Supplementary material for :

Ignorance is Bliss: A Complexity Perspective on Adapting Reactive Architectures

by Todd Wareham, Johan Kwisthout, Pim Haselager, and Iris van Rooij

In this supplementary material, we provide proofs of the complexity-theoretic statements appearing in our text. We assume familiarity with basic concepts in both classical [2] and parameterized [1] computational complexity theory. All of our intractability proofs will involve reductions from the following well-known problem:

DOMINATING SET
Input: A graph $G = (V, A)$ and an integer k.
Question: Is there a dominating set in G of size at most k, *i.e.*, is there a subset $V' \subseteq V$, $|V'| \leq k$, such that for each $v \in V$, either $v \in V'$ or $\exists(v, v') \in E$ such that $v' \in V'$?

DOMINATING SET is known to be NP-complete in general [2] and $W[2]$-hard relative to parameter-set $\{k\}$ [1].

Theorem 1 *NA-REC is NP-hard*

Proof: Given an instance $I = \langle G, k\rangle$ of DOMINATING SET, construct the following instance $I' = \langle W, A, M, s, l\rangle$ of NA-REC: Let $E = \{U, D, L, A, B, \%, V_1, V_2, \ldots V_{|V|}, -, F\}$, with F being freespace and all other squares being obstacles. World W is a ring-shaped track of freespaces surrounded by obstacles on both the inner and outer sides of the track. This track can be divided into north, east, south, and west regions, and each region has inner and outer sides. The track is specified as follows:

- the east region consists of D-squares on both sides.
- the south region consists of L-squares on both sides.
- the west region consists of U-squares on both sides.
- The north region has an initial pair of A-squares on the left and a final pair of B-squares on the right. In between are $|V|$ blocks of length $|V|$ if $|V|$ is odd and $|V| + 1$ otherwise apiece separated by pairs of A-squares. Given an arbitrary order on the vertices in V, square i in the top side of each block corresponds to vertex i under the ordering; when $|V|$ is even, the vertices in V are split into two even-length sub-blocks with a middle spacer-square. Block i, $1 \leq i \leq |V|$, corresponds to a particular vertex $v_i \in V$, and encodes the vertices adjacent to v_i (including v_i itself) on the top side of the block (with vertex-positions not in the neighbourhood (or the middle spacer-square, if $|V|$ is even) marked with $-$-squares) and all $\%$-squares on the bottom side of the block.

A graph G and its associated world are shown in Figure 1. Note that regions are modified at corners of the track to ensure that an architecture can change direction at the corners under the

```
(a)

1   4
|\  |
| \ |
|  \|
2---3

(b)

UUUA12-3-A12-3-A12-34A--34BBBB
U............................B
U.UA%%%%%A%%%%%A%%%%%A%%%%BB.B
U.U                        D.D
L.LLLLLLLLLLLLLLLLLLLLLLLLLD.D
L............................D
LLLLLLLLLLLLLLLLLLLLLLLLLLDDDD
```

```
(c)

e(A)? -> E ------>
e(%)? -> N--++++
            ||||
e(1)? -> E -i---->
             |||
e(2)? -> E --i--->
              ||
e(3)? -> E -i-i-->
               |
e(4)? -> E ----i->
e(U)? -> N ------>
e(L)? -> W ------>
e(D)? -> S ------>
e(B)? -> E ------>
```

Figure 1: Illustration of reduction from DOMINATING SET to NA-REC given in Theorem 1. a) Sample graph G. b) World W constructed from G. c) Subsumption architecture A associated with G and W. Note that as the given graph G has 4 vertices, i.e., $|V| = 4$, by the construction specified in the reduction, the perceptual radius r of A is 2, i.e., $r = \lfloor \frac{|V|}{2} \rfloor = \lfloor \frac{4}{2} \rfloor = 2$. For clarity, freespace squares are denoted by a period instead of F in (b).

layer-ordering and visibility constraints below. Architecture A has perception-radius $r = \lfloor |V|/2 \rfloor$ and consists of $|V| + 6$ layers, such that the layers in descending order are:

- A layer that issues a E action if an A-square is detected;
- A layer that issues a N action if a $\%$-square is detected;
- $|V|$ layers, one for each vertex $v \in V$, that issue E actions if the square corresponding to v is detected;
- A layer that issues a N action if a U-square is detected;
- A layer that issues a W action if a L-square is detected;
- A layer that issues a S action if a D-square is detected; and
- A layer that issues a E action if a B-square is detected.

The subsumption links are output-inhibition links from the topmost layer to each of the vertex layers. Such an architecture for an example graph G and associated world W is shown in part (c) of Figure 1. Finally, let $M = \emptyset$, $s = k$, and $l = 0$. Observe that this construction can be done in time polynomial in the size of the given instance I of DOMINATING SET.

The following observations will be useful:

1. Both A and any A' created by subsumption-link modification can only move in a clockwise fashion in W.

2. A can move past any freespace in W except the middle freespace in each vertex block. This is so because in each such middle freespace, neither of the A or B-squares surrounding the vertex-block which would allow A to move forward can be sensed (as $r = \lfloor |V|/2 \rfloor$).

3. The only way any A' created from A by subsumption-link modification can progress past the middle freespace in a vertex block is to remove the inhibition links from the topmost layer to one or more of the vertex layers in A corresponding to a vertex-square that is present in that block (which could be sensed under the given r).

The above implies that to make any A' derived from A by subsumption-link modification fully navigable for W, those link modifications must enable A' to progress past the middle freespace of each vertex-block in W.

To prove that this construction is a reduction, we must show that the answer to the given instance of DOMINATING SET is "Yes" if and only if the constructed instance of NA-REC has an associated A' that is fully navigable for W. Let us consider the two implications separately. If the answer to the given instance of DOMINATING SET is "Yes", then there is a dominating set V' such that $V' \leq k$. Construct A' from A by deleting the subsumption links from the topmost layer to all $|V'| \leq k = s$ vertex layers corresponding to vertices in V'. These layers will now be active whenever they detect squares corresponding to vertices in V' in the vertex blocks in W, which, by the observations above, will allow A' to navigate between any two freespaces in W. Conversely, if there is an A' that is fully navigable for W, A' was constructed from A by removing at most s subsumption links from A; let V' be the set of vertices in G corresponding to the now-active vertex layers in A'. By the construction of W, each vertex $v \in V$ is either in V' or is adjacent to a vertex in V', implying that V' is a dominating set of size at most $s = k$ in G. ∎

Theorem 2 *NA-DES is NP-hard*

Proof (sketch): Follows by a slight variant of the reduction in Theorem 1 which deletes M and sets $f = 1$. The proof of correctness of this reduction is identical to that for Theorem 1. ∎

Result 1 *NA-REC and NA-DES are NP-hard.*

Proof: Follows from Theorems 1 and 2. ∎

Corollary 1 *NA-REC is fp-intractable for parameter-set* $\{s, f, l, |M|\}$.

Proof: Follows from the $W[2]$-hardness of DOMINATING SET for parameter-set $\{k\}$ and the reduction in Theorem 1, in which $s = k$, $f = 1$, and $|M| = l = 0$. ∎

Theorem 3 *NA-REC is fp-intractable for parameter-set* $\{s, f, l, |L|\}$.

Proof (sketch): Modify the reduction in Theorem 1 to let W be such that the bottom-side squares in the vertex blocks are now the same as the top-side squares, *i.e.*, eliminate the %-squares, A consist of the lowest five layers and the topmost layer of the original A, M consist of the vertex-layers from the original A, $s = 0$, and $l = k$. The result then follows from the $W[2]$-hardness of DOMINATING SET for parameter-set $\{k\}$ and the reduction above from DOMINATING SET to NA-REC in which $s = 0$, $f = 1$, $l = k$, and $|L| \leq k + 6$. ∎

Result 2 *NA-REC is fp-intractable for parameter-sets* $\{s, f, l, |M|\}$ *and* $\{s, f, l, |L|\}$.

Proof: Follows from Corollary 1 and Theorem 3. ∎

Lemma 1 *Given an architecture* A*, a world* W*, and initial and final positions* s *and* d *with* W*, whether or not* A *can navigate from* s *to* d *can be computed in* $O(|W||A|)$ *time.*

Proof: The action computed by A at a position p in W can be determined in $O(|A|)$ time. Observe that the behavior of A at p is fixed, in that regardless of whether p has been encountered once or more by A, the same action is generated. Hence, when started at s, if A doubles back on any previously-entered square, A can never encounter d. As A can travel to at most $|W| - 2$ different squares before entering d. it can be determined in $|W|$ moves whether A can reach d from s. ∎

Theorem 4 *NA-REC is fp-tractable for parameter-set* $\{|L|, |M|\}$.

Proof (sketch): The number of subsumption link-configurations of A is a function of $|L|$, *i.e.*, $3^{|L|(|L|-1)/2}$. As $s \leq |L|$ and $l \leq |M|$, the number of possible A' that can be generated from A is a function of $|L|$ and $|M|$. To complete the proof, observe that each such configuration A' of A can be evaluated wrt W in polynomial time (as there are $|W|(|W| - 1)/2$ source-destination pairs in W and the reachability for each pair can be determined in $O(|A||W|)$ time by Lemma 1) and checked to see if A and A' differ by at most s subsumption-links in $O(max(|A|, |A'|))$ time. ∎

Theorem 5 *NA-REC is fp-tractable for parameter-set* $\{|E|\}$.

Proof (sketch): Both $|L|$ and $|M|$ are bounded by the number of possible Boolean functions over $|E|$ times the number of possible actions a layer may generate, *i.e.*, $|L| \leq 2^{2^{|E|}} \times 4$. Substituting this $|E|$-expression for each occurrence of $|L|$ and $|M|$ in the runtime of the algorithm described in Theorem 4 gives an algorithm for NA-REC that is fp-tractable for $\{|E|\}$. ∎

Result 3 *NA-REC is fp-tractable for parameter-set* $\{|L|, |M|\}$ *and and* $\{|E|\}$.

Proof: Follows from Theorems 4 and 5. ∎

Result 4 *NA-DES is fp-intractable for parameter-set* $\{s, f, l, |L|\}$.

Proof (sketch): Modify the reduction in Theorem 3 to eliminate M and explicitly set $f = 1$. The result then follows from the $W[2]$-hardness of DOMINATING SET for parameter-set $\{k\}$ and the reduction above from DOMINATING SET to NA-DES in which $s = 0$, $f = 1$, $l = 0$, and $|L| \leq k + 6$. ∎

Result 5 *NA-DES is fp-tractable for parameter-set* $\{|E|, f\}$.

Proof (sketch): Consider the following algorithm: By comparison against all possible values of the $exists$-predicates for E, possible layers over E that are not already part of A can be isolated; call this set of layers P. This set P can be further reduced to those layers that have trigger-condition formulas of length at most f over E; call this set P'. Evaluate all A' created by adding at most l layers from P' to and modifying at most s subsumption-links of A to see if any are fully navigable for W and output accordingly. To complete the proof, note that the non-polynomial quantities in the runtime of this algorithm (*i.e.*, the number of possible layers over E ($2^{2^{|E|}} \times 4$), the number of possible values of the $exists$-predicates for E ($2^{|E|}$), the number of trigger-condition formulas of length at most f ($\leq f \times |E|^f$)) are all functions of $|E|$ and f. ∎

References

[1] Downey, R.G. and Fellows, M.. (1999) *Parameterized Complexity*. Springer; Berlin.

[2] Garey, M.R. and Johnson, D.S. (1979) *Computers and Intractability: A Guide to the Theory of NP-Completeness.* W.H. Freeman, San Francisco.