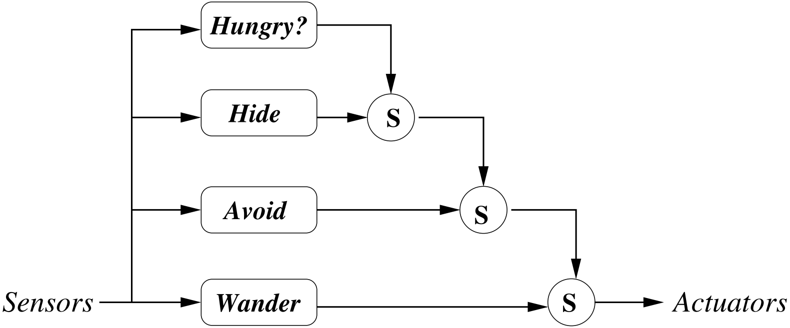
Robot Motion Planning (Take I) (Cont'd)

3D ROBOT MOTION PLANNING
Input: An environment E with obstacles, a robot R, and initial and final configurations c_I and c_F of R in E.
Output: A sequence of moves of R from c_I to c_F in E that does not collide with an obstacle, if such a sequence exists, and special symbol $\perp$ otherwise.

- Is $PSPACE$-complete in general; however, robots often have a small number k of joints (3 for robot arm, ≤ 20 for robot hand).
- Unfortunately, is fp-intractable for parameter-set $\{k, X\}$, where X is **lots** of other problem-aspects (Cesati and Wareham, 1995).

Robot Motion Planning (Take II)

- Let's step back to 2D motion planning and only require the robot to **react** from second to second based on what it sees to get from c_I to c_F.
- Reactive "cockroach" robot:



Robot Motion Planning (Take II) (Cont'd)

2D REACTIVE ROBOT ADAPTATION
Input: An environment E with obstacles, a reactive robot R, and initial and final configurations c_I and c_F of R in E.
Output: A modified reactive robot R' that can move from c_I to c_F in E and does not collide with an obstacle, if such a robot exists, and special symbol $\perp$ otherwise.

- Consider several types of allowable modification, *e.g.*, change linkages between layers, add / delete layers relative to a library, add / delete layers in general.

Robot Motion Planning (Take II) (Cont'd)

- Is NP-complete in general, even for simplest types of modifications; however, reactive robots often have a small number $l \leq 10$ of layers.
- Unfortunately, is fp-intractable for parameter-set $\{l, X\}$, where X is **lots** of other problem-aspects (Wareham et al, 2011).
- ... However, is fp-tractable when the number of sensor-recognizable distinctions in the world is small, *i.e.*, ignorance is (computational) bliss.
- Lots of work remains to be done ...

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