Computer Science 6789 (Winter 2014): Assignment #3
Due: 10:30 AM on Thursday, April 3, 2014

For each of the following parameterized problems, either prove that this problem is fixed-parameter tractable (by giving an fp-tractable algorithm) or prove that this problem is fp-intractable (by showing that it is either hard for or cannot be included in some class in the W-hierarchy above FPT). In the latter case, include reductions where appropriate and state any problem-class-relationships, *e.g.*, $FPT \neq W[1]$, that are required to prove fp-intractability.

1. **(15 marks)**

   $\{k\}$-Skolem Packing
   Input: An undirected graph $G = (V, E)$ and a positive integer k.
   Question: Does G have a Skolem packing of size $\leq k$, *i.e.*, Is there a subset $E' \subseteq E$, $|E'| \leq k$, such that for each $e \in E$, either $e \in E'$ or e shares an endpoint with at least one $e' \in E'$?

2. **(15 marks)**

   $\{d\}$-l-Clique Configuration
   Input: An undirected graph $G = (V, E)$ and positive integers k, l, and d.
   Question: Does G contain a set C of at most k l-cliques such that each clique in C is connected by edges to at most d other cliques in C and each l-Clique in G is either in C or connected by an edge to a clique in C?

3. **(15 marks)**

   $\{k\}$-Erdös Subgraph
   Input: An undirected graph $G = (V, E)$, a set S of subgraphs of G, and positive integers k and l.
   Question: Is there a subgraph G' of G containing at most k vertex- and edge-disjoint paths of length l such that each subgraph in S contains at least one of the length-l paths in G'?

4. **(15 marks)** Define a **finite-state set transducer (FSST)** $A = \langle Q, I, s, F, \delta \rangle$ as a finite-state automaton with state-set Q, item-set I, start-state $s \in Q$, accepting states $F \subseteq Q$, and state-transition relation $\delta \subseteq Q \times 2^I \times Q \times 2^I$ where 2^I is the set of all subsets of I. Informally, the computation of a FSST starts in state s with a given initial set $I_0 \subseteq I$, and at each step with the FSST in state q and associated item-set $I' \subseteq I$, executes a state-transition (q, X, q', Y) if $X \subseteq I'$ by re-setting the state to q' and the associated item-set to $(I' - X) \cup Y$. An FSST A **accepts a subset** $I' \subseteq I$ if there is some sequence of transitions which, starting from s and $I_0 = I'$, result in some $f \in F$ and some $I'' \subseteq I$.

   $\{k\}$-FSST Computation
   Input: An FSST $A = \langle Q, I, s, F, \delta \rangle$, set $I' \subseteq I$, and a positive integer k.
   Question: Can A accept I' by executing a sequence of at most k transitions?

5. **(15 marks)**

$\{k, d\}$-Thue Convolution
Input: An undirected graph $G = (V, E)$ of maximum vertex-degree d and a positive integer k.
Question: Is there a subset $E' \subseteq E$, $|E'| \leq k$, such that for each $v \in V$, there is at least one edge in E' with v as an endpoint?

Bonus Question (15 marks):

What is the parameterized complexity of $\{k, |I|\}$-FSST Computation? Prove your answer.